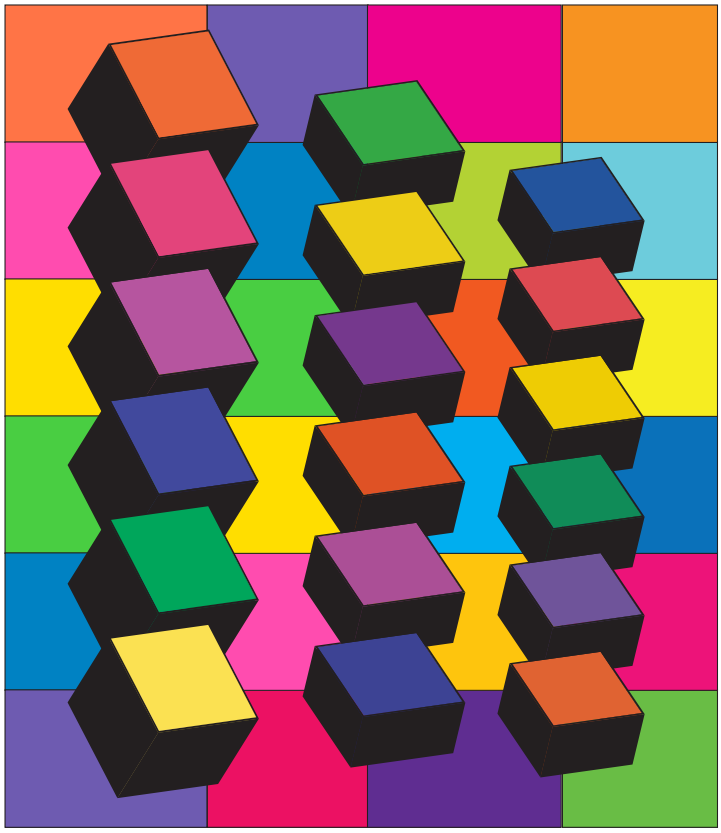
Student Explorations in Mathematics

The Algebra of Patterns



Mathematicians often use functions to describe patterns. In this activity, you will build figures that follow a pattern and describe how the figures change. By the end of the unit, you will have a clearer understanding of what a function is and how to use one to describe a pattern.

In a sequence, the first figure will be referred to as the stage 1 figure, the second will be stage 2, and so on. So, being asked for the stage 15 figure means you would create the fifteenth figure in the pattern. The variable *n* represents the stage number.

1. A student used tiles to build the sequence below. From one stage to the next, what changes? Following the pattern, use tiles to build stage 4. How many tiles did you use?

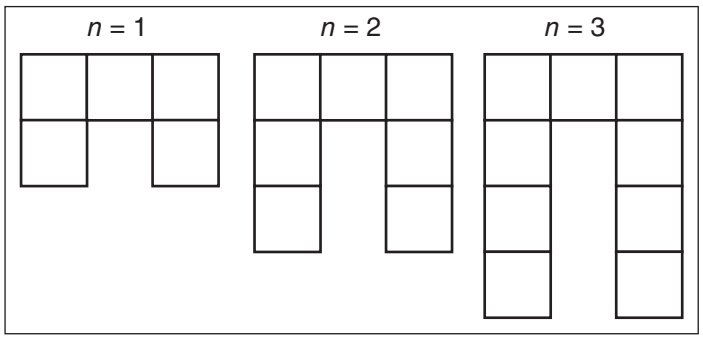


Figure 1

2. Use your tiles to build the stage 9 figure of this growing pattern. How many tiles did you use? Explain how you know this is the correct number of tiles.

3. How many tiles would be in the stage 100 figure? Use words to describe how you know.

4. How many tiles would be in the stage *n* figure? Write your description in words. This is called a *verbal description of the sequence*.

5. Choose one of the verbal descriptions generated by your class. Use that model to write an algebraic expression for the number of tiles in the stage *n* figure.

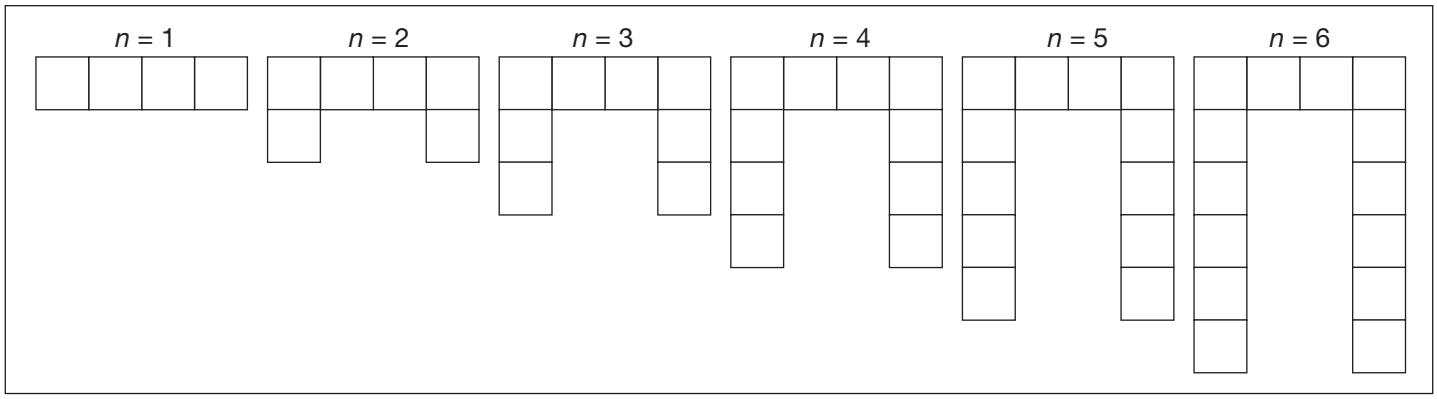


Figure 2

6. Match each expression displayed in the classroom to the correct verbal description.

Figure 2 above shows the first six stages of a growing sequence.

7. Describe the number of tiles in the stage *n* figure with both a verbal description and an algebraic expression.

8. How is this set of figures similar to the first set you worked with? How is it different?

Figure 3 below shows the first five stages of a growing sequence.

9. a. Describe the number of tiles in the stage *n* figure with a verbal description and with an algebraic expression.

   b. Compare and contrast this sequence with the first sequence of this activity.

10. Create a sequence of diagrams that can be described by the expression $4n + 3$. Which stage has 47 tiles?

Mathematicians use functions to describe how patterns change. Using physical models, verbal descriptions, and mathematical symbols, you have described how the patterns in this activity change. Two other ways to display the information are in tables and graphs. Functions can be represented by equations, tables, words, or graphs.

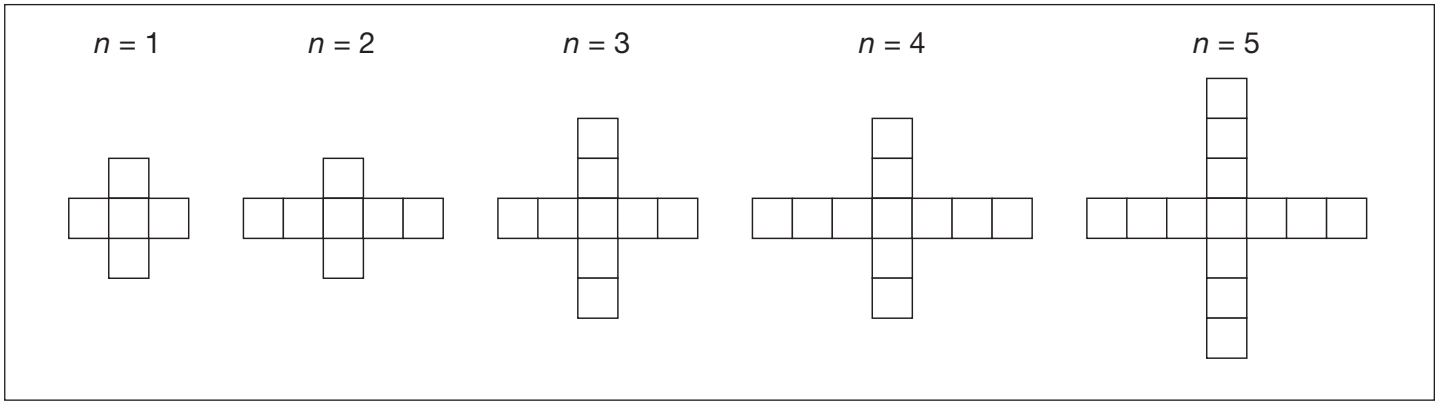


Figure 3

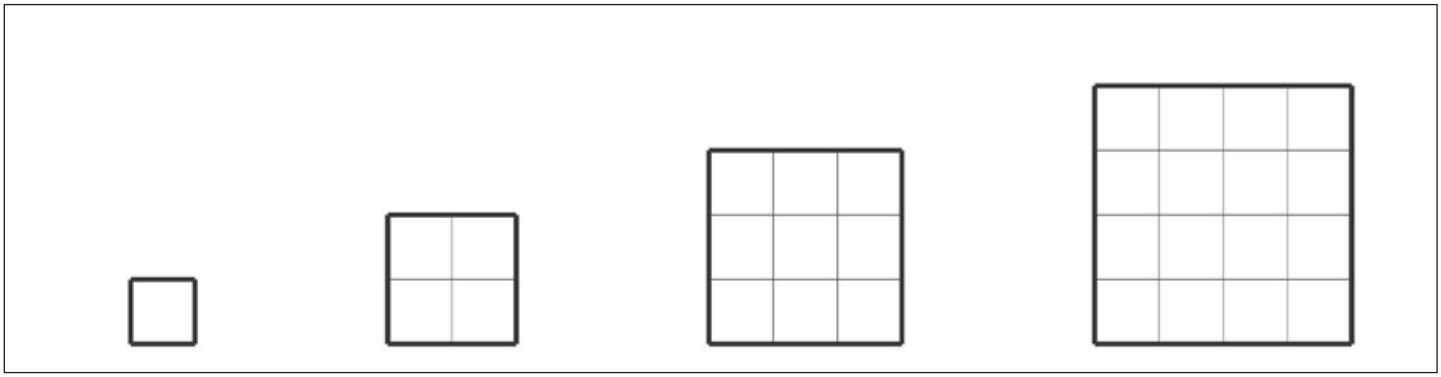


Figure 4

11. Consider the sequence consisting of the squares of the counting numbers (see **fig. 4** above). Complete **table 1** below, where the input, *n*, is the stage number and the output is the number of tiles in the *n*th drawing.

The *domain* of a function is a list of all input values (the first column of **table 1**). The *range* is a list of all of the output values.

12. What kind of numbers make up the domain of the function in question 11? What about the range?

13. Plot the points from **table 1** onto a set of coordinate axes (see **fig. 5**). Use the domain for the horizontal axis and the range for the vertical axis. Should the points be connected?

Table 1

n	tiles
1	1
2	4
3	
4	
5	
6	
7	

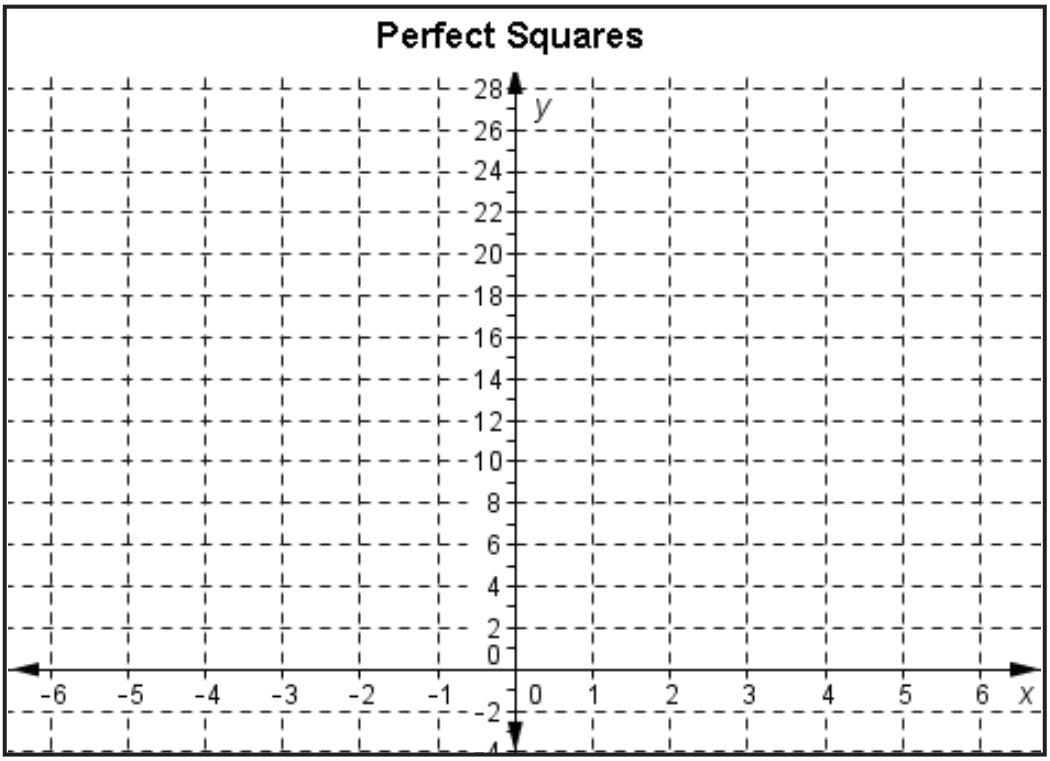
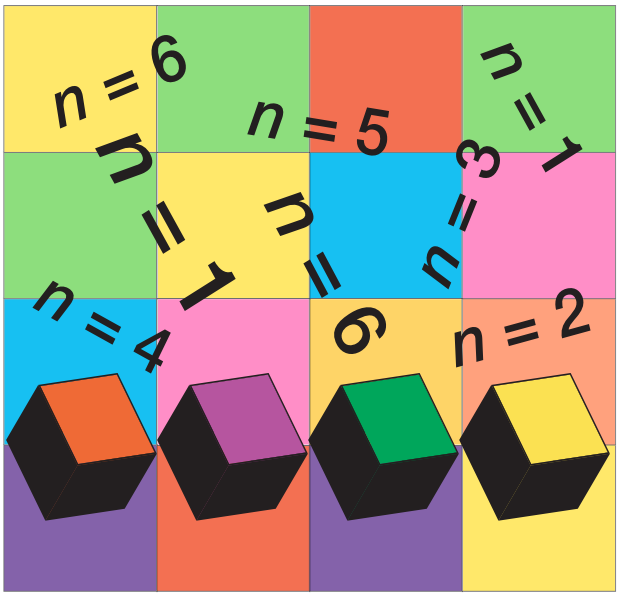


Figure 5

14. We will assume the given function is continuous, so we do connect the data points. Extend the graph to include negative inputs. What are the domain and range of the new function you have drawn?



Can you . . .

- create a sequence of diagrams that can be modeled by the expression $5n - 2$?
- create a sequence of diagrams that can be modeled by the expression $n^2 + 4$?
- create a sequence of diagrams that can be modeled by the expression $(n + 2)^2$?
- create a sequence of diagrams that can be modeled by the expression $\sqrt{n} + 3$?

Did you know . . .

- that three common types of functions are linear, quadratic, and exponential? Find out how each of these functions is defined.

Sources

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Murdock, Jerald, Ellen Kamischke, and Eric Kamischke. 2001. *Discovering Advanced Algebra: An Investigative Approach*, vol.1. Peabody, MA: Key Curriculum Press. http://books.google.com/books/about/Discovering_Advanced_Algebra.html?id=_JGqGgAACAAJ

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