Week of: April 6-April 10

Grade: 7 **Content:** Math

Learning Objective:

Greetings 7th graders! We hope you are safe and well with your families!

This is the second week of Carnegie Math work on the BPS website. If you'd like to go back and try the other work (introduction to adding integers), it would be a great challenge.

This week we are going to continue adding integers using different methods.

We've included a video link to review adding and subtracting integers using a number line and adding without using a number line in case you get stuck. This work will not be graded, just do your best and have fun!

Video Links:

Khan Academy: https://www.khanacademy.org/math/arithmetic-home/negative-numbers/add-sub-neg-number-line/v/adding-negative-numbers-on-number-line-examples

Khan Academy: https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-add-negatives-intro/v/adding-negative-numbers

Practice Activities:

On-Line

All students now have access to an on-line program called Mathia!

Mathia- If you are already in Mathia, please continue to work in the program.

If you are new to Mathia: Please see the log-in information attached.

Printable Resources:

Skills Practice: see the attached practice pages

Module 2: Operating with Signed Numbers

TOPIC 1: ADDING AND SUBTRACTING RATIONAL NUMBERS

In this topic, students use number lines and two-color counters to model addition and subtraction of integers before developing rules for determining the sum and difference of signed numbers. Students are expected to make connections among the representations used. After they understand what it means to add and subtract integers, students apply the rules to the set of rational numbers.

Where have we been?

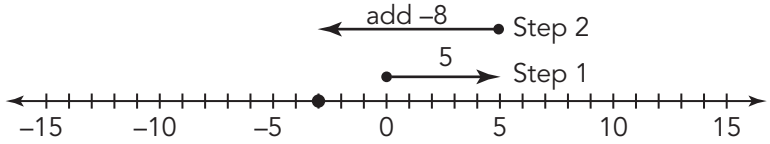
In grade 6, students learned how to represent positive and negative rational numbers on a number line. They also know that $-p$ is p units from 0 on the number line and that $|p| = |-p| = p$. Students used number lines to model the distance from 0 and to model the distance between two rational numbers represented on vertical or horizontal number lines.

Where are we going?

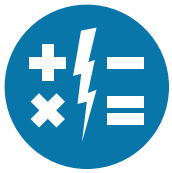
Students will develop a strong conceptual foundation for adding and subtracting with rational numbers to provide the foundation for manipulating and representing increasingly complex numeric and algebraic expressions in later lessons and future courses and grades.

Using a Number Line to Model Adding and Subtracting Integers

A number line can be used to model adding and subtracting negative numbers. This number line models the sum $5 + (-8)$.



Myth: Students only use 10% of their brains.



Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of the movies *Lucy* (2014) and *Limitless* (2011). Both films ask the audience: Imagine what you could accomplish if you could use 100% of your brain!

Well, this isn't Hollywood, and you're stuck with an ordinary brain. The good news is that you **do** use 100% of your brain. As you look around the room, your visual cortex is busy assembling images; your motor cortex is busy moving your neck; and all of the associative areas recognize the objects that you see. Meanwhile, the corpus callosum, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts...like mathematics!

#mathmythbusted

Talking Points

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to reason using signed numbers.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Terms

absolute value

The absolute value of a number is its distance from 0 on a number line.

additive inverse

The additive inverse of a number is the opposite of the number: $-x$ is the additive inverse of x. Two numbers with the sum of zero are called additive inverses.

zero pair

A zero pair is a pair of numbers whose sum is zero. The value of negative 1 plus positive 1 is zero. So, negative 1 and positive 1 together are a zero pair.

2

Walk the Line

Adding Integers, Part I

WARM UP

A large hotel has a ground floor (street level) and 26 floors of guest rooms above street level, which can be modeled by positive integers. There are 5 floors of parking below street level, which can be modeled by negative integers. In this hotel, street level is represented by zero.

Write an integer addition problem that models the hotel elevator's motion in each case.

1. The elevator starts at street level, goes up 7 floors, and then goes down 3 floors.
2. The elevator starts at street level, goes up 10 floors, and then goes down 12 floors.
3. The elevator starts at street level, goes down 4 floors, and then goes up 11 floors.
4. The elevator starts at street level, goes down 2 floors, goes up 5 floors, and finally goes down 3 floors.

LEARNING GOALS

- Model the addition of integers on a number line.
- Develop a rule for adding integers.
- Identify $p + q$ as the number located a distance of $|q|$ from p.

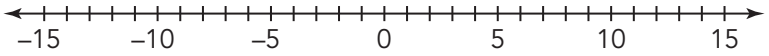
KEY TERM

- absolute value

You have been adding and subtracting positive numbers most of your life. In elementary school, you learned how to add numbers using a number line. How can a number line be a helpful tool in adding positive and negative numbers?

Getting on Line

Use the number line and determine the number described by each. Explain your reasoning.



1. the number that is 7 more than −9

2. the number that is 2 more than −6

3. the number that is 10 more than −8

4. the number that is 10 less than 6

5. the number that is 5 less than −4

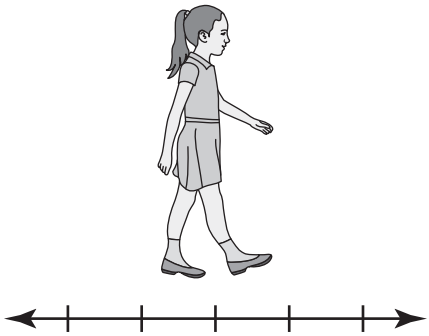
6. the number that is 2 less than −4

ACTIVITY
2.1 Walking the Number Line

Walking a number line can help you to add positive and negative numbers.

Walk the number line for an addition sentence:

- Start at zero and walk to the value of the first term of the expression.
- To indicate addition, turn to face up the number line, towards the greater positive numbers.
- Walk forward if adding a positive number or walk backward if adding a negative number.



Your teacher will select a classmate to walk the line for each of the given problems. Help your classmate by preparing the directions that are needed.

1. Complete the table.

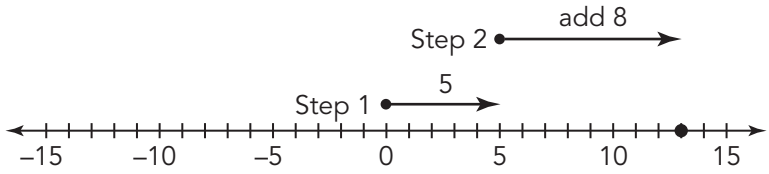
	Where You Start	Direction You Face	Walk Backwards or Forwards	Final Location
1 + 3				
0 + (−4)				
−3 + 5				
−1 + (−4)				

This worked example represents the movement created by walking the number line.

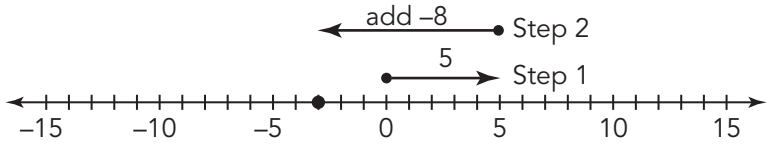
WORKED EXAMPLE

A number line can be used to model integer addition.
When adding a positive integer, move to the right on a number line.
When adding a negative integer, move to the left on a number line.

Example 1: The number line shows how to determine 5 + 8.



Example 2: The number line shows how to determine 5 + (−8).



Compare the first steps in each example.

2. What distance is shown by the first term in each example?

Remember that the **absolute value** of a number is its distance from 0.

3. Describe the graphical representation of the first term. Where does it start and in which direction does it move? How does this movement represent walking the line?

4. What is the absolute value of the first term in each example?

Compare the second steps in each example.

5. What distance is shown by the second term in each example?

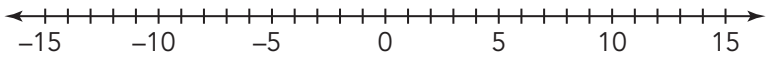
6. Why did the arrows for the second terms both start at the endpoints of the first terms but then continue in opposite directions? Explain your reasoning.

7. What is the absolute value of the second term in each example?

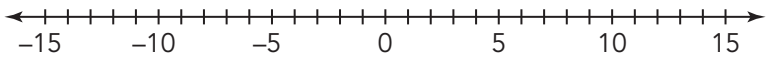
8. Use the number line to determine each sum. Show your work.

Think about walking the line as you model these sums on the number line.

a. −3 + 7 = _____

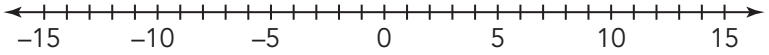


b. 3 + (−7) = _____

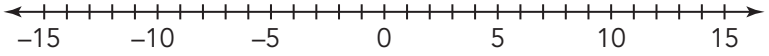


NOTES

c. $-3 + (-7) =$ _____



d. $3 + 7 =$ _____



Notice that the first term in each expression in parts (a) through (d) was either 3 or (−3).

9. What do you notice about the distances shown by these terms on the number lines?

10. What is the absolute value of each term?

Notice that the second term in each expression was either 7 or (−7).

11. What do you notice about the distances shown by these terms on the number lines?

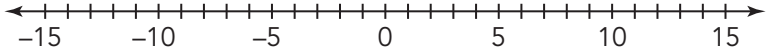
12. What is the absolute value of each term?

ACTIVITY
2.2 Adding on Number Lines

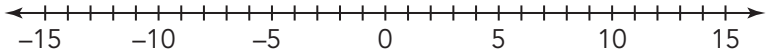
Now that you have the feel for how to move on the number line when adding negative numbers, it is time to practice with more examples.

Use the number line to determine each sum. Show your work.

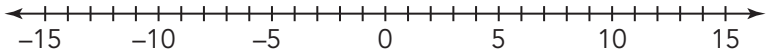
1. −9 + 5 = _____



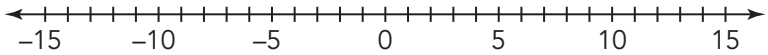
2. 9 + (−5) = _____



3. −9 + (−5) = _____



4. 9 + 5 = _____



Notice that the first term in each expression in Questions 1 through 4 was either 9 or (−9).

5. What do you notice about the distances shown by these terms on the number lines?

6. What is the absolute value of each term?

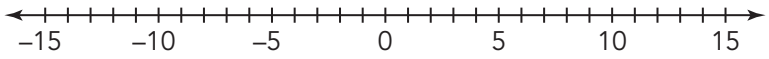
Notice that the second term in each expression was either 5 or (−5).

7. What do you notice about the distances shown by these terms on the number lines?

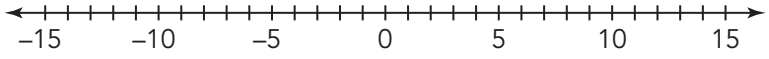
8. What is the absolute value of each term?

Use the number line to determine each sum. Show your work.

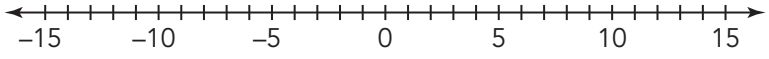
9. $-8 + 2 =$ _____



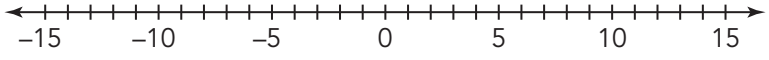
10. $8 + (-2) =$ _____



11. $-8 + (-2) =$ _____



12. $8 + 2 =$ _____



3

Two-Color Counters

Adding Integers, Part II

WARM UP

Use a number line to determine each sum. Then write a sentence to describe the movement you used on the number line to compute the sum of the two integers.

1. $-2 + 1$

2. $-5 + 8$

3. $-2 + (-3)$

4. $4 + (-6)$

LEARNING GOALS

- Describe situations in which opposite quantities combine to make 0.
- Model the addition of integers using two-color counters.
- Develop a rule for adding integers.
- Apply previous understandings of addition and subtraction to add rational numbers.

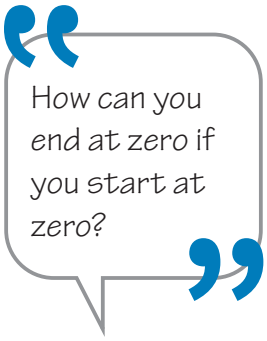
KEY TERM

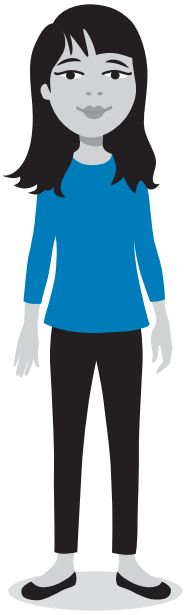
- additive inverses

You know how to use a number line to model adding positive and negative numbers. Do the patterns you noticed from the number line model apply to other models for adding positive and negative numbers?

Creating Zero

Use a number line to illustrate how the sum of two numbers can be zero.





1. **Write 3 examples of number sentences that sum to zero and draw the number line models to support your solutions.**

2. **What pattern do you notice?**

3. **Describe a real-life situation in which two numbers would sum to zero. Write the number sentence that could be used to represent the situation.**

ACTIVITY
3.1 Additive Inverses

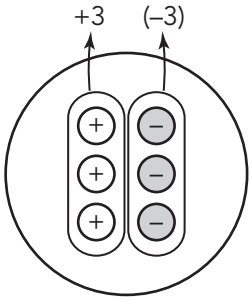
Addition of integers can also be modeled using two-color counters that represent positive (+) charges and negative (–) charges. One color, usually red, represents the negative number, or negative charge. The other color, usually yellow, represents the positive number, or positive charge. In this book, gray shading will represent the negative number, and no shading will represent the positive number.

Two numbers with the sum of zero are called **additive inverses.**

⊖ = –1 ⊕ = +1

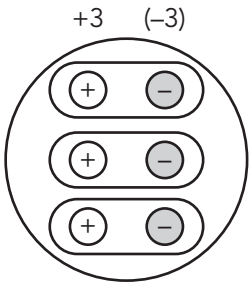
WORKED EXAMPLE

You can model the expression 3 + (–3) in different ways using two-color counters:



Three positive charges and three negative charges have no charge.

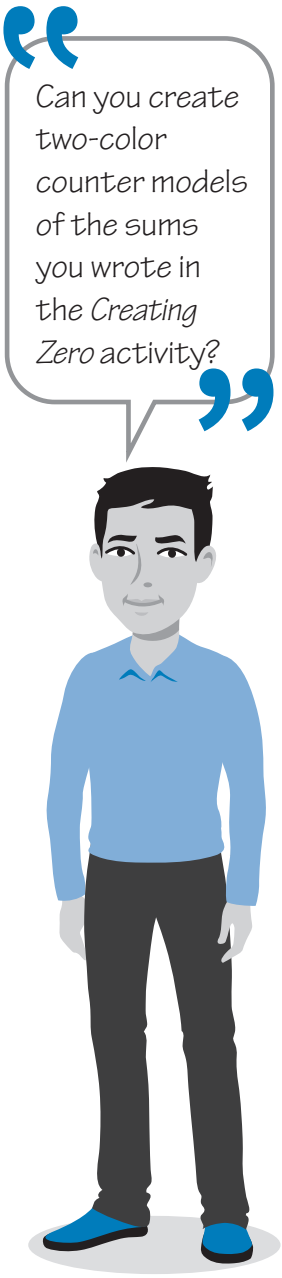
3 + (–3) = 0



Each positive charge is paired with a negative charge.

Each pair of positive and negative charges has no charge.

3 + (–3) = 0



NOTES

1. What is the value of each ⊕ and ⊖ pair in the second model?

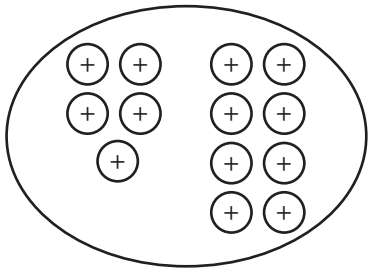
2. Describe how you can change the numbers of ⊕ and ⊖ counters in the model but leave the sum unchanged.

ACTIVITY
3.2 Adding Integers with Two-Color Counters

Let's consider two examples where integers are added using two-color counters.

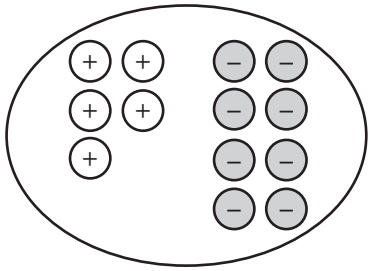
WORKED EXAMPLE

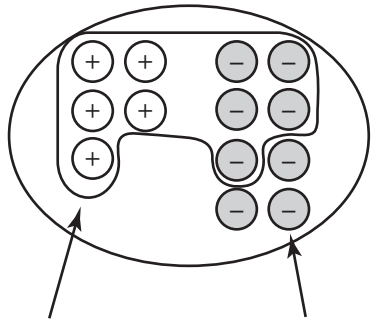
Example 1: 5 + 8



There are 13 positive counters in the model. The sum is 13.

Example 2: 5 + (−8)





There are five (+) (−) pairs. The value of those pairs is 0.

There are 3 (−), or negative counters, remaining.

There are 3 negative counters remaining. The sum of 5 + (−8) is −3.

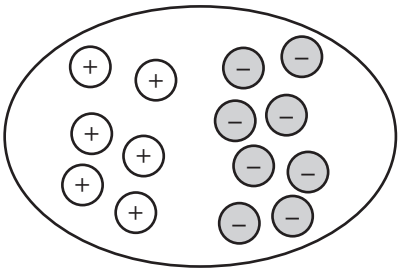
1. Create another model to represent a sum of −3. Write the appropriate number sentence.

2. Share your model with your classmates. How are they the same? How are they different?

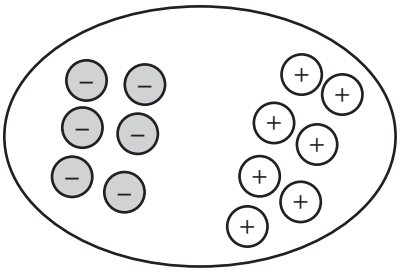
In an addition sentence, the terms being added together are called addends.

3. Write a number sentence to represent each model.

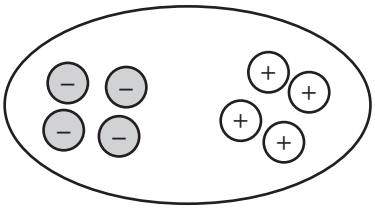
a.



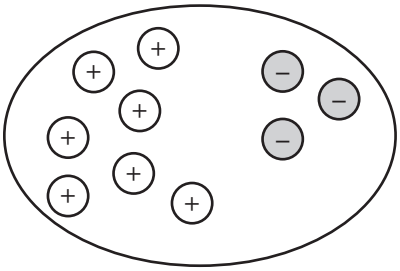
b.



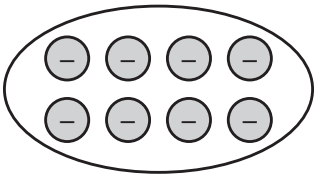
c.



d.



4. Does the order in which you wrote the integers in your number sentence matter? How do you know?



The students were then asked to write a number sentence for the given model.

Ava

$-8 + 0 = -8$

Landon

$-1 + (-7) = -8$

5. **Analyze the number sentences written by Ava and Landon.**

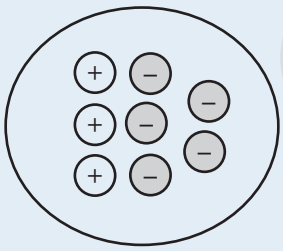
   a. **Explain why both number sentences are correct.**

   b. **Write an additional number sentence that could describe the model.**

6. **Write each number sentence in Question 5 a second way.**

Ava and Landon used two-color counters to represent the number sentence $3 + (-5)$.

7. **The students placed the same counters on their desks, but they reported different sums. Ava reported the sum as 8 and Landon said the sum was −2. Use the model to explain who is correct. What was the error made by the incorrect student?**



NOTES

8. Draw a model for each, and then complete the number sentence.

a. $-9 + (-4) =$ _____

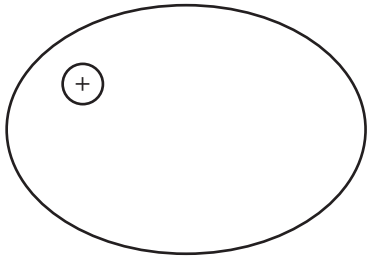
b. $-9 + 4 =$ _____

c. $9 + (-4) =$ _____

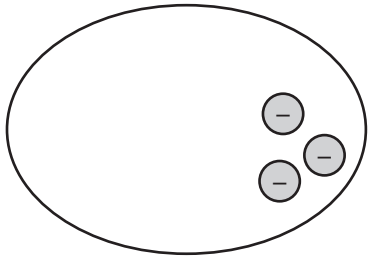
d. $9 + 4 =$ _____

9. Complete the model to determine the unknown integer.

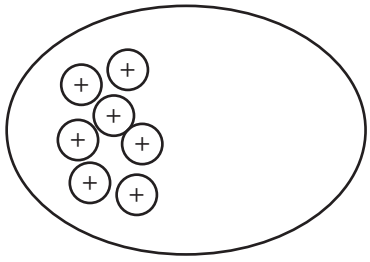
a. $1 +$ _____ $= -4$



b. $-3 +$ _____ $= 7$



c. $7 +$ _____ $= -1$



10. Describe the set of integers that makes each sentence true.

a. What integer(s) when added to −7 give a sum greater than 0?

b. What integer(s) when added to −7 give a sum less than 0?

c. What integer(s) when added to −7 give a sum of 0?

You have now used two models to represent adding integers.

11. For each problem, draw both models to represent the number sentences and determine the sums.

a. (−6) + 13

b. 8 + (−13)

c. (−3) + (−7)

d. 2 + 9

Think about how the absolute values of the addends compare with each other.

12. Explain the similarities and differences of the models in helping you determine the sum of two integers.

TOPIC 1

Adding and Subtracting Rational Numbers

Death Valley has some very high and very low elevations: Badwater Basin is the point of the lowest elevation in North America, at 282 feet below sea level, while Telescope Peak in the Panamint Range has an elevation of 11,043 feet. Photo by Tuxyso / Wikimedia Commons, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=28603629

Adding and Subtracting Rational Numbers

Topic 1 Overview — 9 Days

How is *Adding and Subtracting Rational Numbers* organized?

In *Adding and Subtracting Rational Numbers*, students use number lines and two-color counters to model addition and subtraction of integers before developing rules for the sum and difference of signed numbers. Students begin by walking a number line to visualize adding integers. They transition from physical movement to modeling the movement on number lines. Students then use two-color counters as a second model and use their experiences to develop rules for adding integers.

To build a conceptual understanding of subtracting integers, students again use physical motion, modeling movement on number lines, and two-color counters. With both addition and subtraction, the two different representations are used to highlight connections between numbers, including additive inverses and zero pairs, and to build understanding. Students are expected to make connections among the representations used. After they understand what it means to add and subtract integers, students apply the rules to the set of rational numbers.

What is the entry point for students?

In grade 6, students learned how to represent positive and negative rational numbers on a number line. They also know that $-p$ is p units from 0 on the number line and that $|p| = |-p| = p$. Students used number lines to model the distance from 0 and to model the distance between two rational numbers represented on vertical or horizontal number lines. *Adding and Subtracting Rational Numbers* begins with students playing Math Football, a game used to model the sum of a positive and negative integer using a number line. Students use the context of the game and what they know about number lines to begin making conjectures about adding and subtracting integers. As students progress through the topic, they draw on their experiences of physically walking a number line and modeling on a number line to develop rules for adding and subtracting integers.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Adding and Subtracting Rational Numbers* if they can:

- Demonstrate addition and subtraction on a horizontal or vertical number line diagram.
- Describe situations where opposite quantities combine to make 0.
- Explain how a number and its opposite have a sum of 0 and are additive inverses.
- Explain why the sum of $p + q$ is located a distance of $|q|$ in the positive or negative direction from p on a number line.
- Explain the distance between two rational numbers on a number line in terms of the absolute value of their difference.
- Solve real-world and mathematical problems by adding and subtracting rational numbers.

- Apply properties of operations as strategies to add and subtract rational numbers.
- Add and subtract rational numbers.
- Solve multi-step real-world and mathematical problems with numbers in any form (whole numbers, fractions, and decimals), using tools strategically.

Why is *Adding and Subtracting Rational Numbers* important?

Rational numbers are the primary set of numbers with which students will work in their educational and non-educational lives. It is essential that students develop a strong conceptual foundation for adding and subtracting with rational numbers, beyond the procedural knowledge of the oft-forgotten rules, to provide the foundation for manipulating and representing increasingly complex numeric and algebraic expressions. In high school, students will focus more on expressions and equations than on numbers, including rational expressions, equations, and functions.

How do the activities in *Adding and Subtracting Rational Numbers* promote student expertise in the mathematical practice standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the habits of mind should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

A primary focus in *Adding and Subtracting Rational Numbers* is recognizing patterns and developing algorithms through the repeated use of physical and visual tools. Students are expected to use tools (e.g., physical number line, drawn number lines, and two-color counters) to explore patterns in the sums and differences of integers. From their results, students should recognize patterns and generate rules for adding and subtracting integers. They reason about the properties of numbers to generalize the rules for integers into rules for all rational numbers.

Materials Needed

- Scissors
- Red and black number cubes
- Masking tape
- Marker
- Two-color counters
- Tape or glue sticks

Digital Access

For all digital files aligned to this topic, login at www.carnegielearning.com/c2.

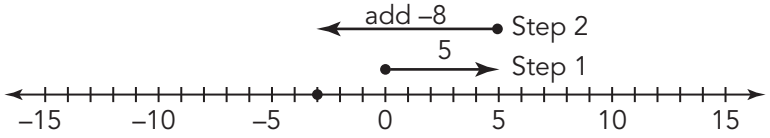
Learning Together: 8 Days

Lesson	Lesson Name	Standards	Days	Highlights	Spaced Review
1.1	Math Football Using Models to Understand Integer Addition	7.NS.1	1	A math football game is used to model the sums of positive and negative integers. Students use number cubes to generate the integers. They then use that information and write integer number sentences.	7.RP.2 7.RP.3 7.G.1
1.2	Walk the Line Adding Integers, Part I	7.NS.1b	2	Students explore patterns for adding two integers using a number line. They focus on the absolute values of the numbers being added and develop informal rules for adding integers.	7.G.4
1.3	Two-Color Counters Adding Integers, Part II	7.NS.1a 7.NS.1b	2	Students use two-color counters to develop rules for adding integers. They model adding positive and negative integers with the two-color counters. They use a graphic organizer to represent how to add additive inverses using a variety of representations.	7.RP.2c 7.RP.3 7.NS.1
1.4	What's the Difference? Subtracting Integers	7.NS.1c	2	Students use number lines and two-color counters to model subtraction of signed numbers. They develop and apply rules for subtracting integers.	6.EE.7 7.G.1
1.5	All Mixed Up Adding and Subtracting Rational Numbers	7.NS.3	1	Students apply their knowledge of adding and subtracting positive and negative integers to the set of rational numbers.	7.RP.1 7.RP.2b 7.NS.1d

Representations Used to Develop Conceptual Understanding of Procedures

Adding Integers on a Number Line

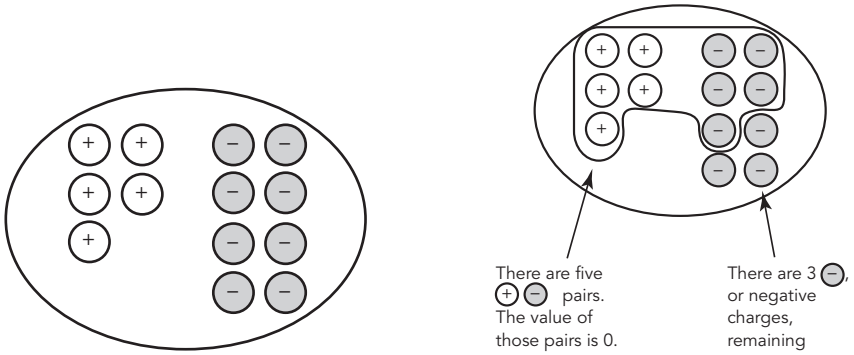
$5 + (-8)$



$5 + (-8) = -3$

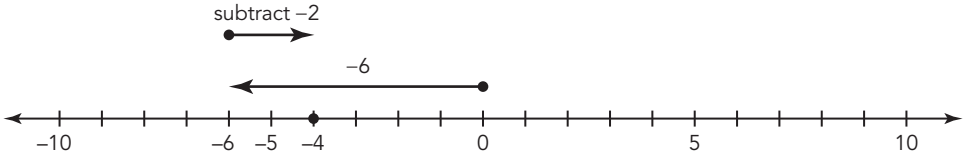
Adding Integers with Two-Color Counters

$5 + (-8)$



Subtracting Integers on a Number Line

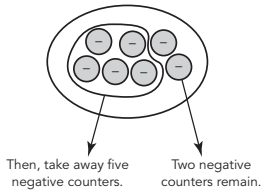
$6 - (-2)$



$-(-2) = -4$

Subtracting Integers with Two-Color Counters

$-7 - (-5)$



$-7 - (-5) = -2$

Learning Individually with MATHia: Approximately 1 Day

In MATHia software, students practice adding and subtracting integers using a number line.

At Carnegie Learning, we believe it is our responsibility to continuously enhance MATHia to better support your students' learning needs. We leverage the learning data in MATHia to regularly improve both the learning experience and content. To get the latest MATHia content alignment map, please go to: www.carnegielearning.com/c2/mag.

Learning Independently with Skills Practice: Approximately 1 Day

Problem Set	Overview
Using Number Lines to Add and Subtract Integers	Students represent sums and differences on number lines and determine the value of the sum or difference.
Adding and Subtracting Negative Integers	Students add and subtract integers in mathematical and contextual situations.

Assessments

There are six assessments aligned to this topic: a Pre-test, a Post-test, End of Topic Test (Form A and Form B), Standardized Test Practice, and a Performance Task.

Log into your account at CarnegieLearning.com to access Edulastic, an online assessment tool, and Lessoneer.com, an online implementation platform.

Module 2: Operating with Signed Numbers

TOPIC 1: ADDING AND SUBTRACTING RATIONAL NUMBERS

In this topic, students use number lines and two-color counters to model addition and subtraction of integers before developing rules for determining the sum and difference of signed numbers. Students are expected to make connections among the representations used. After they understand what it means to add and subtract integers, students apply the rules to the set of rational numbers.

Where have we been?

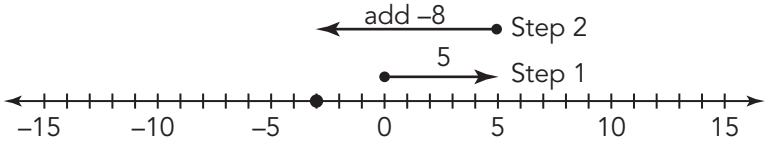
In grade 6, students learned how to represent positive and negative rational numbers on a number line. They also know that $-p$ is p units from 0 on the number line and that $|p| = |-p| = p$. Students used number lines to model the distance from 0 and to model the distance between two rational numbers represented on vertical or horizontal number lines.

Where are we going?

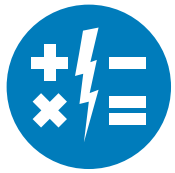
Students will develop a strong conceptual foundation for adding and subtracting with rational numbers to provide the foundation for manipulating and representing increasingly complex numeric and algebraic expressions in later lessons and future courses and grades.

Using a Number Line to Model Adding and Subtracting Integers

A number line can be used to model adding and subtracting negative numbers. This number line models the sum $5 + (-8)$.



Myth: Students only use 10% of their brains.



Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of the movies *Lucy* (2014) and *Limitless* (2011). Both films ask the audience: Imagine what you could accomplish if you could use 100% of your brain!

Well, this isn't Hollywood, and you're stuck with an ordinary brain. The good news is that you **do** use 100% of your brain. As you look around the room, your visual cortex is busy assembling images; your motor cortex is busy moving your neck; and all of the associative areas recognize the objects that you see. Meanwhile, the corpus callosum, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts...like mathematics!

#mathmythbusted

Talking Points

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to reason using signed numbers.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Terms

absolute value

The absolute value of a number is its distance from 0 on a number line.

additive inverse

The additive inverse of a number is the opposite of the number: $-x$ is the additive inverse of x. Two numbers with the sum of zero are called additive inverses.

zero pair

A zero pair is a pair of numbers whose sum is zero. The value of negative 1 plus positive 1 is zero. So, negative 1 and positive 1 together are a zero pair.

Math Football

1

Using Models to Understand Integer Addition

MATERIALS

Scissors
Red and black number cubes

Lesson Overview

A math football game is used to model the sum of positive and negative integers. Rules for the game and a game board are provided. Students use number cubes to generate the integers. They then take that same information and write integer number sentences.

Grade 7 The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Essential Ideas

- A model can be used to represent the sum of a positive and negative integer, two negative integers, or two positive integers.
- Information from a model can be rewritten as an equation.

Lesson Structure and Pacing: 1 Day

Day 1

Engage

Getting Started: Hut! Hut! Hike!

Students familiarize themselves with the rules for Math Football—a game they will play throughout this lesson. Students then read through and answer a question about an example play.

Develop

Activity 1.1: Signed Numbers as Values with Directions

Students play the game Math Football introduced in the Getting Started activity. They investigate the effects of adding positive and negative integers on the direction a point is moved on a number line, as well as on the magnitude of the move.

Activity 1.2: Writing Equations with Signed Numbers

Students explore writing equations with positive and negative values. They play the Math Football game again, this time focusing on interpreting their moves as equations. Students investigate patterns in integer sums that consistently move a point on a number line in one direction or another.

Demonstrate

Talk the Talk: Mission: Possible, and Impossible

Students generalize what they have learned from playing the Math Football game about adding positive and negative integers. Students investigate patterns in the addends and how these affect the sum.

Getting Started: Hut! Hut! Hike!

ENGAGE

Facilitation Notes

In this activity, the rules for a game establish the foundations for combining positive and negative numbers. Students interpret an example of game play.

Ask a student to read the introduction before Question 1 aloud. Discuss the rules and scoring procedures and complete Question 1 as a class.

Questions to ask

- Where does each player start?
- Where are the end zones?
- What color cube tells you how many yards to move forward or up the field?
- Which cube can be associated with positive numbers?
- Which cube can be associated with negative numbers?
- What color cube tells you how many yards to move backward or down the field?
- How does a player score 6 points?
- How does a player lose 2 points?
- What happens if the numbers you roll take you further than the end zone? Do you still score 6 points?

Summary

Positive and negative numbers can be modeled using real-world situations.

Activity 1.1

Signed Numbers as Values with Directions

DEVELOP

Facilitation Notes

In this activity, students cut out a game board and use it to play the game introduced in the Getting Started activity. They answer questions relevant to the game that associate directional distance to adding positive and negative integers.

Ask students to cut out the game board, play Math Football, and then complete Question 1 with a partner. For the first half of the game, have students position the board between them so that their scoring end zone is to their right. Share responses as a class.

Differentiation strategy

To support students who struggle, they may benefit by playing the entire game with their scoring end zone in the positive direction.

As students work, look for

- Confusion with the directions associated with the red cube and black cube.
- Two separate moves by a player, one for each number cube.
- One move by a player that is the result of the two number cubes.

Questions to ask

- Why do you want the black cube to show the greater value when approaching the Home Team end zone?
- Why do you want the red cube to show the greater value when approaching the Away Team end zone?
- What is an example of a roll that would cause you to move backward or to the left?
- What is an example of a roll that would cause you to move forward or to the right?
- What is an example of a roll that would cause you to have no gain in yardage?
- What is an example of two values that would send you back to the yard line where you began?
- What is an example of a roll that would cause you to move the least distance?
- What is an example of a roll that would cause you to move the greatest distance?
- How is the first half of the game related to the number line?
- How is the second half of the game related to the number line?

Summary

Real-world situations can be used to model the directional distance on a number line associated with positive and negative integers.

Activity 1.2

Writing Equations with Signed Numbers

Facilitation Notes

In this activity, students write equations for different situations containing positive and negative integers associated with the number cube rolls of game play.

Ask a student to read the introduction before Question 1 aloud then complete Question 1 as a class.

Questions to ask

- Why are plus signs used between all the terms in the number sentences?
- Why are parentheses used in some terms of the number sentences?
- Would you get the same result if you reordered the second and third terms of the number sentences? Explain.
- Does (−6) represent the roll of the red number cube or the roll of the black number cube in this situation?
- Does (+2) represent the roll of the red number cube or the roll of the black number cube in this situation?
- Create an original number sentence that yields a negative result.

Have students play Math Football again. This time, note that partners are playing together on the same team. They are working together to get to the Home end zone in the first half and then to the Visiting end zone in the second half. After they play Math Football again, have students work with a partner or in groups to complete Questions 2 and 3. Share responses as a class.

Questions to ask

- What is an another equation to represent this situation?
- If you combine the terms in a different order, will you get the same result?
- What properties allow you to combine terms in a different order?
- If an integer is added to its opposite, what is the result?
- Provide an example with different signs that yields a positive result.
- Provide an example with different signs that yields a negative result.

Differentiation strategy

To extend the activity, have students combine five integers at a time. Discuss the strategy of combining all the positive values, combining all the negative values, and then combining those two results.

Summary

Equations can be used to describe the relationship between positive and negative integers.

Talk the Talk: Mission: Possible, and Impossible

DEMONSTRATE

Facilitation Notes

In this activity, students generalize the results of rolling different combinations with the red and black color cubes. This is an informal introduction of the rules used to combine signed numbers.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Why does a larger number on the black cube than on the red cube always result in a positive integer?
- Why does a larger number on the red cube than on the black cube always result in a negative integer?
- Which roll represents combining two positive integers?
- Which roll represents combining two negative integers?
- Which roll represents combining one negative and one positive integer?
- Which roll can yield a positive result?
- Which roll can yield a negative result?

Summary

Rules can be applied to combining positive and negative integers. The sum of two negative numbers is always a negative number. The sum of two positive numbers is always a positive number. The sum of a positive number and negative number is the the sign of the larger number.

1

Math Football

Using Models to Understand Integer Addition

WARM UP

Sketch a number line and plot each value.

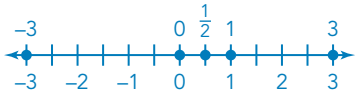
1. -3
2. 0
3. 1
4. $\frac{1}{2}$
5. 3

LEARNING GOALS

- Represent numbers as positive and negative integers.
- Use a number line diagram to represent the sum of positive and negative integers.

You have learned about negative numbers and can plot locations on a number line. Does addition and subtraction work the same with negative numbers as with positive numbers?

Warm Up Answers



LESSON 1: Math Football • M2-7

Answers

1. The Home Team player is closest to the Home end zone.

Getting Started

Hut! Hut! Hike!

The playing field and footballs are located at the end of the lesson. You also need two number cubes, one red and one black.

You and a partner are going to play **Math Football**. You will take turns rolling two number cubes to determine how many yards you can advance the football toward your end zone.

Player 1 will be the Home Team and Player 2 will be the Visiting Team. In the first half, the Home Team will move toward the Home end zone, and the Visiting Team will move toward the Visiting end zone.

Rules

Players both start at the zero yard line and take turns. On your turn, roll two number cubes, one red and one black. The number on each cube represents a number of yards. Move your football to the left the number of yards shown on the red cube. Move your football to the right the number of yards shown on the black cube. Start each of your next turns from the ending position of your previous turn.

Scoring

When players reach their end zone, they score 6 points. If players reach their opponent's end zone, they lose 2 points. An end zone begins on either the +10 or −10 yard line.

Example:

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position
First Turn	Home Team	0	Red 3 and Black 5	+2
	Visiting Team	0	Red 5 and Black 6	+1
Second Turn	Home Team	+2	Red 1 and Black 6	+7
	Visiting Team	+1	Red 6 and Black 2	−3

1. Read through the table. After two turns, which player is closest to their end zone?

M2-8 • TOPIC 1: Adding and Subtracting Rational Numbers

ACTIVITY
1.1

Signed Numbers as Values with Directions

NOTES

Let's play Math Football. Begin by selecting the home or visiting team. Your teacher will set the length of time for each half. You will play two halves. Make sure to switch ends at half-time with the Home Team moving toward the Visiting end zone, and the Visiting Team moving toward the Home end zone. Good luck!

1. **Once your game is finished, answer each question.**

   a. **When you are trying to get to the Home end zone, which number cube do you want to show the greater value? Explain your reasoning.**

   b. **When you are trying to get to the Visiting end zone, which number cube do you want to show the greater value? Explain your reasoning.**

   c. **Did you ever find yourself back at the same position you ended on your previous turn? Describe the values on the cubes that would cause this to happen.**

   d. **Describe the roll that causes you to move your football the greatest distance either left or right.**

Answers

1a. I wanted the black cube to show the greater value when moving toward the Home end zone.

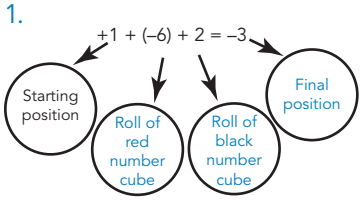
1b. I wanted the red cube to show the greater value when moving toward the Visiting end zone.

1c. If the same number is rolled on both cubes, I don't move.

1d. Rolling a 6 on one cube and 1 on the other would move me 5 spaces in some direction.

Answers

1.



2a. Rolls where the black cube was greater than the red cube moved me closer to Home.

2b. Rolls where the red cube was greater than the black cube moved me closer to Visiting.

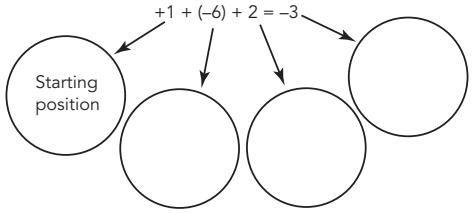
ACTIVITY 1.2

Writing Equations with Signed Numbers

You can write equations to describe the results of number cube rolls. Think of the result of rolling the red number cube as a negative number and the result of rolling the black number cube as a positive number.

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position	Number Sentence
First Turn	Home Team	0	Red 3 and Black 5	+2	0 + (−3) + 5 = +2
	Visiting Team	0	Red 5 and Black 6	+1	0 + (−5) + 6 = +1
Second Turn	Home Team	+2	Red 1 and Black 6	+7	+2 + (−1) + 6 = +7
	Visiting Team	+1	Red 6 and Black 2	−3	+1 + (−6) + 2 = −3

1. Describe each part of the number sentence for the second turn of the Visiting Team player.



Play Math Football again. But this time, work with your partner to get to the Home end zone together in the first half and the Visiting end zone in the second half. Write equations to record your moves.

2. Think about the number cube rolls you made in the game.

a. What kind of rolls move you closer to the Home end zone?

b. What kind of rolls move you closer to the Visiting end zone?

M2-10 • TOPIC 1: Adding and Subtracting Rational Numbers

3. Write an equation for each situation. Use the game board for help.

a. The Home Team player starts at the zero yard line and rolls a red 6 and a black 2. What is the ending position?

Equation: _____

b. The Visiting Team player starts at the zero yard line and rolls a red 5 and a black 4. What is the ending position?

Equation: _____

c. The Home Team player starts at the 5 yard line and rolls a red 2 and a black 2. What is the ending position?

Equation: _____

d. The Visiting Team player starts at the −5 yard line and rolls a red 4 and a black 6. What is the ending position?

Equation: _____

e. Suppose the Home Team player is at the +8 yard line. Complete the table and write two equations that will put the player into the Home end zone.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Equation
+8			
+8			

f. Suppose the Visiting Team player is at the −8 yard line. Complete the table and write two equations that will put the player into the Visiting end zone.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Equation
−8			
−8			

Answers

3a. $0 + (-6) + 2 = -4$

3b. $0 + (-5) + 4 = -1$

3c. $5 + (-2) + 2 = 5$

3d. $-5 + (-4) + 6 = -3$

3e., 3f. See tables below.

3e.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Equation
+8	−1	+3	+8 + (−1) + 3 = 10
+8	−2	+5	+8 + (−2) + 5 = 11

3f.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Equation
+8	−4	+2	−8 + (−4) + 2 = −10
+8	−6	+3	−8 + (−6) + 3 = −11

Answers

1a. a move in the positive direction

1b. a move in the negative direction

1c. a move in the positive direction

1d. it could be positive, negative, or 0

1e. a move in the negative direction

2. No. With two black cubes, the sum will always be positive.

3. No. With two red cubes, the sum will always be negative.

NOTES

TALK the TALK

Mission: Possible, and Impossible

Consider the moves you made in the Math Football game.

1. In which direction would you move if you roll:

a. a larger number on the black cube than on the red cube?

b. a larger number on the red cube than on the black cube?

c. two black cubes?

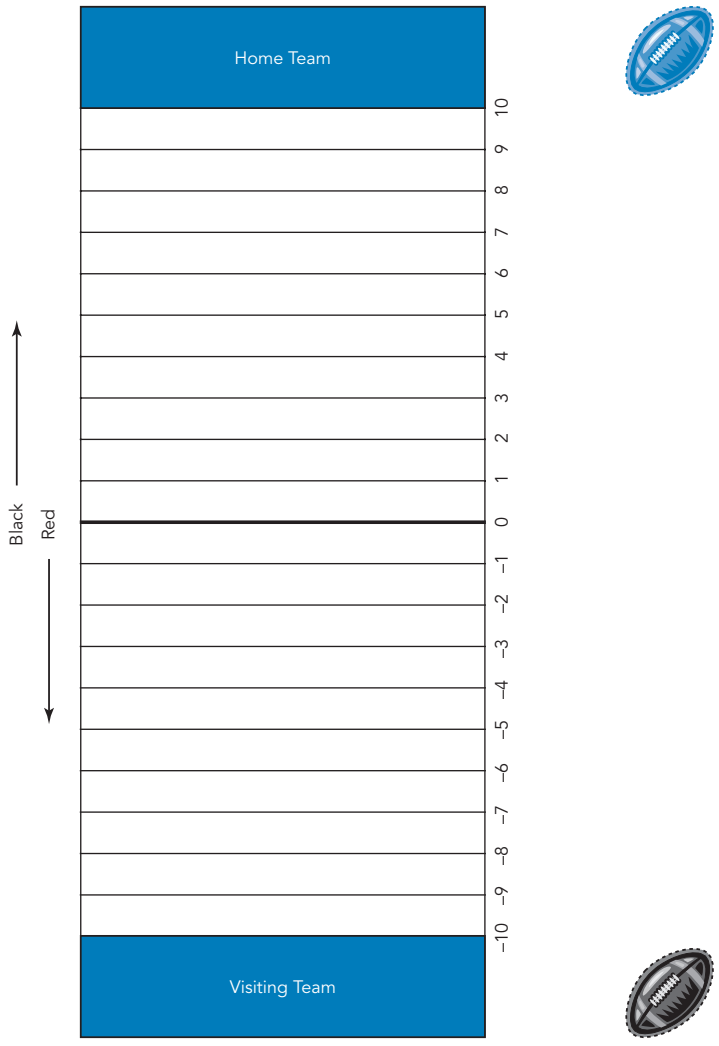
d. a black cube and a red cube?

e. two red cubes?

2. Is it possible to decrease in value if rolling two black cubes? Explain your reasoning.

3. Is it possible to increase in value if rolling two red cubes? Explain your reasoning.

M2-12 • TOPIC 1: Adding and Subtracting Rational Numbers



LESSON 1: Math Football • M2-13

Assignment

Write

In your own words, explain how to decide whether the sum of two numbers is less than, equal to, or greater than 0.

Remember

Combining positive and negative moves together on a line results in a move to the left, a move to the right, or staying in the same position, depending on the size of the positive and negative values.

Practice

1. Determine the ending position by adding and subtracting the indicated steps from each starting position.

Starting Position	Steps Backward	Steps Forward	Ending Position
+3	4	5	
+7	6	2	
+5	2	4	
0	5	8	
−4	3	7	
+1	7	9	
−6	1	5	
−2	5	6	
8	3	1	
−9	2	4	

2. Write an equation to represent the movement indicated by the starting point, steps backward, and steps forward.

Starting Position	Steps Backward	Steps Forward	Equation
+2	4	7	
−7	3	5	
+6	9	4	
+4	6	1	
−5	2	9	
0	5	3	
−3	1	4	
−8	2	6	
0	8	2	
+9	7	8	

Assignment Answers

Write

1. Answers will vary.

Practice

1.

Ending Position
+4
+3
+7
+3
0
+3
−2
−1
+6
−7

2.

Equation
+2 + (−4) + 7 = +5
−7 + (−3) + 5 = −5
+6 + (−9) + 4 = +1
+4 + (−6) + 1 = −1
−5 + (−2) + 9 = +2
0 + (−5) + 3 = −2
−3 + (−1) + 4 = 0
−8 + (−2) + 6 = −4
0 + (−8) + 2 = −6
+9 + (−7) + 8 = +10

Assignment Answers

Stretch

Check students' models:

$-3\frac{1}{2} + (-1\frac{1}{4}) = -4\frac{3}{4}$

Review

1a. $p = 8.91$

1b. $c = 5$

2a. Use the scaling method because it is easy to see that the numerator is multiplied by 10, so the denominator must also be multiplied by 10.

2b. Use the unit rate method because the numerator is a multiple of the denominator, so a unit rate is more efficient to determine.

3a. This is not a scale drawing.
$\frac{10}{25} \neq \frac{5}{20}$

3b. This is a scale drawing.
$\frac{10}{40} = \frac{5}{20}$

Stretch

Draw a model to represent the addition problem $-3\frac{1}{2} + (-1\frac{1}{4})$. Then determine the solution.

Review

1. Solve each proportion.

   a. $\frac{6.6}{p} = \frac{9}{12.15}$

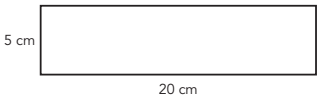
   b. $\frac{8}{10.5} = \frac{c}{6.5625}$

2. Describe which method (scaling, unit rate, or means and extremes) you would use to solve for each variable and explain why.

   a. $\frac{2}{3} = \frac{20}{x}$

   b. $\frac{16}{4} = \frac{100}{x}$

3. Determine if each rectangle is a scale drawing of the given rectangle. Explain why or why not.



a.

b.

Walk the Line

Adding Integers, Part I

2

MATERIALS

- Tape to create a number line on the classroom floor
- Markers to mark the integer values on the floor number line

Lesson Overview

A number line is used to model the sum of two integers. Students begin the lesson by walking a number line on the floor of the classroom. Through a series of activities, students will notice patterns for adding integers. After the kinesthetic activity, students examine a worked example and then practice calculating sums of positive and negative numbers using a number line model. Questions focus students on the distance an integer is from 0 on the number line, or the absolute value of the integer, to anticipate writing a rule for the sum of two integers having different signs. Students then write the rules for the sum of any two integers. Students demonstrate their understandings of the patterns by writing informal rules for adding integers.

Grade 7 The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   b. Understand $p + q$ as the number located a distance of $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Essential Ideas

- On a number line, when adding a positive integer, move to the right.
- On a number line, when adding a negative integer, move to the left.
- When adding two positive integers, the sign of the sum is always positive.
- When adding two negative integers, the sign of the sum is always negative.
- When adding a positive and a negative integer, the sign of the sum is the sign of the number that is the greatest distance from zero on the number line.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Getting on Line

Several word statements are given. Students use a number line to determine the integer described by each statement and explain their reasoning. Students informally consider how to move on the number line to identify values that are specific units more than or less than a given number.

Develop

Activity 2.1: Walking the Number Line

Students physically walk a number line to model addition of positive and negative numbers. After the kinesthetic activity with addition, two examples of adding integers on a number line are provided, and students answer questions that describe the steps taken to compute the sum of the integers. Students translate their walking on the number line to pencil-and-paper models.

Day 2

Activity 2.2: Adding on Number Lines

Students practice adding positive and negative numbers using a number line. Special attention is paid to absolute values and connecting absolute values to the number line model for addition. No formal rules for adding integers are established yet.

Demonstrate

Talk the Talk: Patterns on the Line

Students demonstrate their understanding by writing informal rules for the addition of integers. They use a number line model to determine unknown values in equations.

Getting Started: Getting on Line

Facilitation Notes

In this activity, numbers are described using phrases such as "more than" or "less than" a given value. Students use a number line model to determine each number described.

Emphasize the connection between the left and right movements in the football game students played in the previous activities and the left and right movements on the number line model.

Have students complete Questions 1 through 6 with a partner or in a group. Share responses as a class.

Misconception

When students translate a phrase into mathematical symbols, they sometimes just take the values written in the phrase from left to right instead of interpreting the meaning of the phrase. This can result in a wrong starting point or a mathematical error if subtraction is involved.

Differentiation strategy

To support students who struggle, provide a separate number line for each problem so that they can record and refer to their responses more easily.

As students work, look for

- Errors in order when the phrase "less than" is used. For example, the description "10 less than 6" incorrectly interpreted as $10 + (-6)$ rather than $6 + (-10)$.
- Counting tick marks rather than spaces on the number line.

Questions to ask

- How do you know what value to start at on the number line?
- How do you know whether to move to the left or right?
- What direction do the words "more than" imply with respect to a number line?
- What direction do the words "less than" imply with respect to a number line?
- If you started at the wrong number in a "more than" problem, would you still get the right answer? Why or why not?
- If you started at the wrong number in a "less than" problem, would you still get the right answer? Why or why not?
- How is this related to the football example in the previous lesson?
- How do you write this expression using mathematical symbols?
- How would you change your expression into an equation?

Summary

A number line and reasoning are helpful when thinking about addition and subtraction of integers.

DEVELOP

Activity 2.1
Walking the Number Line

Facilitation Notes

In this activity, walking a number line is used to model the addition of positive and negative integers. Students examine worked examples and use a number line to determine sums of signed numbers.

Prior to class, use masking tape to set up a human number line from at least −5 to 5. Spaces between tick marks should be about one step in width.

Ask a student to read the introduction aloud. Have another student restate the rules in their own words. Select a student to model the first addition problem by walking the line. Complete Question 1 as a class. Select individual students to model each problem. Note there are empty spaces in the chart so that teachers or students can make up additional problems to solve using the human number line.

Have students read and discuss the worked examples with their partner or in groups. Review the worked examples as a class.

Differentiation strategy

To support students who struggle, have two blank number lines to redo each example after the class discussion. Modifications may be to have students draw the two steps side by side on the number line rather than one above the other. Then as they show the 5 and 8, have students show each individual "jump" as they count the spaces.

Have students complete Questions 2 through 4 with a partner or in a group. Share responses as a class.

Questions to ask

- When computing the sum of two or more integers using a number line, where do you always start?
- When computing the sum of two or more integers using a number line, when you start at zero, how do you know which direction, left or right, to move next?
- How do you know which direction, left or right, to move, to combine the second term?

- On a number line, what is the sign of the first term, if you move from zero on the number line, to the left?
- On a number line, what is the sign of the first term, if you move from zero on the number line, to the right?

Have students complete Questions 5 through 7 with a partner. Share responses as a class.

Questions to ask

- On a number line, what is the sign of the second term, if you move from the location of the first term, to the left?
- On a number line, what is the sign of the second term, if you move from the location of the first term, to the right?
- What information does the absolute value of a term give you, with respect to the graphical representation?

Have students complete Question 8 with their group or partner. As students are working, invite them to use the human number line to model the problem if necessary or to check their answers. Share responses as a class.

Questions to ask

- If the sign of the first term was positive and the sign of the second term was positive, which directions did you move on the number line?
- If the sign of the first term was positive and the sign of the second term was negative, which directions did you move on the number line?
- If the sign of the first term was negative and the sign of the second term was negative, which directions did you move on the number line?
- If the sign of the first term was negative and the sign of the second term was positive, which directions did you move on the number line?

Have students answer Questions 9 through 12. Share responses as a class.

Questions to ask

- Why does moving to the left on a number line leave you with a smaller number?
- Why does moving to the right on a number line leave you with a larger number?
- What information does the absolute value of each term give you?
- The absolute value of the two integers used in each part of this question are the same, why aren't the sums of the two integers the same?

Summary

Number lines can be used to model the addition of signed numbers.

Activity 2.2
Adding on Number Lines

Facilitation Notes

In this activity, students use number lines to determine sums of signed numbers. Patterns are emphasized using absolute values of the positive and negative integers.

Have students work with a partner or in groups to complete Questions 1 through 4. As a class, discuss Questions 5 through 8. Then, have students practice adding integers in groups by completing Questions 9 through 16. Again, invite students to model the problems or check their solutions with the human number line. Share responses as a class.

Questions to ask

- What rule might you use when working with a number line to add two positive integers?
- What rule might you use when working with a number line to add two negative integers?
- What rule might you use when working with a number line to add a positive and a negative integer?
- Could a number line be used to compute the sum of three or more integers?
- How could these problems be solved without using a number line?

Summary

Number lines can be used to model the addition of signed numbers.

DEMONSTRATE

Talk the Talk: Patterns on the Line

Facilitation Notes

In this activity, students note patterns involved when combining positive and negative integers. They also use number line models to complete number sentences.

Have students work individually to complete Questions 1 and 2. Discuss the patterns that they noticed and whether they will generalize to fractions and decimals. Then, have students work with a partner or in groups to complete Question 3. Share responses as a class.

Questions to ask

- When is the sum of a positive number and a negative number a positive answer?

- When is the sum of a positive number and a negative number a negative answer?
- When is the sum of a positive number and a positive number a positive answer?
- When is the sum of a positive number and a positive number a negative answer?
- When is the sum of a negative number and a negative number a positive answer?
- When is the sum of a negative number and a negative number a negative answer?
- When combining two integers, if you are always moving to the left, what does this tell you about the sign of the answer?
- When combining two integers, if you are always moving to the right, what does this tell you about the sign of the answer?
- When combining two integers, if you are moving to the left and then moving to the right, what does this tell you about the sign of the answer?
- When combining two integers, if you are moving to the left and then moving more to the right, what does this tell you about the sign of the answer?
- When combining two integers, if you are moving to the right and then moving more to the left, what does this tell you about the sign of the answer?

Summary

The sum of two negative numbers is always a negative number. The sum of two positive numbers is always a positive number. The sum of a positive number and negative number is the the sign of the largest number.

2 Walk the Line

Adding Integers, Part I

WARM UP

A large hotel has a ground floor (street level) and 26 floors of guest rooms above street level, which can be modeled by positive integers. There are 5 floors of parking below street level, which can be modeled by negative integers. In this hotel, street level is represented by zero.

Write an integer addition problem that models the hotel elevator's motion in each case.

1. The elevator starts at street level, goes up 7 floors, and then goes down 3 floors.
2. The elevator starts at street level, goes up 10 floors, and then goes down 12 floors.
3. The elevator starts at street level, goes down 4 floors, and then goes up 11 floors.
4. The elevator starts at street level, goes down 2 floors, goes up 5 floors, and finally goes down 3 floors.

LEARNING GOALS

- Model the addition of integers on a number line.
- Develop a rule for adding integers.
- Identify $p + q$ as the number located a distance of $|q|$ from p.

KEY TERM

- absolute value

You have been adding and subtracting positive numbers most of your life. In elementary school, you learned how to add numbers using a number line. How can a number line be a helpful tool in adding positive and negative numbers?

Warm Up Answers

1. $0 + 7 + (-3)$
2. $0 + 10 + (-12)$
3. $0 + (-4) + 11$
4. $0 + (-2) + 5 + (-3)$

LESSON 2: Walk the Line • M2-17

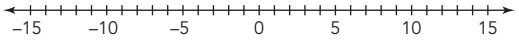
Answers

1. The number that is 7 more than −9 is −2. Go to −9 on the number line, and then move 7 units to the right.
2. The number that is 2 more than −6 is −4. Go to −6 on the number line, and then move 2 units to the right.
3. The number that is 10 more than −8 is 2. Go to −8 on the number line, and then move 10 units to the right.
4. The number that is 10 less than 6 is −4. Go to 6 on the number line, and then move 10 units to the left.
5. The number that is 5 less than −4 is −9. Go to −4 on the number line, and then move 5 units to the left.
6. The number that is 2 less than −4 is −6. Go to −4 on the number line, and then move 2 units to the left.

Getting Started

Getting on Line

Use the number line and determine the number described by each. Explain your reasoning.



1. the number that is 7 more than −9

2. the number that is 2 more than −6

3. the number that is 10 more than −8

4. the number that is 10 less than 6

5. the number that is 5 less than −4

6. the number that is 2 less than −4

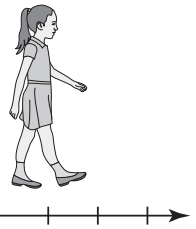
M2-18 • TOPIC 1: Adding and Subtracting Rational Numbers

ACTIVITY
2.1 Walking the Number Line

Walking a number line can help you to add positive and negative numbers.

Walk the number line for an addition sentence:

- Start at zero and walk to the value of the first term of the expression.
- To indicate addition, turn to face up the number line, towards the greater positive numbers.
- Walk forward if adding a positive number or walk backward if adding a negative number.



Your teacher will select a classmate to walk the line for each of the given problems. Help your classmate by preparing the directions that are needed.

1. Complete the table.

	Where You Start	Direction You Face	Walk Backwards or Forwards	Final Location
1 + 3				
0 + (−4)				
−3 + 5				
−1 + (−4)				

Answers

1. See table below.

1.

	Where You Start	Direction You Face	Walk Backwards or Forwards	Final Location
1 + 3	1	up	forwards	4
0 + (−4)	0	up	backwards	−4
−3 + 5	−3	up	forwards	2
−1 + (−4)	−1	up	backwards	−5

Answers

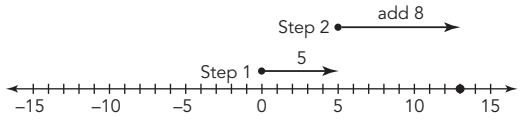
2. The distance shown by the first term in each example is the same: 5 units.
3. The graphical representation for the first term begins at 0 and moves to the right. It moves to the right because the first term is positive.
4. The absolute value of 5 is 5.

This worked example represents the movement created by walking the number line.

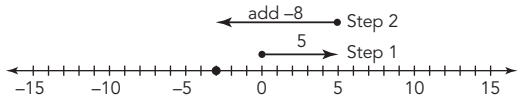
WORKED EXAMPLE

A number line can be used to model integer addition.
When adding a positive integer, move to the right on a number line.
When adding a negative integer, move to the left on a number line.

Example 1: The number line shows how to determine 5 + 8.



Example 2: The number line shows how to determine 5 + (−8).



Compare the first steps in each example.

2. What distance is shown by the first term in each example?

Remember that the **absolute value** of a number is its distance from 0.

3. Describe the graphical representation of the first term. Where does it start and in which direction does it move? How does this movement represent walking the line?

4. What is the absolute value of the first term in each example?

M2-20 • TOPIC 1: Adding and Subtracting Rational Numbers

Compare the second steps in each example.

5. What distance is shown by the second term in each example?

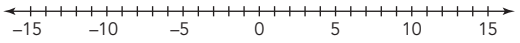
6. Why did the arrows for the second terms both start at the endpoints of the first terms but then continue in opposite directions? Explain your reasoning.

7. What is the absolute value of the second term in each example?

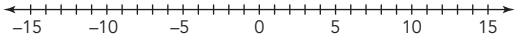
8. Use the number line to determine each sum. Show your work.

Think about walking the line as you model these sums on the number line.

a. −3 + 7 = _____



b. 3 + (−7) = _____



Answers

5. The distance shown by the second term in each example is the same: 8 units.

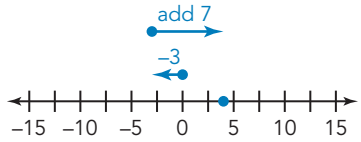
6. The arrows are drawn in opposite directions because the numbers are opposites of each other. Positive 8 tells me to move to the right; negative 8 tells me to go in the opposite direction, or move to the left.

7. $|8| = 8$

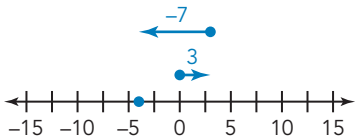
$|-8| = 8$

The absolute values are both 8.

8a. 4

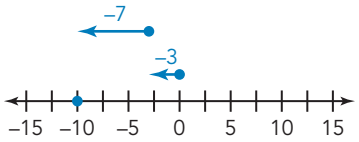


8b. −4

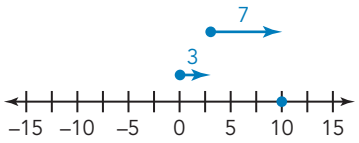


Answers

8c. −10



8d. 10



9. The distances are the same: 3 units.

10. |3| = 3

|−3| = 3

The absolute values are equal: 3.

11. The distances are the same: 7 units.

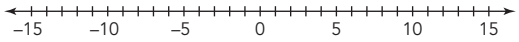
12. |7| = 7

|−7| = 7

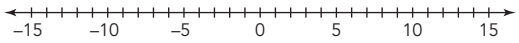
The absolute values are equal: 7.

NOTES

c. −3 + (−7) = _____



d. 3 + 7 = _____



Notice that the first term in each expression in parts (a) through (d) was either 3 or (−3).

9. What do you notice about the distances shown by these terms on the number lines?

10. What is the absolute value of each term?

Notice that the second term in each expression was either 7 or (−7).

11. What do you notice about the distances shown by these terms on the number lines?

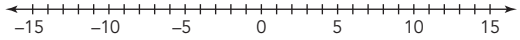
12. What is the absolute value of each term?

ACTIVITY
2.2 Adding on Number Lines

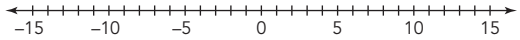
Now that you have the feel for how to move on the number line when adding negative numbers, it is time to practice with more examples.

Use the number line to determine each sum. Show your work.

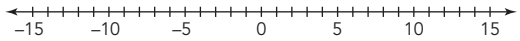
1. −9 + 5 = _____



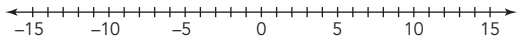
2. 9 + (−5) = _____



3. −9 + (−5) = _____

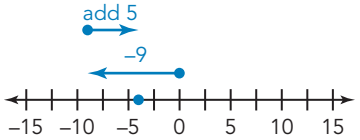


4. 9 + 5 = _____

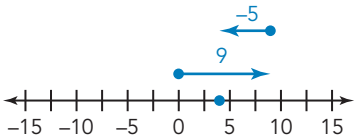


Answers

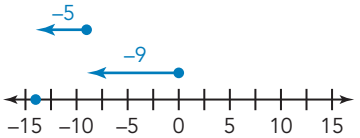
1. −4



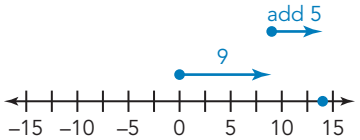
2. 4



3. −14



4. 14



Answers

5. The distances are the same: 9 units.
6. $|9| = 9$

   $|-9| = 9$

   The absolute values are equal: 9.
7. The distances are the same: 5 units.
8. $|5| = 5$

   $|-5| = 5$

   The absolute values are equal: 5.

Notice that the first term in each expression in Questions 1 through 4 was either 9 or (−9).

5. What do you notice about the distances shown by these terms on the number lines?

6. What is the absolute value of each term?

Notice that the second term in each expression was either 5 or (−5).

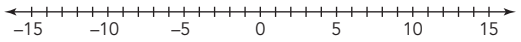
7. What do you notice about the distances shown by these terms on the number lines?

8. What is the absolute value of each term?

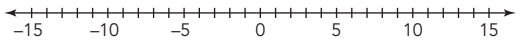
M2-24 • TOPIC 1: Adding and Subtracting Rational Numbers

Use the number line to determine each sum. Show your work.

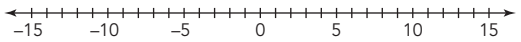
9. −8 + 2 = _____



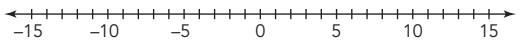
10. 8 + (−2) = _____



11. −8 + (−2) = _____

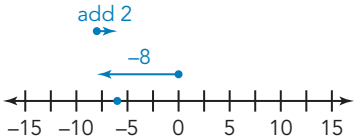


12. 8 + 2 = _____

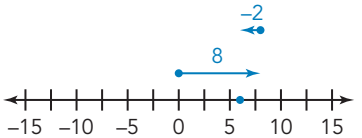


Answers

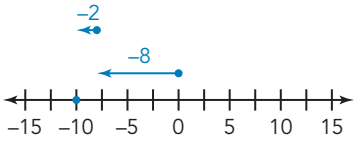
9. −6



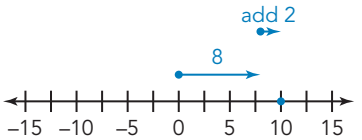
10. 6



11. −10

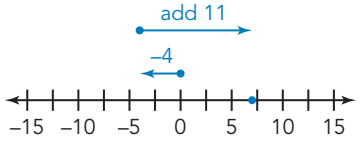


12. 10

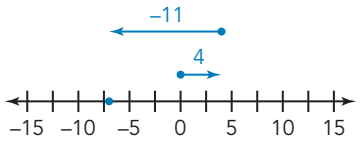


Answers

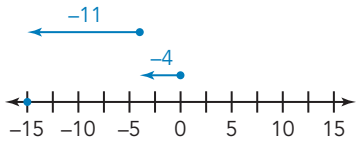
13. 7



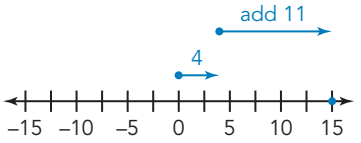
14. –7



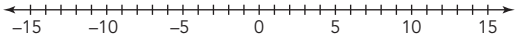
15. –15



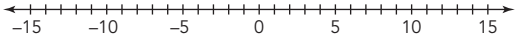
16. 15



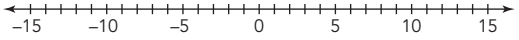
13. $-4 + 11 =$ _____



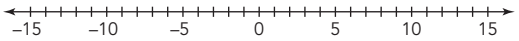
14. $4 + (-11) =$ _____



15. $-4 + (-11) =$ _____



16. $4 + 11 =$ _____



NOTES

TALK the TALK

Patterns on the Line

Demonstrate what you have learned about adding two numbers using a number line.

1. **Describe the patterns from the *Adding on Number Lines* activity, when you:**

   a. **add two positive numbers.**

   b. **add two negative numbers.**

   c. **add a negative and a positive number.**

2. **Do you think these patterns will hold true for all numbers, even fractions and decimals? Explain your reasoning.**

Answers

1a. The sum is always positive.

1b. The sum is always negative.

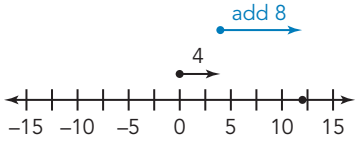
1c. When the negative number has the greatest distance from zero, the sum of the two numbers is negative. When the positive number has the greatest distance from zero, the sum of the two numbers is positive.

2. Answers will vary.

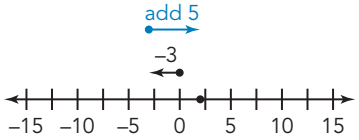
LESSON 2: Walk the Line • M2-27

Answers

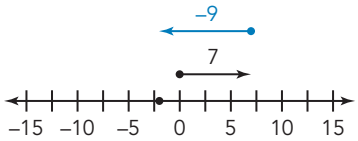
3a.



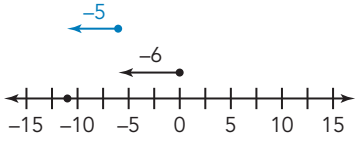
3b.



3c.



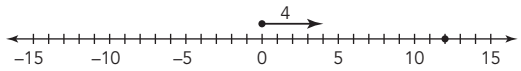
3d.



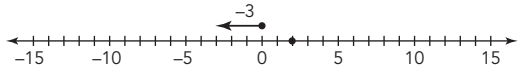
NOTES

3. Complete each number line model and number sentence.

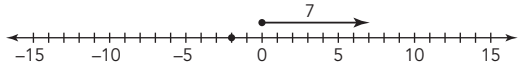
a. 4 + _____ = 12



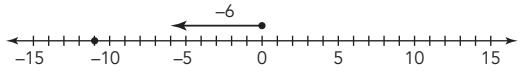
b. –3 + _____ = 2



c. 7 + _____ = –2



d. –6 + _____ = –11



M2-28 • TOPIC 1: Adding and Subtracting Rational Numbers

Assignment

Write

Explain how walking the line is the same as representing addition and subtraction on the number line.

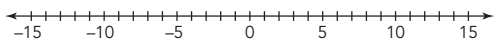
Remember

When adding a positive integer on a number line, move to the right on the number line. When adding a negative integer, move to the left on the number line.

Practice

Use the number line to determine each sum. Show your work.

1. $-6 + 4$
2. $-9 + (-2)$
3. $13 + (-12)$
4. $7 + (-14)$
5. $7 + (-1)$
6. $3 + (-13)$
7. $8 + (-8)$
8. $-2 + 8$
9. $-13 + 3$
10. $0 + (-12)$



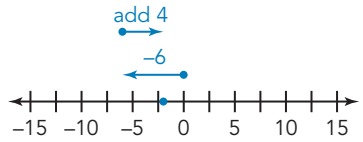
Assignment Answers

Write

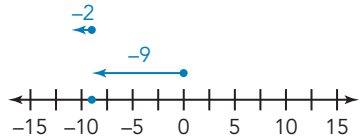
Answers will vary.

Practice

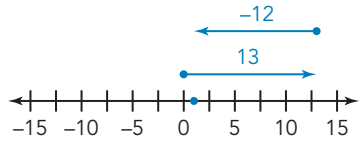
1. -2



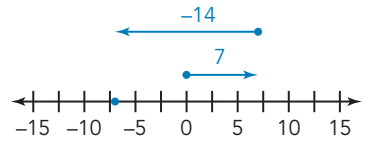
2. -11



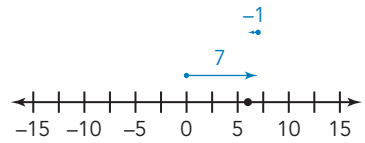
3. 1



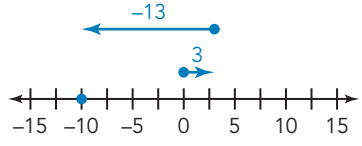
4. -7



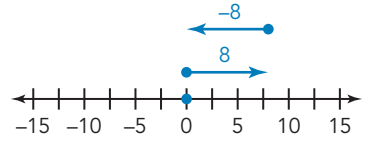
5. 6



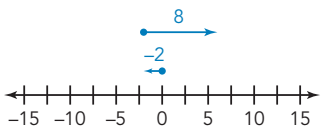
6. -10



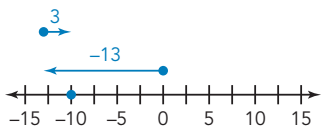
7. 0



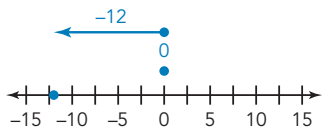
8. 6



9. -10

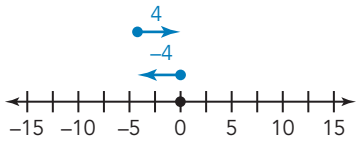


10. -12

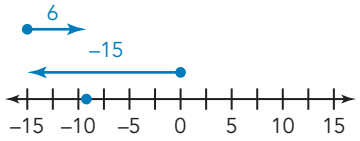


Assignment Answers

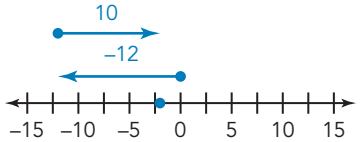
1. 4



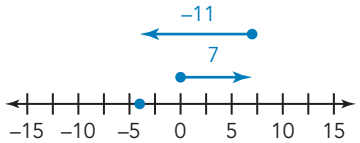
2. 6



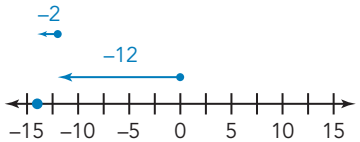
3. –12



4. 7

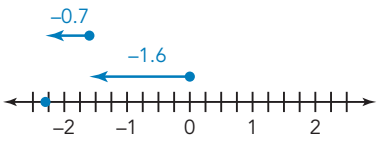


5. –2

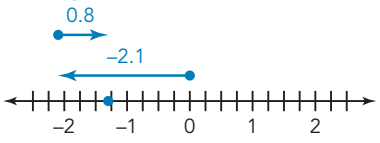


Stretch

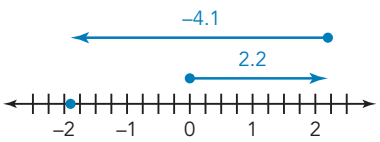
1. –2.3



2. –1.3

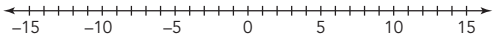


3. –1.9

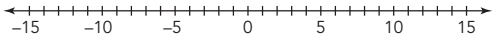


Complete each number line model and number sentence.

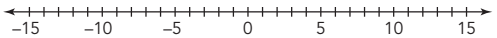
1. –4 + _____ = 0



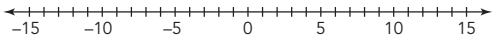
2. –15 + _____ = –9



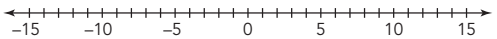
3. _____ + 10 = –2



4. _____ + (–11) = –4



5. –12 + _____ = –14



Stretch

Draw a number line model to determine each sum.

1. –1.6 + –0.7
2. –2.1 + 0.8
3. 2.2 + –4.1

Review

Northern Tier Gardens has hired you for a summer job installing water gardens. They have circular water garden pools available in a variety of sizes. The manager has asked you to create a table to show the circumference and area of the company's various water garden pools. Use 3.14 for π and round each answer to the nearest hundredth.

Garden Name	Radius (feet)	Diameter (feet)	Area (square feet)	Circumference (feet)
Atlantic	2.5	5		
Pacifica	6	12		
Mediterranean	1.75	3.5		
Baltica	1	2		
Japanesque	2.25	4.5		
Floridian	3.25	6.5		

Review

Garden Name	Area (square feet)	Circumference (feet)
Atlantic	19.63	15.70
Pacifica	113.04	37.68
Mediterranean	9.62	10.99
Baltica	3.14	6.28
Japanesque	15.90	14.13
Floridian	33.17	20.41

Two-Color Counters

3

Adding Integers, Part II

MATERIALS

Two-color counters
Scissors
Tape or glue sticks

Lesson Overview

Through a series of activities with two-color counters, students will develop rules for adding integers. Students determine that to have a sum of zero, two integers must have opposite signs but the same absolute value. Examples of modeling the sum of two integers with opposite signs using two-color counters is provided and the counters are paired together, one positive counter with one negative counter, until no possible pairs remain. The resulting counters determine the sum of the integers. Several models are given and students write a number sentence to represent each model. Students critique reasoning about using the two-color counters to model adding integers. They draw models for given number sentences and create number sentences for given models. They create a graphic organizer to represent the sum of additive inverses using a variety of representations.

Grade 7 The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0.
   b. Understand $p + q$ as the number located a distance of $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Essential Ideas

- Opposite quantities in real-life situations combine to make 0.
- Two numbers with the sum of zero are called additive inverses.
- When two integers have the same sign and are added together, the sign of the sum is the sign of both integers.
- When two integers have the opposite sign and are added together, the integers are subtracted and the sign of the sum is the sign of the integer with the greater absolute value.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Creating Zero

Students write number sentences and draw number line models to illustrate how integers can sum to zero. They should notice that the numbers had to be opposites in order to have a sum of zero.

Develop

Activity 3.1: Additive Inverses

Students generate real-world examples in which opposite quantities combine to make 0. The additive inverse is defined as two numbers with the sum of zero. Two-color counters that represent positive charges (+) and negative charges (–) are introduced as a model for calculating the sum of two integers.

Activity 3.2: Adding Integers with Two-Color Counters

Students examine two worked examples using the two-color counter model and create an alternate model to represent the same sum. They analyze student reasoning about using the two-color counters. Then, students are given models and write number sentences to describe each model. Students then create models for each of several given number sentences.

Day 2

Activity 3.3: Rules for Adding Integers

Students use the examples from the lesson to write rules for the sign of the sum of integers and for the value of the sum of two integers. Then they sort number sentences that contain integers with absolute values larger than those in the prior problems according to the sign of their sums, and then they calculate the sums. Finally, students determine missing addends in integer addition problems.

Demonstrate

Talk the Talk: Summarizing Sums

Students create a graphic organizer to represent the sum of additive inverses by writing a number sentence in words, using a number line to model the integers, and using a two-color counter to model the integers. They also create their own number sentences, if possible, under given conditions.

Getting Started: Creating Zero

ENGAGE

Facilitation Notes

In this activity, number lines are used to model the sum of two numbers when the sum is equal to zero.

Have students work with a partner or in groups to complete Questions 1 through 3. Share responses as a class.

Differentiation strategy

Ask for volunteers to demonstrate their examples on the human number line (created for the previous lesson). Record the number sentences created by students on the board. Select a variety of examples: some that added a negative number to a positive, e.g., $2 + (-2)$, and some that added a positive number to a negative number, e.g., $(-2) + 2$.

Questions to ask

- In terms of walking the line, what would it mean for the sum to be zero?
- When using a number line, the starting place is always zero. How would someone end back at zero if they added two numbers?
- What do you notice about the numbers you added?
- What is the absolute value of each of the numbers you added?
- How did you create number sentences that summed to zero?
- What is the sum of any integer and its opposite?
- Why is the sum of any integer and its opposite always equal to zero?
- What is an example in real life of combining a number with its opposite?

Summary

The sum of a number and its opposite is zero.

Activity 3.1
Additive Inverses

DEVELOP

Facilitation Notes

In this activity, the additive inverse is defined and two-color counters are used as a model for calculating the sum of two integers.

Have a student read the definition of additive inverse and the introductory paragraph aloud. Review the worked example and complete Question 1 as a class.

Provide each student with a set of two-color counters. Allow them to lay out the counters and physically group them in the same ways as in the examples. Manipulation of concrete objects, when later linked to an abstract concept, strengthens student retention of the abstract concept.

Have students complete Question 2 independently. Have students share their responses with their partner or group.

Questions to ask

- Why are the two-color counters grouped differently in the bottom diagram?
- When computing the sum of two integers using a two-color counter model, if the sum is zero, what is true about the number of positive charges and the number of negative charges?
- How many different two-color counter models can you come up with that have a sum of zero?

Summary

Two-color counters can be used to model addition of signed numbers. Two numbers with the sum of zero are additive inverses.

Activity 3.2

Adding Integers with Two-Color Counters

Facilitation Notes

In this activity, the sum of two numbers is modeled using two-color counters. Students write number sentences for given models and draw models for given number sentences.

Review the first two worked examples as a class and complete Questions 1 and 2.

Differentiation strategies

- Provide students with sets of two-color counters to act out the examples. Students may find it helpful to remove pairs of positive and negative two-color counters from their desk model.
- If student sets of two-color counters are not available, students can write + and – signs in rows to model problems. For example, 5 + (−8) could be modeled as:

+ + + + +

− − − − − − − −

They could circle or cross out each positive and negative pair. The result is −3.

Questions to ask

- What part of the diagram gives you the answer?
- Is there another way you could have grouped the positive and negative charges?
- Is there another way you could have written the number sentence?
- How many different models representing a sum of –3 are possible?
- How can you quickly tell if your classmate's model is correct?
- When computing the sum of two integers using a two-color counter model, what are the steps?

Have students work with a partner or in groups to complete Questions 3 and 4. Share responses as a class.

Questions to ask

- Glancing quickly at a two-color counter model, how can you conclude the sum of the two integers will be negative?
- Glancing quickly at a two-color counter model, how can you conclude the sum of the two integers will be positive?
- What do all two-color counter models resulting in a negative sum have in common?
- What do all two-color counter models resulting in a positive sum have in common?
- Given a sum, how many two-color counter models can be created to represent the sum?
- Is there more than one number sentence that would represent this two-color counter model?
- How many pairs can you circle in this two-color counter model?
- If nothing can be paired in the two-color counter model, what does this mean?
- If nothing can be paired in the two-color counter model, what can you conclude about the sum of the integers?
- Can you determine the sign of the sum without circling the pairs in this two-color counter model?
- Can the two-color counter model be used to add more than two integers? If so, how?

Have students analyze Ava's and Landon's work. Ask students to work with a partner or in a group to complete Questions 5 through 7. Share responses as a class.

Ask students to work with a partner or in a group to complete Questions 8 through 10. Share responses as a class. Have students demonstrate each answer using their two-color counters.

Questions to ask

- How is calculating sums with two-color counters similar to calculating sums with a number line?
- What pattern(s) are you noticing?

Ask students to work with a partner or in a group to complete Questions 11 and 12. Share responses as a class.

Summary

Two-color counters and number lines can be used to model addition of signed numbers and write number sentences.

Activity 3.3
Rules for Adding Integers

Facilitation Notes

In this activity, students write rules for adding signed numbers. A sorting activity focuses on the sum of large integers. They also determine missing addends in integer addition problems.

Ask students to work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Is there another way to write this rule? If so, what is it?
- Will this rule work for all integers? Why or why not?
- How are these rules related to the patterns you noticed with adding integers on a number line in the previous activity?

Have students work with a partner or in a group to cut out the number sentence cards and complete Question 4. Note that students should not be calculating the sums until they have sorted them in part (b). They should be analyzing the addends to build their number sense and estimating skills. Then have students work with a partner or in groups to complete Question 5. Share responses as a class.

Questions to ask

- What rule was used to determine the sign of the sum in this situation?
- Why wouldn't it be practical to use a two-color counter model to compute this sum?
- Why wouldn't it be practical to use a number line model to compute this sum?

- Glancing at the number sentence, how can you quickly determine the sign of the sum?
- Is it easier to compute the sum or an addend? Why?

Summary

Rules can be used to determine the sum of integers. The sum of two negative numbers is always a negative number. The sum of two positive numbers is always a positive number. The sum of a positive number and negative number is the the sign of the largest number.

Talk the Talk: Summarizing Sums

DEMONSTRATE

Facilitation Notes

In this activity, a graphic organizer is used to represent the sum of additive inverses in multiple ways and students also write number sentences, given specific conditions.

Ask students to work with a partner or in a group to complete Questions 1 and 2. Share responses as a class. Encourage students to use models to justify their solutions in Question 2.

Differentiation strategy

For students who struggle, provide a number phrase, such as $-2 + 2$, to get them started.

Summary

Relationships between integers can be described using number sentences, real life situations, mathematical words, number line models, and two-color counter models.

Two-Color Counters

3

Adding Integers, Part II

WARM UP

Use a number line to determine each sum. Then write a sentence to describe the movement you used on the number line to compute the sum of the two integers.

1. $-2 + 1$
2. $-5 + 8$
3. $-2 + (-3)$
4. $4 + (-6)$

LEARNING GOALS

- Describe situations in which opposite quantities combine to make 0.
- Model the addition of integers using two-color counters.
- Develop a rule for adding integers.
- Apply previous understandings of addition and subtraction to add rational numbers.

KEY TERM

- additive inverses

You know how to use a number line to model adding positive and negative numbers. Do the patterns you noticed from the number line model apply to other models for adding positive and negative numbers?

Warm Up Answers

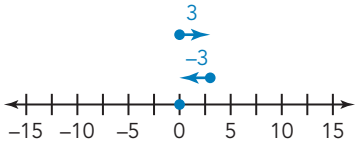
1. I started at zero, moved left 2 units to −2, then moved right 1 unit, to stop at −1.
2. I started at zero, moved left 5 units to −5, then moved right 8 units, to stop at 3.
3. I started at zero, moved left 2 units to −2, then moved left 3 units, to stop at −5.
4. I started at zero, moved right 4 units to 4, then moved left 6 units, to stop at −2.

LESSON 3: Two-Color Counters • M2-31

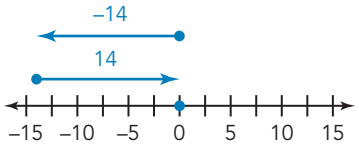
Answers

1. Answers will vary. Examples are provided.

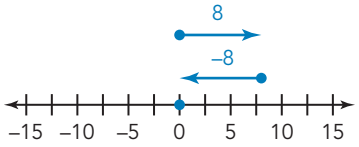
$3 + (-3) = 0$



$(-14) + 14 = 0$



$8 + (-8) = 0$



2. Sample answer. To form zero on the number line by adding two numbers, I added a number and its opposite.
3. Answers will vary.

Getting Started

Creating Zero

Use a number line to illustrate how the sum of two numbers can be zero.

1. **Write 3 examples of number sentences that sum to zero and draw the number line models to support your solutions.**

2. **What pattern do you notice?**

3. **Describe a real-life situation in which two numbers would sum to zero. Write the number sentence that could be used to represent the situation.**

M2-32 • TOPIC 1: Adding and Subtracting Rational Numbers

ACTIVITY
3.1 Additive Inverses

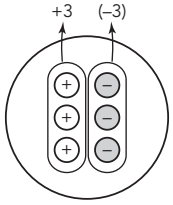
Addition of integers can also be modeled using two-color counters that represent positive (+) charges and negative (–) charges. One color, usually red, represents the negative number, or negative charge. The other color, usually yellow, represents the positive number, or positive charge. In this book, gray shading will represent the negative number, and no shading will represent the positive number.

Two numbers with the sum of zero are called **additive inverses.**

⊖ = –1 ⊕ = +1

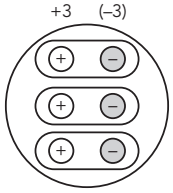
WORKED EXAMPLE

You can model the expression 3 + (–3) in different ways using two-color counters:



Three positive charges and three negative charges have no charge.

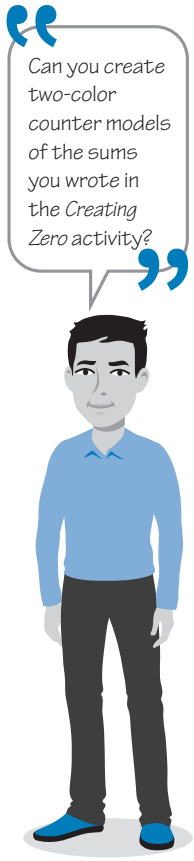
3 + (–3) = 0



Each positive charge is paired with a negative charge.

Each pair of positive and negative charges has no charge.

3 + (–3) = 0



LESSON 3: Two-Color Counters • M2-33

Answers

1. The value of each positive and negative pair is 0.
2. I can add the same number of positive counters and the same number of negative counters.

NOTES

1. **What is the value of each ⊕ and ⊖ pair in the second model?**

2. **Describe how you can change the numbers of ⊕ and ⊖ counters in the model but leave the sum unchanged.**

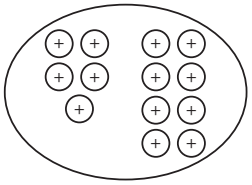
M2-34 • TOPIC 1: Adding and Subtracting Rational Numbers

ACTIVITY
3.2 Adding Integers with Two-Color Counters

Let's consider two examples where integers are added using two-color counters.

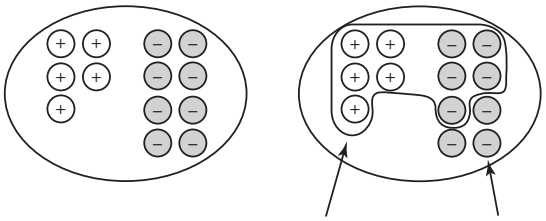
WORKED EXAMPLE

Example 1: 5 + 8



There are 13 positive counters in the model. The sum is 13.

Example 2: 5 + (−8)



There are five (+) (−) pairs. The value of those pairs is 0.

There are 3 (−), or negative counters, remaining.

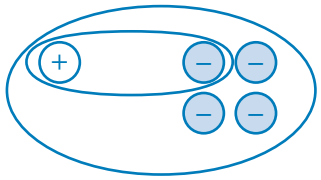
There are 3 negative counters remaining. The sum of 5 + (−8) is −3.

1. Create another model to represent a sum of −3. Write the appropriate number sentence.

2. Share your model with your classmates. How are they the same? How are they different?

Answers

1. Sample answer.
   1 + (−4)= −3



2. Sample answer.
   They are the same because each model represents a sum of −3, and each model had 3 more negative counters in it than positive counters.
   They are different because everyone chose different numbers to represent the positive and negative counters.

LESSON 3: Two-Color Counters • M2-35

Answers

3a. $6 + (-8) = -2$
or $-8 + 6 = -2$

3b. $-6 + 7 = 1$
or $7 + (-6) = 1$

3c. $-4 + 4 = 0$
or $4 + (-4) = 0$

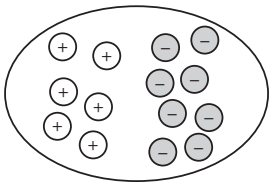
3d. $7 + (-3) = 4$
or $-3 + 7 = 4$

4. The order does not matter because of the Commutative Property of Addition.

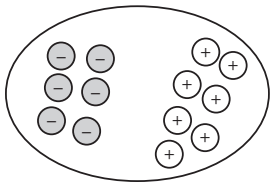
In an addition sentence, the terms being added together are called addends.

3. Write a number sentence to represent each model.

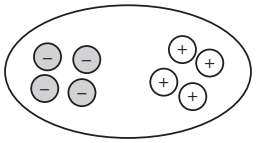
a.



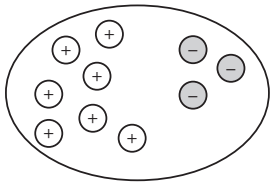
b.



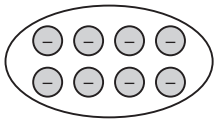
c.



d.



4. Does the order in which you wrote the integers in your number sentence matter? How do you know?



The students were then asked to write a number sentence for the given model.

Ava

$-8 + 0 = -8$

Landon

$-1 + (-7) = -8$

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5. Analyze the number sentences written by Ava and Landon.

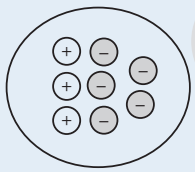
a. Explain why both number sentences are correct.

b. Write an additional number sentence that could describe the model.

6. Write each number sentence in Question 5 a second way.

Ava and Landon used two-color counters to represent the number sentence 3 + (−5).

7. The students placed the same counters on their desks, but they reported different sums. Ava reported the sum as 8 and Landon said the sum was −2. Use the model to explain who is correct. What was the error made by the incorrect student?



Answers

5a. Both number sentences are correct because we do not know what the original addends were. The addends could be any combination of negative numbers and zero that sum to −8.

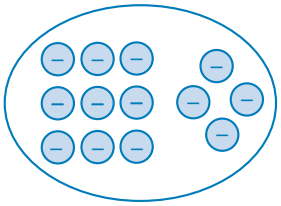
5b. Sample answers.
0 + (−8) = −8,
−2 + (−6) = −8

6. Sample answers based upon Question 5 answers.
−8 + 0 = −8,
−6 + (−2) = −8

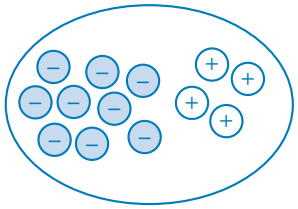
7. Landon is correct. Three pairs of the positive and negative counters, form sums of zero. This leaves two counters with negative charges remaining. So the sum is −2. Ava added all of the counters together, reporting the number of counters. She did not take into account that the signs on the counters/numbers were different.

Answers

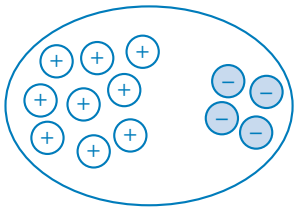
8a. −13



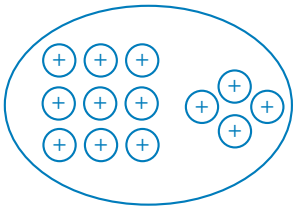
8b. −5



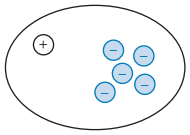
8c. 5



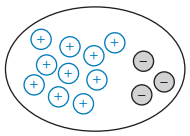
8d. 13



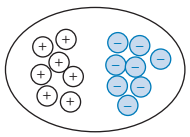
9a. −5



9b. 10



9c. −8



NOTES

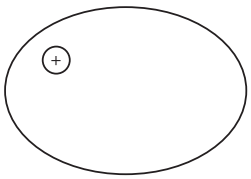
8. Draw a model for each, and then complete the number sentence.

a. −9 + (−4) = _____ **b. −9 + 4 = _____**

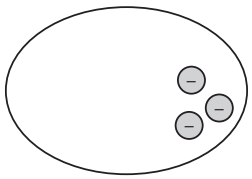
c. 9 + (−4) = _____ **d. 9 + 4 = _____**

9. Complete the model to determine the unknown integer.

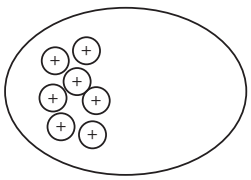
a. 1 + _____ = −4



b. −3 + _____ = 7



c. 7 + _____ = −1



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10. Describe the set of integers that makes each sentence true.

a. What integer(s) when added to −7 give a sum greater than 0?

b. What integer(s) when added to −7 give a sum less than 0?

c. What integer(s) when added to −7 give a sum of 0?

You have now used two models to represent adding integers.

11. For each problem, draw both models to represent the number sentences and determine the sums.

a. (−6) + 13

b. 8 + (−13)

c. (−3) + (−7)

d. 2 + 9

Think about how the absolute values of the addends compare with each other.

12. Explain the similarities and differences of the models in helping you determine the sum of two integers.

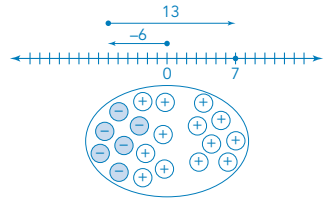
Answers

10a. Any integer greater than 7, or $n > 7$, will give a sum greater than 0 when added to −7.

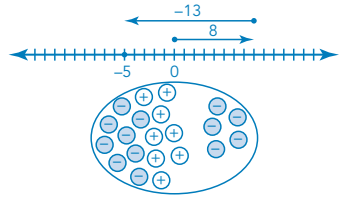
10b. Any integer less than 7, or $n < 7$, will give a sum less than 0 when added to −7.

10c. When $n = 7$, the sum is 0. That is, when 7 is added to −7, the sum is 0.

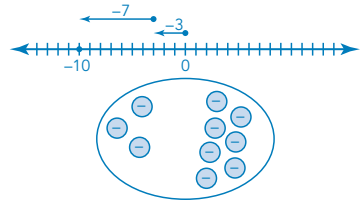
11a. 7



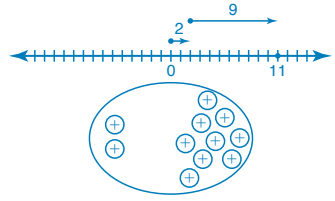
11b. −5



11c. −10



11d. 11



12. Sample answer. If one integer is positive and one integer is negative, the rays on the number line will overlap. This is similar to the counters because the "overlap" is the same as the number of pairs of positive and negative counters. To determine the answer from the counter model, I need to notice which counters remain after I make my pairs and then count how many there are. On the number line model, the final sum can be read directly off the number line.

Answers

1a. The sign of the sum will be positive.

1b. The sign of the sum will be negative.

1c. The sign of the sum will be the same as the sign of the integer with the greater absolute value, or the sign of the number that is a greater distance away from 0.

2. When both of the integers have the same sign, I add the integers and keep the sign of the numbers.

3. When the integers have opposite signs, I subtract the absolute value of the integer with the lesser absolute value from the absolute value of the integer with the greater absolute value and keep the sign of the integer with the greater absolute value.

ACTIVITY 3.3

Rules for Adding Integers

Visual models provide concrete representations of new ideas, like adding signed numbers. But you probably do not want to draw visual models when you have large numbers, lots of addends, fractions, or decimals.

Look back over the activities in this lesson and write rules for adding integers.

1. When adding two integers, what will the sign of the sum be if:

a. both integers are positive?

b. both integers are negative?

c. one integer is positive and one integer is negative?

2. Write a rule that states how to determine the sum of any two integers that have the same sign.

3. Write a rule that states how to determine the sum of any two integers that have opposite signs.

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Cut out the sums provided at the end of the lesson.

NOTES

4. Without computing the sums, sort the number sentences into two piles: those that have a positive sum and those that have a negative sum.

a. How can you decide which number sentences have a positive sum and which have a negative sum?

b. Tape or glue the number sentences in the space provided.

Positive Sums **Negative Sums**

c. Use your rules to determine the sum of each number sentence.

Answers

4a. Answers will vary.
If the signs were the same, the sum had the same sign as the numbers. If the signs were different, I thought about the absolute values of the numbers. If the positive number had a greater absolute value, the sum would be positive. If the negative number had a greater absolute value, the sum would be negative.

4b., 4c. See below.

4b., 4c.

Positive Sums	Negative Sums
$-48 + 60 = 12$	$-58 + 24 = -34$
$26 + (-13) = 13$	$-35 + (-15) = -50$
$67 + 119 = 186$	$-33 + (-12) = -45$
$153 + (-37) = 116$	$-105 + 25 = -80$
$18 + (-17) = 1$	$21 + (-56) = -35$

Answers

5a. 59
5b. −14
5c. −32
5d. −12
5e. −13
5f. −21

5. Determine each unknown addend.

a. _____ + (−25) = 34

b. _____ + 26 = 12

c. 8 + _____ = −24

d. −12 + _____ = −24

e. −15 + _____ = −28

f. _____ + 18 = −3

TALK the TALK

Summarizing Sums

1. Use the graphic organizer provided to represent additive inverses. Write an example, using both a number sentence and a real-life situation. Then represent your number sentence in words, using a number line model, and using a two-color counter model.

Answers

1. Answers will vary. An example is provided.

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EXAMPLE: NUMBER SENTENCE AND REAL-LIFE SITUATION

IN WORDS

ADDITIVE INVERSES AND ZERO

NUMBER LINE MODEL

TWO-COLOR COUNTER MODEL

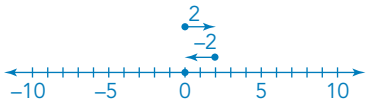
Answers

Sample answer.
Number Sentence and Real-Life:
$2 + (-2) = 0$ or $-2 + 2 = 0$.
If we have a rain deficit of 2 inches but it rains 2 inches this week, we will have a rain deficit of 0 inches.

In Words:
When I add any two opposite numbers, the sum is 0.

Number Line Model:

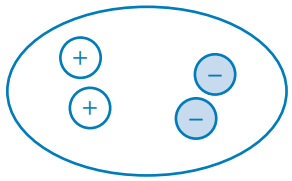
$2 + (-2) = 0$



In any number line model, when the distances of two numbers are equal but in opposite directions, the result is 0.

Two-Color Counter Model:

$2 + (-2) = 0$



In any two-color counter model, when there are the same number of positive and negative counters, the result is 0.

Answers

2a. Answers will vary.
$2 + 7 = 9$

2b. This is not possible.
Reasoning will vary.

2c. This is not possible.
Reasoning will vary.

2d. Answers will vary.
$-2 + (-7) = -9$

2e. Answers will vary.
$-2 + 7 = 5$

2f. Answers will vary.
$2 + (-7) = -5$

NOTES

2. Write a number sentence that meets the given conditions. If it is not possible to create the number sentence, explain why not.

a. Two positive addends with a positive sum.

b. Two positive addends with a negative sum.

c. Two negative addends with a positive sum.

d. Two negative addends with a negative sum.

e. A positive addend and a negative addend with a positive sum.

f. A positive addend and a negative addend with a negative sum.

M2-44 • TOPIC 1: Adding and Subtracting Rational Numbers

Rules for Adding Integers Cutouts

−58 + 24	−35 + (−15)
−33 + (−12)	−48 + 60
26 + (−13)	67 + 119
−105 + 25	153 + (−37)
21 + (−56)	18 + (−17)

LESSON 3: Two-Color Counters • M2-45

Assignment

Write

Define the term *additive inverse* in your own words.

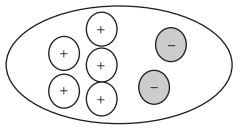
Remember

When two integers have the same sign and are added together, the sign of the sum is the sign of both integers.
When two integers have opposite signs and are added together, the absolute values of the integers are subtracted and the sign of the sum is the sign of the integer with the greater absolute value.

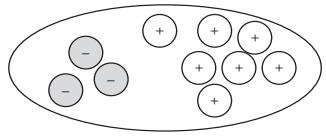
Practice

1. Write a number sentence for each two-color counter model. Then determine the sum.

a.

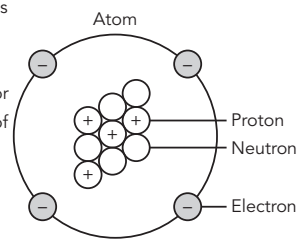


b.

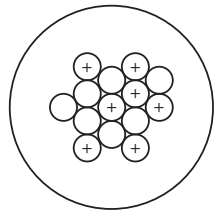


2. Draw a two-color counter model for each number sentence. Then determine the sum.
   a. 3 + (−6)
   b. −7 + (−4)
   c. 2 + 5
   d. 10 + (−8)

3. An atom is made up of protons, neutrons, and electrons. The protons carry a positive (+) charge and make up the nucleus of an atom with the neutrons. Neutrons do not carry a charge. The electrons carry a negative (−) charge and circle the nucleus. Atoms have no positive or negative charge. This means that they must have the same number of protons and electrons. A partial model of a nitrogen atom is shown.



   a. How many electrons should be drawn on the model of a nitrogen atom so that it has the same number of protons and electrons? How did you know?
   b. Complete the model of the nitrogen atom by drawing in the electrons.



   c. Write a number sentence to represent the sum of the number of protons and electrons in a nitrogen atom.
   d. Use a number line to show the sum of the number of protons and electrons in the nitrogen atom.

Determine each sum.

4. 45 + (−27)
5. 32 + (−98)
6. −153 + 74
7. −63 + (−41)
8. 527 + (−289)
9. −32 + 98
10. −47 + (−95)
11. −51 + 134

Assignment Answers

Write

Additive inverses are two numbers with the sum of zero.

Practice

1a. 5 + (−2) = 3 or −2 + 5 = 3

1b. −3 + 7 = 4 or 7 + (−3) = 4

2a. −3

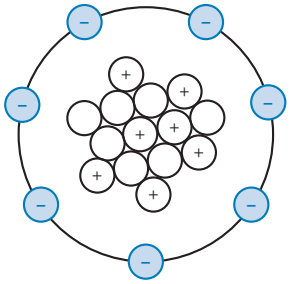
2b. −11

2c. 7

2d. 2

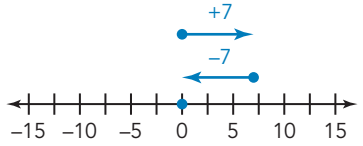
3a. There are 7 protons in a Nitrogen atom. So, 7 electrons need to be drawn on the model.

3b.



3c. 7 + (−7) = 0 or −7 + 7 = 0

3d.



4. 18
5. −66
6. −79
7. −104
8. 238
9. 66
10. −142
11. 83

Assignment Answers

Stretch

1. $-29\frac{7}{8}$
2. $37\frac{3}{5}$
3. 43.592
4. −160.23

Review

1. 1
2. −7
3. $259.25
4. $222.75
5. 13.5
6. 33.6

Stretch

Determine each sum.

1. $21\frac{3}{8} + \left(-51\frac{1}{4}\right)$
2. $-65\frac{2}{5} + 103$
3. −34.528 + 78.12
4. 863.78 + (−1024.01)

Review

Use a number line to determine each sum.

1. −3 + 4
2. −3 + (−4)

Calculate the sale price of each item.

3. A pair of headphones is on sale for 15% off the original price of $305.
4. A hoverboard is on sale for 10% off the original price of $247.50.

Solve each proportion.

5. $\frac{3}{4} = \frac{x}{18}$
6. $\frac{5}{8} = \frac{21}{x}$

M2-48 • TOPIC 1: Adding and Subtracting Rational Numbers

What's the Difference?

Subtracting Integers

4

MATERIALS

Marker
Masking tape
Two-color counters

Lesson Overview

Number lines and two-color counters are used to model subtraction of signed numbers. Through a series of activities, students will develop rules for subtracting integers. As in the lesson on adding signed numbers, the number line method is used to model the difference between two integers. Students then learn how to use zero pairs when performing subtraction using the two-color counter method. Students analyze real-world situations that require calculating the distance between two signed numbers. They build on what they already know about absolute value to determine the distance.

Grade 7 The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   c. Understand the subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Essential Ideas

- Subtraction of integers can be modeled using a number line and two-color counters.
- Subtracting two negative integers is similar to adding two integers with opposite signs.
- Subtracting a positive integer from a positive integer is similar to adding two integers with opposite signs.
- Subtracting a positive integer from a negative integer is similar to adding two negative integers.
- Subtracting two integers is the same as adding the opposite of the subtrahend, the number you are subtracting.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Take It Away

Students use number lines to model real-world situations involving subtraction.

Develop

Activity 4.1: Subtracting Integers on a Number Line

Students walk the line to model subtraction of positive and negative integers. After the kinesthetic activity with subtraction, examples of subtraction on pencil-and-paper number lines are provided and students explain the drawn models. They then create a model for a number sentence that describes a subtraction problem and calculate the difference. Next, students analyze number sentences to look for patterns. They determine unknown integers in number sentences and compute absolute values of differences.

Activity 4.2: Subtracting Integers with Two-Color Counters

The two-color counter model is used to compute the difference between two integers. Zero pairs are introduced to support subtracting a larger integer from a smaller integer. Examples of using this model are provided and students complete partially drawn models. They then create a model for a number sentence that describes a subtraction problem and calculate the difference.

Day 2

Activity 4.3: Analyzing Integer Subtraction

Students analyze number sentences to look for patterns. They compare two strategies for subtracting integers to realize that subtracting a number is equivalent to adding the opposite of the subtrahend. They determine unknown integers in number sentences.

Activity 4.4: Distance Between Rational Numbers

Students use a number line and absolute value to determine the distance between two numbers. They build on their understanding of absolute value from sixth grade to learn that the distance between two numbers on a number line is the absolute value of their difference.

Demonstrate

Talk the Talk: Determining the Difference

Students decide whether subtraction sentences are always true, sometimes true, or never true and use examples to justify their reasoning. Several questions focus students on the relationships between integer addition and subtraction. Finally, students write a rule for subtracting positive and negative integers.

Getting Started: Take It Away

ENGAGE

Facilitation Notes

In this activity, students use number lines to model real-world situations involving subtraction.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- Is the situation of owing money associated with a positive value or negative value?
- Is borrowing money associated with a positive value or negative value?
- Is taking away debt considered positive or negative?
- If it gets colder, does the temperature increase or decrease?
- If it gets warmer, does the temperature increase or decrease?
- Is money on a credit card associated with a positive value or negative value?
- Is purchases using a credit card associated with a positive value or negative value?
- Is digging a hole associated with a positive value or negative value?
- Is filling in a hole associated with a positive value or negative value?

Summary

Real-world situations can be modeled using subtraction problems.

Activity 4.1

Subtracting Integers on a Number Line

DEVELOP

Facilitation Notes

In this activity, walking a number line is used to model the subtraction of positive and negative integers. Students examine worked examples and use a number line to determine differences of signed numbers.

Ask a student to read the introduction aloud. Discuss how walking the line to model a subtraction problem differs from walking the line to model an addition problem. Select a student to model the first subtraction problem by walking the line. Complete Question 1 as a class. Select individual students to model each problem. Note there are empty spaces in the chart

so that teachers or students can make up additional problems to solve using the human number line.

Analyze the worked examples as a class and complete Questions 2 and 3.

Questions to ask

- How is Example 1 similar to Example 2?
- How is Example 1 different from Example 2?
- How is Example 3 similar to Example 4?
- How is Example 3 different from Example 4?
- What do all four examples have in common?

Have students work with a partner or in a group to complete Questions 4 and 5. It may benefit students to have them walk the human number line to determine a difference or to check an answer. Share responses as a class.

Questions to ask

- What is the first step toward solving this problem?
- Is there more than one way to begin solving this problem?
- Is the first step the same for a subtraction problem and an addition problem when using the number line model?
- What is the second step toward solving this problem?
- How is the second step different for subtraction, when comparing it to the second step you used when computing the sum of two integers, with respect to the number line model?
- How do you know if the arrows should be pointing in different directions when using the number line model?
- How do you know if the arrows should be pointing in the same direction when using the number line model?
- If the difference between the two integers is zero, how would you describe the arrows on the number line model?

Summary

Number lines can be used to model the subtraction of signed numbers.

Activity 4.2

Subtracting Integers with Two-Color Counters

Facilitation Notes

In this activity, the difference between two numbers is modeled using two-color counters. Students use zero pairs when subtracting integers with both positive and negative counters, and the take away model when

subtracting integers that are either all positive or all negative. They also draw models for given number sentences.

Ask a student to read the introduction aloud. Analyze the Examples 1 and 2 in the first worked example. Answer Question 1 as a class. Have a student read the information after Question 1 and review the definition of *zero pair*. Have students analyze Example 3 in the next worked example and answer Question 2 with their partner or in groups. Share results as a class.

Differentiation strategy

Provide students with sets of two-color counters to act out the examples. Students may find it helpful to remove pairs of positive and negative two-color counters from their desk model.

Questions to ask

- Does the first example show subtracting integers having the same sign?
- Does the second example show subtracting integers having the same sign?
- Does the first example use only positive integers or only negative integers?
- Does the second example use only positive integers or only negative integers?
- How do you know how many counters to start with, in the model?
- How do you know how many counters to take away, in the model?
- How do you know if you need to add zero pairs to the model?
- How many zero pairs can be added to the model?
- Is it possible to add too many zero pairs to the model? Explain.
- Is it possible to not add enough zero pairs to the model? Explain.
- What happens if you do not add enough zero pairs to the model?
- What happens if you add more zero pairs than you need to the model?
- How is taking away denoted in the model?
- Where is the answer or the difference between the two integers in the model?
- What is the difference between a subtraction sign and a negative sign?

Review Example 4 as a class and have students work in pairs or groups to complete Question 3.

Questions to ask

- Glancing quickly at a two-color counter model, how can you conclude the difference between the two integers will be negative?
- Glancing quickly at a two-color counter model, how can you conclude the difference between the two integers will be positive?

- What do all two-color counter models resulting in a negative difference have in common?
- What do all two-color counter models resulting in a positive difference have in common?
- Given a difference, how many two-color counter models can be created to represent the difference?
- Is there more than one number sentence that would represent this two-color counter model?
- How many pairs can you circle in this two-color counter model?
- If nothing can be paired in the two-color counter model, what does this mean?
- Can you determine the sign of the difference without circling the pairs in this two-color counter model?

Ask students to work with a partner or in a group to complete Questions 4 through 7. Share responses as a class.

Questions to ask

- Is there a difference between the integer 4 and −4? Or are they the same integer?
- Explain how the integers 4 and −4 are different, and how they are alike.
- Can this type of model be used to compute the difference for any subtraction problem?
- What is an example of a subtraction problem that would not be easily solved using this model? Explain.
- Can a subtraction problem have more than one correct answer?
- Can more than one subtraction problem give you the same answer?

Summary

Two-color counters can be used to model subtraction problems involving positive and negative integers.

Activity 4.3
Analyzing Integer Subtraction

Facilitation Notes

In this activity, students analyze number sentences to look for patterns. They compare two strategies for subtracting integers to realize that subtracting a number is equivalent to adding the opposite of the subtrahend. They determine unknown integers in number sentences.

Read the introduction aloud and have students analyze the series of number sentences. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- Using a number line, if you begin at −13 and go to −12, would this be considered an increase or a decrease?
- Using a number line, if you begin at −12 and go to −13, would this be considered an increase or a decrease?

Ask students read and compare Cara's and Neveah's methods, then complete Questions 2 and 3. Share responses as a class.

Questions to ask

- How is Cara's movement on the number line similar to Neveah's strategy?

Have students work with a partner or in a group to complete Questions 4 and 5, encouraging but not requiring them to use Neveah's strategy.

Questions to ask

- When you subtract a negative number from another number, is this the same as adding the number?
- How is subtracting a negative number similar to adding the number?
- Did you need a two-color counter model or a number line model to compute the difference?
- Why would it be impractical to use a two-color counter model to compute the difference?
- Why would it be impractical to use a number line model to compute the difference?
- How did you determine the unknown integer?
- Can you tell the sign of the unknown integer by looking at the problem?
- How can you tell the sign of the unknown integer by looking at the problem?
- Under what circumstance is the sign of the unknown integer negative? Positive?
- At a glance, can you tell if the difference between two integers is positive? Negative?
- At a glance, can you tell if the difference between two integers is zero?

Summary

Subtraction and addition are inverse operations, so subtracting a number, *a*, from another number, *b*, is the same as adding the opposite of *a* to *b*.

Activity 4.4
Distance Between Rational Numbers

Facilitation Notes

In this activity, students solve for total distance in real-world situations using two methods: by adding the absolute values of each number and by calculating the distance between two numbers on a number line (calculating the absolute value of the difference of the numbers).

Ask a student to read the information aloud and complete Question 1 as a class.

Questions to ask

- Is distance always a positive number? Why or why not?
- What is the height of the hill above ground level?
- How is the height of the hill above ground level represented on the number line?
- What is the depth of the tunnel below ground level?
- How is the depth of the tunnel below ground level represented on the number line?

Review Christian's and Mya's methods as a class. Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

Questions to ask

- How is the absolute value of a positive number determined?
- How is the absolute value of a negative number determined?
- How is the total vertical distance determined?
- Did Christian use subtraction to determine the total vertical distance?
- Did Mya use subtraction to determine the total vertical distance?
- How does Christian's answer compare to Mya's answer?
- Which method do you prefer? Why?
- What answer does Carson get using Christian's method?
- What answer does Carson get using Mya's method?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

Questions to ask

- What does BCE represent?
- What does CE represent?
- Is the integer that represents the start of the Olympic Games positive or negative?
- Is the integer that represents the end of the Olympic Games positive or negative?

- Is the integer that represents the start of the Modern Olympics positive or negative?
- How did you adjust your answer in part (c)?
- How was absolute value used to answer this question?
- What is the absolute value of −31?
- What is the absolute value of 79?

Summary

The distance between two numbers on the number line is the absolute value of their difference.

Talk the Talk: Determining the Difference

DEMONSTRATE

Facilitation Notes

In this activity, students answer questions related to the subtraction of integers. Generalizations are made and rules are written that involve subtraction of negative and positive integers.

Ask students to work with a partner or in a group to complete Questions 1 through 7. Encourage students to use the models to explain their thinking. Share responses as a class.

Questions to ask

- Is there another way to write this rule?
- What is an example that shows that a positive − positive is not a positive result?
- What is an example that shows that a negative − positive is a negative result?
- What is an example that shows that a negative − negative is not a negative result?
- Which number is considered the minuend in this situation?
- Which number is considered the subtrahend in this situation?

Summary

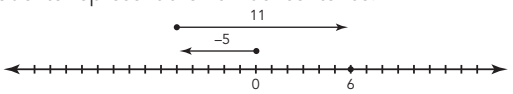
Subtracting two integers is the same as adding the opposite of the number you are subtracting.

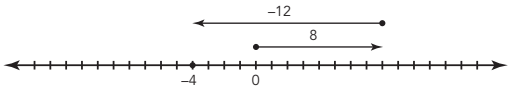
4 What's the Difference?

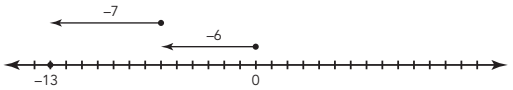
Subtracting Integers

WARM UP

For each number line model, write the number sentence described by the model and draw a two-color counter model to represent the number sentence.

1. 

2. 

3. 

LEARNING GOALS

- Model subtraction of integers on a number line.
- Model subtraction of integers using two-color counters.
- Develop a rule for subtracting integers.
- Apply previous understandings of addition and subtraction to subtract rational numbers.

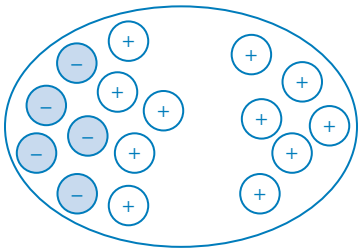
KEY TERM

- zero pair

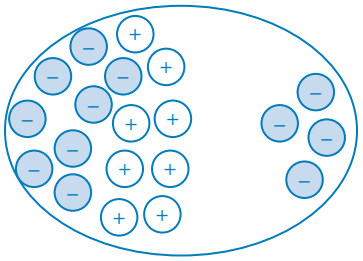
You have added integers using number lines and two-color counter models. How can you use these models to subtract integers?

Warm Up Answers

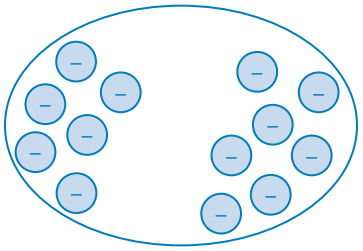
1. $(-5) + 11 = 6$



2. $8 + (-12) = -4$



3. $(-6) + (-7) = -13$



Answers

1a. Start at −10. Move to the left 5 units.

1b. Start at −10. Move to the right 5 units.

2a. Start at −7. Move to the left 12 units.

2b. Start at −7. Move to the right 12 units.

3a. Start at −65. Move to the right 24 units.

3b. Start at −65. Move to the left 24 units.

4a. Start at −20. Move to the right 6 units.

4b. Start at −20. Move to the left 4 units.

Getting Started

Take It Away

Each situation described has two different conclusions. Describe how you might model each on a number line.

1. You owe your friend $10.

a. You borrow an additional $5.

b. Your friend takes away $5 of that debt.

2. The temperature is −7°.

a. Overnight it gets 12° colder.

b. During the day, it gets 12° warmer.

3. You have charged $65 on a credit card.

a. You return an item purchased with that card that cost $24.

b. You purchase an additional item with that card that cost $24.

4. You dug a hole in the ground that is 20 inches deep. Your dog sees the pile of dirt and thinks it's a game.

a. He knocks 6 inches of dirt back into the hole.

b. He digs the hole another 4 inches deeper.

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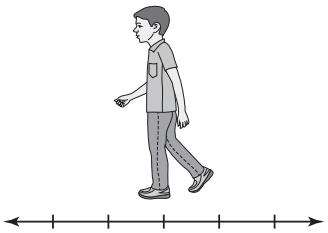
ACTIVITY
4.1

Subtracting Integers on a Number Line

Think about how you moved on the number line when you were learning to add positive and negative numbers in the previous lesson. Let's walk the line to generate rules for subtracting integers.

Walk the number line for a subtraction sentence:

- Start at zero and walk to the value of the first term of the expression.
- To indicate subtraction, turn to face down the number line, towards the lesser negative numbers.
- Walk forward if subtracting a positive number or walk backward if subtracting a negative number.



Your teacher will select a classmate to walk the line for each of the given problems. Help your classmate by preparing the directions that are needed.

1. Complete the table.

	Where You Start	Direction You Face on the Number Line	Walk Backwards or Forwards	Final Location
1 − 3				
0 − (−4)				
−3 − 5				
−1 − (−4)				

Answers

1. See table below.

1.

	Where You Start	Direction You Face on the Number Line	Walk Backwards or Forwards	Final Location
1 − 3	1	down	forwards	−2
0 − (−4)	0	down	backwards	4
−3 − 5	−3	down	forwards	−8
−1 − (−4)	−1	down	backwards	3

Answers

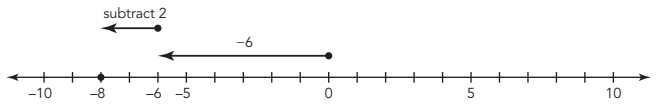
2. Cara went to 6. Because the problem says to subtract (−2), she went in the opposite direction of adding (−2), or to the right two units, and ended at 8.

Cara thought about how she could take what she learned from walking the line and create a number line model on paper. She said, "Subtraction means to move in the opposite direction."

Analyze Cara's examples.

WORKED EXAMPLE

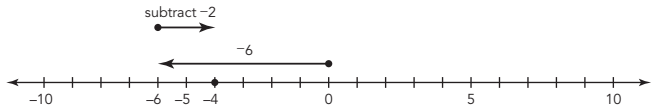
Example 1: −6 − (+2)



First, I moved to −6. Then, I went in the opposite direction of adding (+2) because I am subtracting (+2). So, I went two units to the left and ended up at −8.

−6 − (+2) = −8

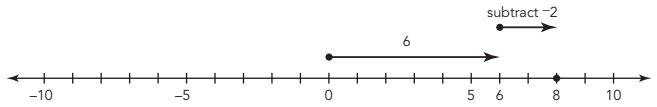
Example 2: −6 − (−2)



In this problem, I started by moving to −6. Because I am subtracting (−2), I went in the opposite direction of adding −2. So, I moved right two units and ended up at −4.

−6 − (−2) = −4

Example 3: 6 − (−2)



2. 6 − (−2) = _____

Explain the movement Cara modeled on the number line to determine the answer.

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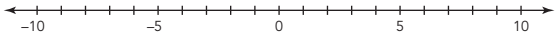
Example 4: 6 − (+2)

3. 6 − (+2) = _____

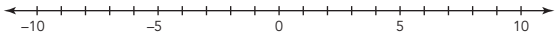
Explain the movement Cara modeled on the number line to determine the answer.

4. Use the number line to complete each number sentence.

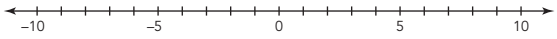
a. −4 − (−3) = _____



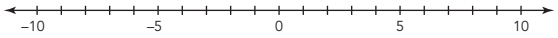
b. −4 − (−4) = _____



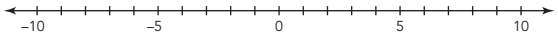
c. −4 − 3 = _____

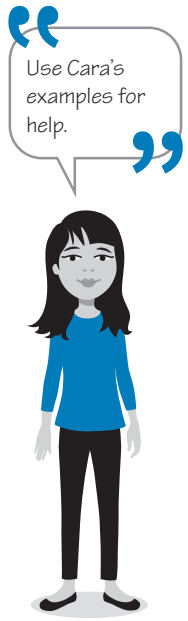


d. −4 − 4 = _____



e. 4 − (−3) = _____

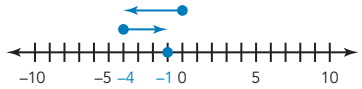




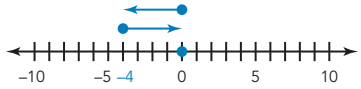
Answers

3. Cara went to 6. Because the problem says to subtract (+2), she went in the opposite direction of adding (+2), or to the left two units, and ended at 4.

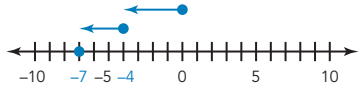
4a. −1



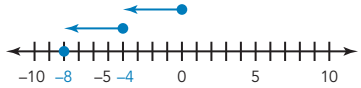
4b. 0



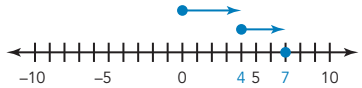
4c. −7



4d. −8

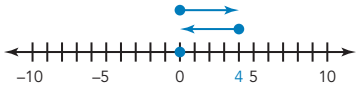


4e. 7

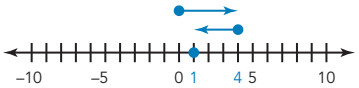


Answers

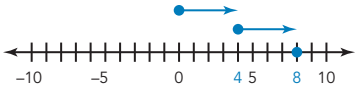
4f. 0



4g. 1



4h. 8



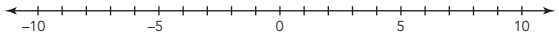
5a. adding two integers with opposite signs

5b. adding two integers with opposite signs

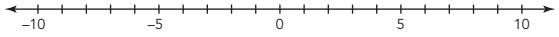
5c. adding two negative integers

5d. adding two positive integers

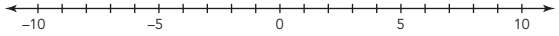
f. 4 – 4 = _____



g. 4 – 3 = _____



h. 4 – (–4) = _____



5. What patterns did you notice when subtracting the integers in Question 4? Describe an addition problem that is similar to each subtraction problem.

a. Subtracting two negative integers

b. Subtracting two positive integers

c. Subtracting a positive integer from a negative integer

d. Subtracting a negative integer from a positive integer

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ACTIVITY
4.2

Subtracting Integers with Two-Color Counters

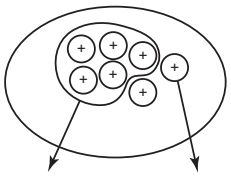
The number line model and the two-color counter model used in the addition of integers can also be used to investigate the subtraction of integers.

Subtraction can mean to "take away" objects from a set. Subtraction can also mean a comparison of two numbers, or the "difference between them."

WORKED EXAMPLE

Using just positive or just negative counters, you can show subtraction using the "take away" model.

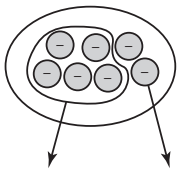
Example 1: $7 - 5$
First, start with seven positive counters.



Then, take away five positive counters. Two positive counters remain.

$$7 - 5 = 2$$

Example 2: $-7 - (-5)$
First, start with seven negative counters.



Then, take away five negative counters. Two negative counters remain.

$$-7 - (-5) = -2$$

1. **How are Examples 1 and 2 similar? How are these examples different?**

Answers

1. Both examples show subtracting integers with the same sign. Example 1 shows the difference between two positive integers and Example 2 shows the difference between two negative integers.

LESSON 4: What's the Difference? • M2-55

Answers

2. The total number of counters in the model changed but the value inside the model has not changed.

To subtract integers using both positive and negative counters, you will need to use *zero pairs*.

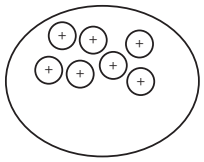
$$\oplus + \ominus = 0$$

Recall that the value of a $\ominus$ and $\oplus$ pair is zero. So, together they form a **zero pair**. You can add as many pairs as you need and not change the value.

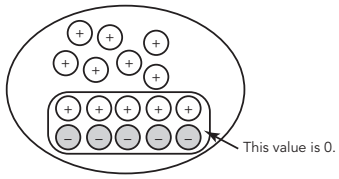
WORKED EXAMPLE

Example 3: $7 - (-5)$

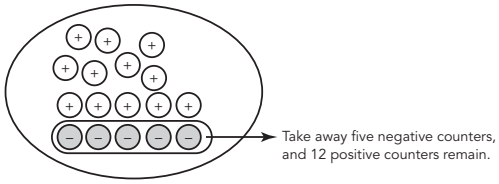
Start with seven positive counters.



The expression says to subtract five negative counters, but there are no negative counters in the first model. Insert five negative counters into the model. So that you don't change the value, you must also insert five positive counters.



Now, you can subtract, or take away, the five negative counters.



$7 - (-5) = 12$

2. Why is the second model equivalent to the original model?

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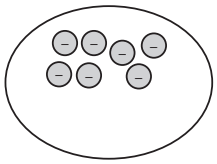
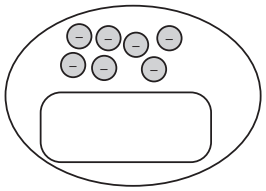
Example 4: −7 − 5
Start with seven negative counters.



3. The expression says to subtract five positive counters, but there are no positive counters in the first model.

a. How can you insert positive counters into the model and not change the value?

b. Complete the model.

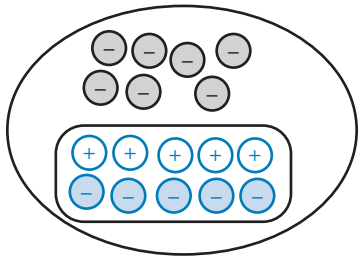


c. Now, subtract, or take away, the five positive counters. Determine the difference.

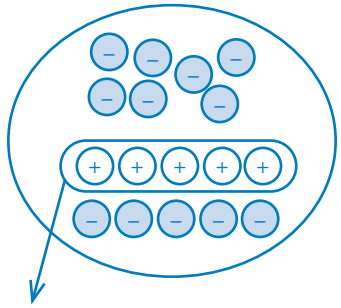
Answers

3a. I can insert five positive counters and five negative counters and not change the value of the model.

3b.



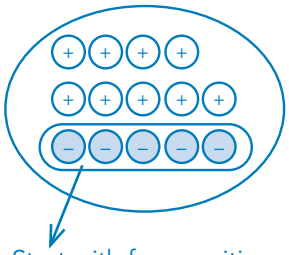
3c.



Remove the five positive counters, and 12 negative counters remain.

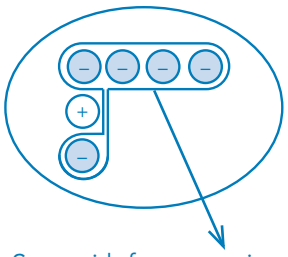
Answers

4a.



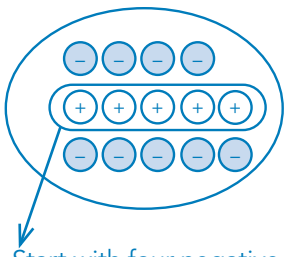
Start with four positive counters, and add five zero pairs. Then, subtract five negative counters. The result is nine positive counters.

4b.



Start with four negative counters, and add one zero pair. Then, subtract five negative counters. The result is one positive counter.

4c.



Start with four negative counters, and add five zero pairs. Then, subtract five positive counters. The result is nine negative counters.

4. Draw a representation for each subtraction problem. Then, calculate the difference.

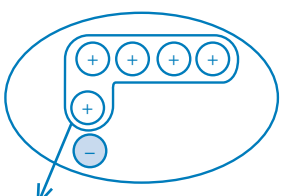
a. 4 − (−5)

b. −4 − (−5)

c. −4 − 5

d. 4 − 5

4d.



Start with four positive counters, and add one zero pair. Then, subtract five positive counters. The result is one negative counter.

NOTES

5. How could you model 0 − (−7)?

   a. Draw a sketch of your model. Then, determine the difference.

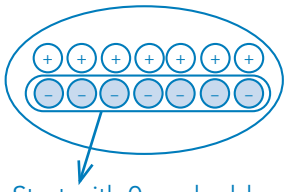
   b. In part (a), does it matter how many zero pairs you add? Explain your reasoning.

6. Does the order in which you subtract two numbers matter? Draw models and provide examples to explain your reasoning.

7. Are the rules you wrote at the end of the previous activity true for the two-color counter models? What else did you learn about subtracting integers?

Answers

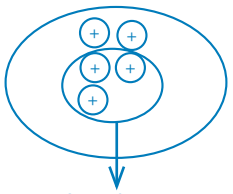
5a.



Start with 0, and add seven zero pairs. Then, subtract seven negative counters. The result is seven positive counters.

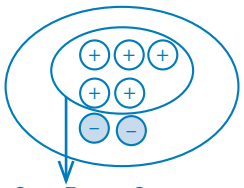
5b. It would not matter how many zero pairs I add. Once I remove the seven negative counters and have seven remaining positive counters, it does not matter how many additional pairs of positive and negative counters are left because their value is zero.

6. Subtraction is not commutative, so the order matters.



5 − 3 = 2

In this expression, I would start with five positive counters and subtract three. The result is two positive counters.



3 − 5 = −2

In this expression, I would start with three positive counters and add two zero pairs. Then, I would subtract five positive counters. The result is two negative counters.

7. Answers will vary.

Answers

1a. As the integer subtracted from 8 decreases, the result increases.

1b. −7

ACTIVITY 4.3

Analyzing Integer Subtraction

You probably have noticed some patterns when subtracting signed numbers on the number line and with two-color counters. Let's explore these patterns to develop a rule.

$-8 - 5 = -13$
$-8 - 4 = -12$
$-8 - 3 = -11$
$-8 - 2 = -10$
$-8 - 1 = -9$
$-8 - 0 = -8$

1. Analyze the number sentences shown.

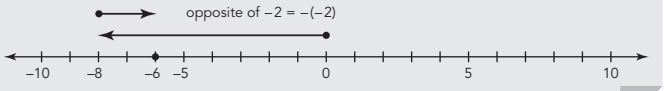
a. What patterns do you see? What happens as the integer subtracted from −8 decreases?

b. From your pattern, predict the answer to −8 − (−1).

Consider the subtraction expression −8 − (−2).

Cara's Method

Start at −8. Since I'm subtracting, you go in the opposite direction of adding (−2), which means I go to the right 2 units. The answer is −6.



Neveah's Method

I see another pattern. Since subtraction is the inverse of addition, you can think of subtraction as adding the opposite number.

−8 − (−2) is the same as −8 + (+2)

−8 + 2 = −6

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2. How is Neveah's method similar to Cara's method?

3. Use Neveah's method to fill in each blank.

   10 − (−4) = 10 + (_____) = _____

4. Determine each difference.

   a. −9 − (−2) =

   b. −3 − (−3) =

   c. −7 − 5 =

   d. 24 − 8 =

   e. −4 − 2 =

   f. 5 − 9 =

   g. −20 − (−30) =

   h. −10 − 18 =

5. Determine the unknown integer in each number sentence.

   a. 3 + _____ = 7

   b. 2 + _____ = −7

   c. _____ + −20 = −10

   d. _____ − 5 = 40

   e. _____ − (−5) = 40

   f. _____ + 5 = 40

   g. 6 + _____ = 52

   h. −6 + _____ = 52

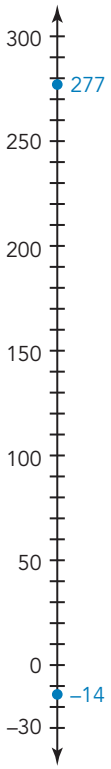
   i. −6 + _____ = −52

Answers

2. Answers will vary.
3. 10 − (−4) = 10 + (+4) = 14
   10 − (−4) = 14
4a. −7
4b. 0
4c. −12
4d. 16
4e. −6
4f. −4
4g. 10
4h. −28
5a. 4
5b. −9
5c. 10
5d. 45
5e. 35
5f. 35
5g. 46
5h. 58
5i. −46

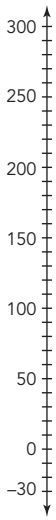
Answers

1.



ACTIVITY 4.4

Distance Between Rational Numbers



Amusement parks are constantly trying to increase the level of thrills on their rides. One way is to make the roller coasters drop faster and farther. A certain roller coaster begins by climbing a hill that is 277 feet above ground. Riders go from the top of that hill to the bottom, which is in a tunnel 14 feet under ground, in approximately 3 seconds!

Determine the vertical distance from the top of the roller coaster to the bottom of the tunnel.

1. Plot the height and depth of the first hill of the roller coaster on the number line.

Consider Christian's and Mya's methods for determining the vertical distance.

Christian

In sixth grade, I learned that you could add the absolute values of each number to calculate the distance.

$$|277| + |-14| = 277 + 14$$

$$= 291$$

The vertical distance is 291 feet.

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Mya

I learned in elementary school that the difference between two numbers on a number line can be determined with subtraction. Because absolute value measures distance, I need the absolute value of the difference.

$$|277 - (-14)| = |277 + (+14)|$$
$$= |291|$$
$$= 291$$

The vertical distance is 291 feet.

2. Describe how Christian and Mya used absolute value differently to determine the vertical distance from the top of the roller coaster to the bottom of the tunnel.

3. Carson wonders if order matters. Instead of calculating the distance from the top to the bottom, he wants to calculate the vertical distance from the bottom to the top. Is Carson correct? Determine if Carson is correct using both Christian's strategy and Mya's strategy.

Answers

2. Christian took the absolute value of each number and then added them together. Mya took the absolute value of the difference between the two numbers.
3. Yes, Carson is correct.

   Christian's strategy: $|-14| + |277| = 14 + 277 = 291$

   Mya's strategy: $|-14 - 277| = |-291| = 291$

LESSON 4: What's the Difference? • M2-63

Answers

4a. Start: −776

End: 393

4b. |−776 − 393| = |−1169| = 1169; 1169 years

4c. |−776 − 1896| = |−2672| = 2672

4d. There were 2671 years between the start of the Ancient Olympics and the start of the Modern Olympics.

5. |−31 − 79| = |−110| = 110

The difference between the low and high temperatures was 110 degrees.

As demonstrated in Mya's strategy, the distance between two numbers on the number line is the absolute value of their difference. Use Mya's strategy to solve each problem.

4. The first recorded Olympic Games began in 776 BCE. Called the *Ancient Olympics*, games were held every four years until being abolished by Roman Emperor Theodosius I in 393 CE.

a. Represent the start and end years of the Ancient Olympic Games as integers.

b. Determine the length of time between the start and end of the Ancient Olympic Games.

c. Determine the length of time between the start of the Ancient Olympics and the Modern Olympics, which began in 1896.

d. If you research the ancient calendar, you will learn that there actually was no Year 0. The calendar went from −1 BCE to 1 CE. Adjust your answer from part (c) to account for this.

5. On February 10, 2011, the temperature in Nowata, OK, hit a low of −31°. Over the course of the next week, the temperature increased to a high of 79°. How many degrees different was the low from the high temperature?

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NOTES

TALK the TALK

Determining the Difference

Use what you have learned about adding and subtracting with integers to think about patterns in addition and subtraction.

1. **Determine whether these subtraction sentences are always true, sometimes true, or never true. Give examples to explain your thinking.**

   a. **positive − positive = positive**

   b. **negative − positive = negative**

   c. **positive − negative = negative**

   d. **negative − negative = negative**

2. **If you subtract two negative integers, will the answer be greater than or less than the number you started with? Explain your thinking.**

3. **What happens when a positive number is subtracted from zero?**

Answers

1a. Sometimes true. $10 - 4 = 6$ but $4 - 10 = -6$

1b. Always true. $-10 - 4 = -14$

1c. Never true. $10 - (-2) = 12$ or $2 - (-10) = 12$

1d. Sometimes true. $-5 - (-2) = -3$ but $-2 - (-5) = 3$

2. The answer will be greater than the minuend, the number you started with. For example, $-10 - (-1) = -9$ and -9 is greater than -10.

3. The result will be the opposite of the subtrahend, the number you subtracted from zero. It will be the negative of that number.

Answers

4. The result will be the opposite of the subtrahend, the number you subtracted from zero. It will be the positive of that number.
5. If both integers are positive, then the result is positive. If both numbers are negative, then the result is negative. If the numbers are opposites, then the result is zero. If you are adding two integers with different signs, then the sign of the number with the greater absolute value determines the sign of the result.
6. Subtracting two integers is the same as adding the opposite of the number you are subtracting.
7. Answers will vary.

NOTES

4. What happens when a negative number is subtracted from zero?

5. Just by looking at the problem, how do you know if the sum of two integers is positive, negative, or zero?

6. How are addition and subtraction of integers related?

7. Write a rule for subtracting positive and negative integers.

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Assignment

Write

Define the term *zero pair* in your own words.

Remember

You can change any subtraction problem to an addition problem without changing the answer. Subtracting two integers is the same as adding the opposite of the subtrahend, the number you are subtracting.

Practice

1. Draw both a model using two-color counters and a model using a number line to represent each number sentence. Then, determine the difference.
   a. $-8 - (-5)$
   b. $-4 - 9$
   c. $2 - (-8)$
   d. $3 - 12$
2. Determine each difference without using a number line.
   a. $7 - (-13)$
   b. $10 - (-1)$
   c. $-16 - 3$
   d. $-9 - 7$
   e. $-1 - (-2)$
   f. $-5 - (-5)$
   g. $19 - (-19)$
   h. $-8 - (-8)$
   i. $40 - (-20)$
   j. $-800 - (-300)$
3. The highest temperature ever recorded on Earth was 136° F at Al Aziziyah, Libya, in Africa. The lowest temperature ever recorded on Earth was −129° F at Vostok Station in Antarctica. Plot each temperature as an integer on a number line, and use absolute value to determine the difference between the two temperatures.
4. The highest point in the United States is Mount McKinley, Alaska, at about 6773 yards above sea level. The lowest point in the United States is the Badwater Basin in Death Valley, California, at about 87 yards below sea level. Plot each elevation as an integer on a number line, and use absolute value to determine the number of yards between in the lowest and highest points.

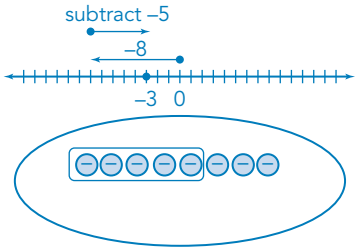
Assignment Answers

Write

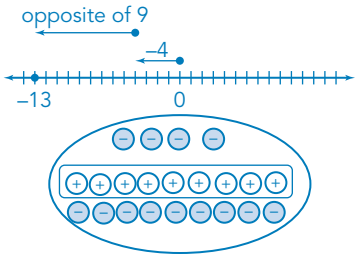
Using two-color counters, a zero pair is a pair consisting of one positive counter and one negative counter. The sum of the counters in a zero pair is equal to zero.

Practice

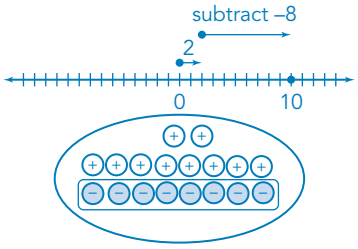
1a. $-8 - (-5) = -3$



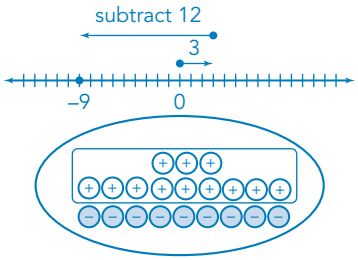
1b. $-4 - {}^{+}9 = -13$



1c. $2 - (-8) = 10$



1d. $3 - {}^{+}12 = -9$



2a. 20
2b. 11
2c. −19
2d. −16
2e. 1
2f. 0
2g. 38
2h. 0
2i. 60
2j. −500

3. Check students' number lines.
   $|-129 - 136| = 265$
   There are 265 degrees between the lowest and highest recorded temperatures.

4. Check students' number lines.
   $|-87 - 6773| = 6860$
   There are 6860 yards between the lowest and highest points in the U.S.

Assignment Answers

Stretch

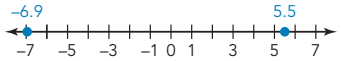
1a. 6.4

1b. −17.1

1c. −3.1

1d. 51.1

2. There are 12.4 miles between the deepest point in the ocean and the highest point in the world.



Review

1a. The height of the clay model was 50.2 inches.

1b. The height of the model was 100.4 inches.

2a. $\frac{3 \text{ mistakes}}{50 \text{ words}} = \frac{x}{300 \text{ words}}$,

$x = 18$

Tommy would be expected to make 18 mistakes when typing 300 words.

2b. $\frac{6 \text{ cans}}{2.50 \text{ dollars}} = \frac{72 \text{ cans}}{x}$,

$x = 30$

Ned will spend $30.00 to buy 72 cans of fruit juice.

3a. $x = 17$

3b. $x = 75$

Stretch

1. Determine each difference without using a number line.
   a. $3.1 - (-3.3)$
   b. $-8.3 - 8.8$
   c. $42.5 - 45.6$
   d. $-28.4 - (-79.5)$
2. The deepest point in the ocean is the Marianas Trench in the Pacific Ocean at about 6.9 miles below sea level. The highest point in the world is Mount Everest in the Himalayan Mountains at about 5.5 miles. Plot each elevation as an rational number on a number line, and use absolute value to determine the number of miles between the deepest point in the ocean and the highest point in the world.

Review

1. The city of Nashville, Tennessee, constructed an exact replica of the Parthenon. In 1982, construction began on a sculpture of Athena Parthenos, which stands 41 feet 10 inches tall.
   a. The sculptor first made a 1 : 10 model from clay. This means that 1 inch on the model is equal to 10 inches on the real statue. What was the height of the clay model?
   b. Later the sculptor made a 1 : 5 model. This means that 1 inch on the model is equal to 5 inches on the real statue. What was the height of the model?
2. Write and solve a proportion to answer each problem. Show all your work.
   a. Tommy types 50 words per minute, with an average of 3 mistakes. How many mistakes would you expect Tommy to make if he typed 300 words? Write your answer using a complete sentence.
   b. Six cans of fruit juice cost $2.50. Ned needs to buy 72 cans for a camping trip for the Outdoor Club. How much will he spend?
3. Solve each equation for x.
   a. $72 = 55 + x$
   b. $\frac{4}{5}x = 60$

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