

Relating Circumference and Area of a Circle

CCSS: 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.



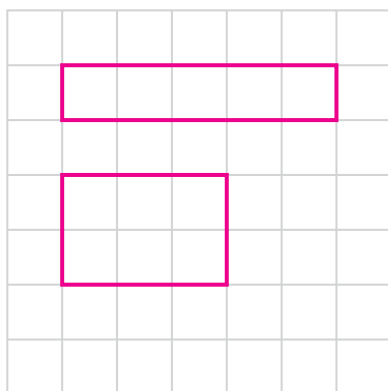
Launch

SAMPLE SOLUTIONS ARE SHOWN BELOW.

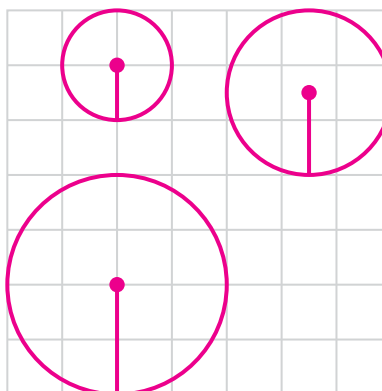
MP3, MP5

Does a greater distance around a shape always mean a greater area? Show one pair of rectangles and one pair of circles where greater distance around does not mean a greater area. If you can't show an example, explain why.

Rectangles



Circles



The perimeter of the top rectangle (12 units) is greater than the perimeter of the bottom rectangle (10 units). But the area of the top rectangle (5 square units) is less than the area of the bottom rectangle (6 square units).

I could not find an example where a circle with a smaller circumference has a greater area. The area always increases as circumference increases.

Reflect Besides size, can you change a circle's basic shape? Can you change a rectangle's basic shape? How might this relate to the problem above?

Sample: You cannot change the shape of a circle. But, you can change the length and width of a rectangle. So, you can make a rectangle long and narrow to cover less area but have a greater perimeter.

Got It?

PART 1 Got It



A circle has a circumference of 22 cm. What is the approximate area of the circle? Use 3.14 for π .

38.5 cm²

PART 1 Got It (1 of 2)



The ratio of the area of a circle to the circumference of a circle $\left(\frac{A}{C}\right)$ is $\frac{11}{1}$. What is the circumference of the circle? Leave your answer in terms of π .

44π units

Got It?

PART 2 Got It (2 of 2)



Use formulas to show why $\frac{A}{C} = \frac{1}{2}r$.

$$\frac{A}{C} = \frac{\pi r^2}{2\pi r} \quad A = \pi r^2 \text{ and } C = 2\pi r.$$

$$= \frac{\pi \cdot r \cdot r}{2 \cdot \pi \cdot r} \quad r^2 = r \cdot r$$

$$= \frac{\cancel{\pi} \cdot \cancel{r} \cdot r}{2 \cdot \cancel{\pi} \cdot \cancel{r}} \quad \text{Divide out the common factors.}$$

$$= \frac{r}{2}$$

$$= \frac{1}{2}r$$

Close and Check

Focus Question

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How are the area of a circle and the circumference of a circle related?

Sample: The area of a circle and the circumference of a circle are both measurements that depend on the radius of the circle. Circumference is the length around a circle ($2\pi r$) while area is a measure of square units inside the circle (πr^2).

Do you know HOW?

1. A circle has a circumference of 38 yd. Find the approximate area of the circle. Use 3.14 for π .

115 yd²

2. The ratio of the area of a circle to the circumference of the circle ($\frac{A}{C}$) is $\frac{7}{1}$. Find the circumference of the circle. Leave your answer in terms of π .

28 π units

3. The ratio of the area of a circle to the circumference of the circle ($\frac{A}{C}$) is $\frac{5}{1}$. Find the area of the circle. Leave your answer in terms of π .

100 π units²

4. The ratio of the area of a circle to the circumference of the circle ($\frac{A}{C}$) is $\frac{6}{1}$. Find the circumference and area of the circle. Leave your answers in terms of π .

C = 24 π units

A = 144 π units²

SAMPLE SOLUTIONS ARE SHOWN BELOW.

Do you UNDERSTAND?

5. **Reasoning** The ratio ($\frac{A}{C}$) of a circle is $\frac{3}{1}$. Explain how to use this information to find the radius and circumference of the circle.

The ratio ($\frac{A}{C}$) is equal to $\frac{1}{2}r$, so $\frac{3}{1} = \frac{1}{2}r$. Multiply both sides by 2. Substitute $r = 6$ into the formula to find the circumference.

6. **Error Analysis** The ratio ($\frac{A}{C}$) of a bike wheel is $\frac{4}{1}$. Your friend says $C = 8\pi$ and $A = 16\pi$. Explain and correct the error your friend made.

My friend used the ratio ($\frac{A}{C}$) = r . The radius r is equal to twice the ratio ($\frac{A}{C}$), so $C = (2)(8)\pi$ or 16π , and $A = \pi 8^2$ or 64π .