Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Link to ISK Educational Aims:

CO Communicate | L Learn | CR Create | S Solve | A Act | CH Character

1 Make sense of problems and persevere in solving them.

CH S L

Mathematically proficient students:

- Understand meaning of a problem and look for entry points to its solution.
- Analyze givens, constraints, relationships
- Plan a solution strategy.
- Consider simpler forms of the original problem in order to gain insight.
- Monitor and evaluate their progress and change course if necessary.
- Older students might transform algebraic expressions to get the information they need.
- Can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.
- Check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"
- Can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

S L CO

Mathematically proficient students:

- Make sense of quantities and their relationships in problem situations.
- Bring two complementary abilities to bear on problems involving quantitative relationships:
  - the ability to *decontextualize—to* abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents
  - the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

- Create a clear representation of the problem at hand including the units involved, the meaning of he problem, not just computation and using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

CO CH S

Mathematically proficient students:

- Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- Make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- Analyze situations by breaking them into cases
- Justify their conclusions, communicate them to others, and respond to the arguments of others.
- Reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- Can construct arguments using concrete referents such as objects, drawings, diagrams, and actions, which can make sense and be correct.
- Can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

S L CO CR

Mathematically proficient students:

- Can apply the mathematics they know to solve problems arising in everyday life (including the workplace).
    - (ES) This might be as simple as writing an addition equation to describe a situation.
    - (MS) Apply proportional reasoning to plan a school event or analyze a problem in the community
    - (HS) Use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- Are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- Are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- Can analyze relationships mathematically to draw conclusions.
- Can interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

S L

Mathematically proficient students:

- Consider the available tools when solving a mathematical problem. (*These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.)*
- Are sufficiently familiar with tools appropriate for their grade to make sound decisions about when each of these tools might be helpful.
- Detect possible errors by strategically using estimation and other mathematical knowledge.
- Are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.
- Are able to use technological tools to explore and deepen their understanding of concepts.
- Know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data.

6 Attend to precision.

L CO

Mathematically proficient students:

- Try to communicate precisely to others
- Use clear definitions in discussion with others and in their own reasoning
- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately
- Are careful about specifying units of measure, and labeling axes
- Calculate accurately and efficiently
- Express numerical answers with a degree of precision appropriate for the problem context
- (ES/MS) Give carefully formulated explanations to each other
- (MS/HS) examine claims and make explicit use of definitions.

7 Look for and make use of structure.

CO CH S

Mathematically proficient students:

- Discern a pattern or structure
  - Example: Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x2* + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7.
- Recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems
- Can step back for an overview and shift perspective
- Can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects
  - Example: students can see 5 – 3(x – *y)2* as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

8 Look for and express regularity in repeated reasoning.

S L

Mathematically proficient students:

- Notice if calculations are repeated, and look both for general methods and for shortcuts
  - (UES): Students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal
  - (MS/HS) By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding $(x–1)(x+1)$,$(x–1)(x^2+x+1)$,and$(x–1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series.
- Notice the regularity, which might lead them to the general formula for the sum of a geometric series
- Maintain oversight of the process, while attending to the details
- Continually evaluate the reasonableness of their intermediate results

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