Date____________________

The Number System:

Types of numbers:

Real numbers: The combined sets of rational and irrational numbers.

Rational Numbers: Any number that can be written as the ratio of two integers in the form $\frac{a}{b}$, where $b \neq 0$.

***Numbers that can be written as fractions.**
(The denominator cannot be zero.)

- **Counting Numbers or Natural Numbers:** Whole numbers greater than zero. Ex. 1, 2, 3, 4, 5 …
- **Whole Numbers:** The set of counting numbers plus zero. Ex. **0**, 1, 2, 3, 4 …
- **Integers:** The set of whole numbers and their opposites. *or* Positive and negative whole numbers.
  Ex. …-3, -2, -1, 0, 1, 2, 3…
- **Fraction**: A number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$.
- **Terminating decimal**: A decimal that terminates or stops. *Ex.* 5.46
- **Repeating decimal**: A decimal in which a digit or sequence of digits keeps repeating.
  Use a bar over the digits to show that they repeat.
  Ex. 356.23232323… = $356.\overline{23}$
  0.7644444444… = $0.76\overline{4}$

Irrational Numbers: Non-repeating, non-terminating decimals.

- **Non-repeating decimal**: A number containing a decimal extension that does not contain a repeating pattern. The decimal extension can be a random series of numbers.

  Ex. 2.48265036512396028 46…

We use three dots to symbolize that the decimal representation continues.

AND

- **Non-Terminating decimal**: A decimal that does not terminate or stop.

  Ex. 67.8998…

Remember, an irrational number is non-repeating, non-terminating!!!

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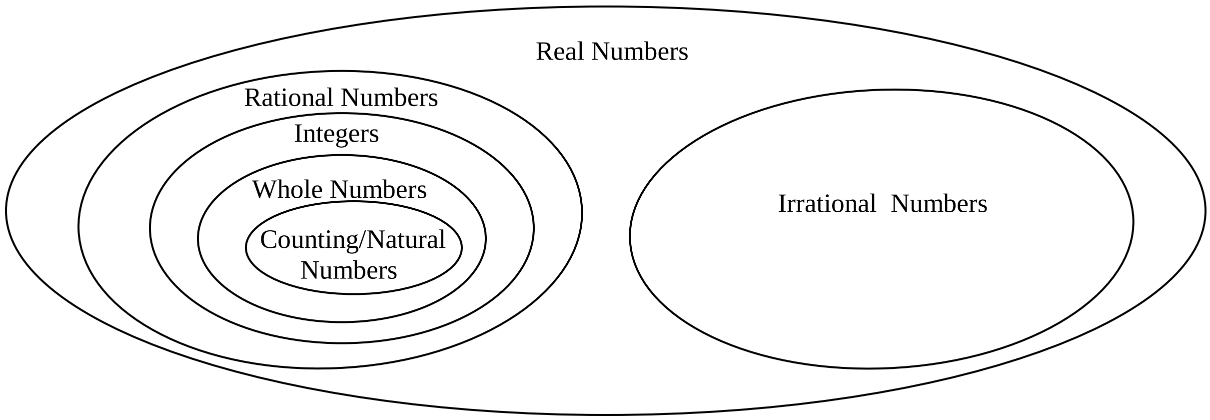
Venn Diagram of types of numbers:



The following are examples of **rational numbers**:

(*numbers that can be written as fractions*)

7, -34, 0.75, ½, $\sqrt{25}$, 0.0043, $.\overline{82}$ (.82828282…)

Whole number: can be written: $\frac{7}{1}$

Integer: can be written: $-\frac{34}{1}$

Terminating decimal: can be written: $\frac{75}{100} = \frac{3}{4}$

Fraction:

$\sqrt{25} = 5$ (the square root of 25 = 5) 5 is a **whole number:** can be written $\frac{5}{1}$

Terminating decimal: Can be written: $\frac{43}{10000}$

Repeating decimal: Part or all of the decimal extension will repeat in a pattern: can be written $\frac{82}{99}$

The following are examples of **irrational numbers:**

$\sqrt{2}$ 0.141141114… π

$\sqrt{2} = 1.414213562…$ Decimal does **not** terminate or repeat.

Decimal does **not** terminate or repeat.

Pi ≈ 3.14159… Decimal does **not** terminate or repeat.

Date__________________

Determine if the following are repeating or non-repeating decimal; then rational or irrational.

		Ex. Repeating	Ex. Rational
1.	73.022222…	____________________	____________________
2.	12.956489564895648…	____________________	____________________
3.	8.7932547560231145…	____________________	____________________
4.	1,654.0000141414…	____________________	____________________

Determine if the following are terminating or non-terminating; then rational or irrational.

		Ex. Terminating	Ex. Rational
5.	16.76443233238	____________________	____________________
6.	17.987…….	____________________	____________________
7.	985.6324…..	____________________	____________________
8.	87.6	____________________	____________________

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Rational Number Operations:

Fractions:

- **Fraction:** The quotient of two numbers or expressions.

- **Proper faction:** A fraction that is less than 1. The numerator is less than the denominator.

  Ex. $\frac{7}{8}$

  Numerator: Dividend.

  Denominator: Divisor.

- **Improper fraction:** A fraction that is greater than 1. The numerator is greater than the denominator. *Ex.* $\frac{13}{5}$

- **Mixed numbers:** A whole number with a fraction. *Ex.* $4\frac{6}{7}$

- **Equivalent fractions:** Two or more fractions with the same value. One fraction can be reduced to the other fraction.

Determining if fractions are equivalent by finding the cross products:

1. Find the cross products. Multiply on the diagonal from bottom to top.
2. Record the products **above the numerators**.
   - If the products are **equal**, the fractions are equivalent.
   - If the products are **not equal**, the fractions are not equivalent.

Ex.) $\overset{30}{\frac{3}{5}} \overset{=}{?} \overset{30}{\frac{6}{10}}$ These fractions are equivalent.

Ex.) $\overset{40}{\frac{2}{5}} \overset{\neq}{?} \overset{50}{\frac{10}{20}}$ These fractions are **not** equivalent.

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Determine if the following fractions are equivalent or not equivalent. ***Show work.***

Ex. 1) $\frac{5}{10}$? $\frac{10}{20}$ ____________________________

Ex. 2) $\frac{8}{10}$? $\frac{12}{15}$ ____________________________

Ex. 3) $\frac{2}{7}$? $\frac{5}{9}$ ____________________________

Finding a missing number in a pair of equivalent fractions.

1. Find the cross product for the two known numbers.
2. Divide the product by the remaining number.
3. The quotient is the missing number.

$84 \div 7 = 12$

Ex. $\frac{4}{7} = \frac{n}{21}$ $n = 12$

Determine the value of the missing number. ***Show work.***

Ex. 1) $\frac{4}{6} = \frac{10}{n}$ $n =$ _______

Ex. 2) $\frac{5}{8} = \frac{n}{12}$ $n =$ _______

Ex. 3) $\frac{6}{n} = \frac{24}{28}$ $n =$ _______

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Reducing fractions:

*Fractions must always be in **lowest terms**.
A fraction is in **lowest terms if the numerator and denominator are **relatively prime** (Their greatest common factor is 1.)

To reduce fractions choose one of the following methods:

A. Reducing by dividing

1. Find the greatest common factor of the numerator and denominator.
2. Divide the numerator by the GCF.
3. Divide the denominator by the GCF.

You may divide by any common factor, however, if you don't use the GCF, you will need to repeat until there are no common factors remaining.

Ex: $\frac{100}{120} \frac{\div 20}{\div 20} = \frac{5}{6}$

B. Prime factorization

1. Give the prime factors for the numerator and denominator (these can be found by making the factor tree).
2. Cross out any factors that are common to both the numerator and the denominator.
3. If there are more than one number left in either the numerator or the denominator, multiply them together.

Ex: $\frac{100}{120} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{5} \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 3 \cdot \cancel{5}} = \frac{5}{6}$

Reduce the fractions – choose either method.

Ex. 1) $\frac{45}{60}$ Ex. 2) $\frac{27}{90}$ Ex. 3) $\frac{125}{300}$

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Adding/Subtracting Fractions:

Follow these steps when adding/subtracting fractions:

Add: $\frac{1}{2} + \frac{3}{5} =$ | *Subtract*: $\frac{11}{15} - \frac{2}{3} =$

Step 1: Write the fractions with common denominator. Use equivalent fractions.

$\frac{5}{10} + \frac{6}{10} =$ | $\frac{11}{15} - \frac{10}{15} =$

Step 2: Add or subtract the numerators. Keep the denominator.

$\frac{5}{10} + \frac{6}{10} = \frac{11}{10}$ | $\frac{11}{15} - \frac{10}{15} = \frac{1}{15}$

Step 3: Reduce if necessary.

$\frac{11}{10} = 1\frac{1}{10}$

Follow these same steps when adding/subtracting mixed numbers. There is no need to change mixed numbers to improper fractions.

Ex. *Add:* $5\frac{2}{3} + 2\frac{3}{4} =$ | *Subtract:* $5\frac{2}{3} - 2\frac{3}{4} =$

Step 1: $5\frac{8}{12} + 2\frac{9}{12} =$ | $5\frac{8}{12} - 2\frac{9}{12} =$

Step 2: $5\frac{8}{12} + 2\frac{9}{12} = 7\frac{17}{12}$ | $5\frac{8}{12} - 2\frac{9}{12} =$

You can't subtract 9 from 8. You need to borrow from the whole number.

$\boxed{4\,\cancel{5}\frac{8}{12} - 2\frac{9}{12}}$

Add the numerator and the denominator to form the new numerator.

$4\frac{20}{12} - 2\frac{9}{12} = 2\frac{11}{12}$

Step 3: $7\frac{17}{12} = 8\frac{5}{12}$

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Add or subtract the following fractions. Show work!

Ex. 1) $\frac{4}{5} + \frac{2}{3} =$

Ex. 2) $4\frac{1}{3} - 1\frac{4}{7} =$

Ex. 3) $8\frac{2}{3} - 6\frac{3}{8} =$

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Changing mixed numbers to improper fractions:

1) Multiply the denominator with the whole number.
2) Add the numerator to the product.
3) The sum becomes the new numerator.
4) Keep the denominator.

Ex.) $\overset{+}{3}\frac{1}{\underset{x}{2}}$ = $2 \times 3 = 6 \rightarrow 6 + 1 = 7 \rightarrow \frac{7}{2}$

(keep the denominator)

Change the following mixed numbers into improper fractions.

Ex. 1) $4\frac{3}{4}$ *Ex*. 2) $7\frac{4}{5}$ *Ex*. 3) $5\frac{1}{3}$

Changing improper fractions to mixed numbers:

Divide the numerator by the denominator.

A) The whole number from the quotient is the whole number for the mixed number.
B) The remainder becomes the new numerator.
C) Keep the denominator.
D) Reduce the fraction if possible.

Ex. $\frac{22}{6} \rightarrow 22 \div 6 = 3 \text{ r } 4 \rightarrow 3\frac{4}{6} = 3\frac{2}{3}$

Change the following improper fractions to mixed numbers.

Ex. 1) $\frac{13}{5}$ *Ex*. 2) $\frac{54}{8}$ *Ex*. 3) $\frac{75}{20}$

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Multiplying Fractions:

Follow these steps when multiplying fractions.

Multiply: $\frac{4}{5}$ x $\frac{5}{6}$ =

Step 1: Multiply the numerators. $\frac{4}{5}$ x $\frac{5}{6}$ = $\frac{20}{30}$

Step 2: Multiply the denominators.

Step 3: Reduce if necessary. $\frac{20}{30}$ = $\frac{2}{3}$

*You can reduce the fractions before multiplying by taking common factors out of any numerator and denominator.

$$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} = \frac{2}{3}$$

Multiply the following fractions.

Ex. 1) $\frac{3}{10}$ x $\frac{5}{9}$ =

Ex. 2) $\frac{2}{5}$ x $\frac{6}{7}$ =

Ex. 3) $\frac{5}{12}$ x $\frac{8}{15}$ =

Multiplying mixed numbers:

Follow these steps when multiplying mixed numbers.

Multiply: $4\frac{1}{5}$ x $3\frac{1}{3}$ =

Step 1:. Convert the mixed numbers to improper fractions. $\frac{21}{5}$ x $\frac{10}{3}$ =

Step 2: Multiply the numerators. $\frac{21}{5}$ x $\frac{10}{3}$ = $\frac{210}{15}$

Step 3:. Multiply the denominators.

Step 4: Reduce if necessary. $\frac{210}{15}$ = 14

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Multiply the following mixed numbers. Show work.

Ex. 1) $3\frac{2}{3} \times 4\frac{1}{11} =$

Ex. 2) $5\frac{3}{5} \cdot 4\frac{2}{7} =$

Dividing fractions:

<u>Reciprocal:</u> Two numbers whose product is 1. Invert (flip) the fraction.

Ex.) $\frac{4}{7} \rightarrow \frac{7}{4}$

* To find the reciprocal of a mixed number, **<u>first make the fraction improper</u>**, then invert.

Give the reciprocal of the fraction.

Ex. 1) $\frac{6}{7} \rightarrow$

Ex. 2) $4\frac{3}{8} \rightarrow$

Ex. 3) $3\frac{2}{3} \rightarrow$

Follow these steps to divide fractions.

Divide: $\frac{2}{5} \div \frac{7}{15} =$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{2}{5} \times \frac{15}{7} = \frac{30}{35}$

$\frac{30}{35} = \frac{6}{7}$

Step 1: Copy the first fraction.

Step 2: Change division to multiplication.

Step 3: Use the reciprocal of the second fraction.

Step 4: Multiply the fractions.

Step 5: Reduce.

Think **KEEP, CHANGE, FLIP** to help you remember how to divide fractions!

(Keep, change, flip)

(Stay, change, flip)

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Dividing mixed numbers:

Divide: $2\frac{1}{2} \div 3\frac{1}{3}$

$\downarrow \quad \downarrow$

$\frac{5}{2} \div \frac{10}{3}$

Follow these steps to divide mixed numbers.

Step 1: Change the mixed numbers to improper fractions.

Step 2: Copy the first fraction.
Step 3: Change division to multiplication.

Step 4: Use the reciprocal of the second fraction.

Step 5: Multiply the fractions.

Step 6: Reduce.

keep change flip

$$\frac{5}{2} \times \frac{3}{10} = \frac{15}{20}$$

$$\frac{15}{20} = \frac{3}{4}$$

Divide the following fractions.

Ex. 1) $\frac{3}{4} \div \frac{5}{8} =$

Ex. 2) $2\frac{2}{5} \div \frac{3}{5} =$

Ex. 3) $5\frac{2}{5} \div 2\frac{1}{4} =$

Ex. 4) $3\frac{3}{8} \div 6\frac{3}{4} =$

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Adding or subtracting decimals:

Follow these steps to add or subtracting decimals.

	Add:	***Subtract:***
	3.24 + 0 .008 + 12	14.9 - 0.76
		You might need to annex zeros to the end of the number.
		$\overset{8\ \ 1}{14.\cancel{9}0}$
Step 1: Write the problem vertically aligning the decimal point.	$\begin{array}{r} 3.240 \\ 0.008 \\ +\ \underline{12.000} \end{array}$	$\begin{array}{r} 14.90 \\ -\ 0\underline{.76} \end{array}$
Step 2: Add/subtract.	15 248	14 14
Step 3: Bring the decimal point straight down.	↓ 15.248	↓ 14.14

Add or subtract the following decimal. ***Re-write the problem vertically.***

Ex. 1) 24.8 + 13.09

Ex. 2) 17.2 + 9.003 + 6 + 3.3

Ex. 3) 65.9 - 4.98

Ex. 4) 4 - .0765

Date____________________

Multiplying Decimals:

Follow these steps to multiply decimals.

Multiply: 5.78 x 2.3

Step 1: Write the problem vertically.
Do not line-up the decimal.

$$\begin{array}{r} 5.78 \\ \underline{\times \quad 2.3} \\ 1734 \\ \underline{+\ 11560} \\ 13294 \\ \\ 13.294 \end{array}$$

There are a total of **3** places behind the decimal in the factors.

Step 2: Multiply.

Step 3: Count the number of places after the decimal in the *factors*. Place the decimal point in the *product* so that it has that same number of places behind the decimal.

I need to place the decimal so that there are **3** places behind the decimal in the product.

Multiply the following decimals. [Notice that we can also show multiplication with a dot, •, or with parenthesis, ( ).]

*Ex.*1) 4.7 x 9.22

Ex. 3) 3.211(0.7)

Ex. 2) 0.48 • 7

Ex. 4) 6.8 x 3.9

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Dividing decimals:

Dividend Divisor

Follow the steps of long division shown below to divide decimals.

Divide: 5.654 ÷ 0.02

Step 1: Rewrite the division problem with the divisor on the outside and the dividend inside of the division symbol.

0.02)5.654

Step 2: Move the decimal in the divisor enough places to form a whole number.

0.02.) 5.654

I move the decimal **2** places in the divisor to make it a whole number.

Step 3: Move the decimal the same number of places in the dividend*.

002.)565.4

So, I must move the decimal **2** places in the dividend.

Step 4: Move the decimal point straight up.

Step 5: Divide.

282.7

2)565.4

Now, move the decimal point straight up.

- In some cases it may be necessary to add zeros when moving the decimal in the dividend.

0.22) 5.6

Step 1: Rewrite the divisor on the outside and the dividend inside of the division symbol.

Step 2: Move the decimal in the divisor enough places to form a whole number.

0.22.) 5.6

I move the decimal **2** places in the divisor to make it a whole number.

Step 3: Move the decimal the same number of places in the dividend*.

022.)560.

So, I must move the decimal **2** places in the dividend.

Step 4: Move the decimal point straight up.

Step 5: Divide.

25.4545… (Repeating decimal! ☺)

22)560.00

Now, move the decimal point straight up.

Add zeros and continue to divide.

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Divide the following decimals. Round to the nearest hundredth when necessary.

Ex. 1) $37 \div 0.3$

Ex. 3) $90.9 \div 0.06$

Ex. 2) $7.431 \div 0.3$

Ex. 4) $34.567 \div 2.54$

Go back and label each answer as terminating or repeating and rational or irrational.

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Converting between Decimals and Fractions:

- **To convert a number written as a decimal into a fraction:**

1) "Say it". By reading the decimal using correct place value language, you will be able to hear the fraction.
2) Reduce.

OR

1) Place the decimal over the appropriate power of ten denominator.

Ex. $0.\underbrace{14}_{\text{2 digits} \rightarrow 10^2} = \frac{14}{100} = \frac{7}{50}$

2) Reduce.

Try these: 1) 0.54 = _______ 2) 3.08 = _______ 3) 0.9 = ________

- **To convert a number written as a fraction into a decimal**:

<u>Method I</u>: Divide the numerator by the denominator following the steps of long division.

Ex. $\frac{4}{5}$

Step 1: Rewrite the problem with the denominator on the outside and the numerator on the inside of the division symbol. $5\overline{)4}$

Step 2: Divide. If the divisor (denominator) does not fit into the dividend (numerator), add a decimal point and a zero to the dividend. $5\overline{)4.\mathbf{0}}$

Step 4: Move the decimal point straight up.

Step 5: Divide.

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \end{array}$$

$\frac{4}{5} = 0.8$

Try these:

Ex. 1) $\frac{7}{8}$ Ex. 2) $\frac{3}{5}$ Ex. 3) $\frac{1}{3}$

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Method II:

2) Create an equivalent fraction with a denominator of a power of ten, then write the decimal in the correct place value.

Ex.) $\frac{3}{5} \text{ x } \frac{2}{2} = \frac{6}{10} \rightarrow 0.6$

Multiply by a factor that will result in a power of 10.

NOTE: This method can only be used if the prime factors of the denominator are only 2s and 5s.

A more detailed description can be found below.

Step 1: Factor the denominator.
Step 2: Multiply the both the numerator and denominator by the number required to give an equal number of 2s and 5s.

See the examples below:

a. **Write the number $\frac{3}{20}$ as a decimal. Describe your process.**

The fractional form is $\frac{3}{20} = \frac{3}{2^2 \times 5}$. The denominator lacks a factor of 5 to be a power of ten. To arrive at the decimal form I multiply the fractional form by $\frac{5}{5}$ to arrive at $\frac{3}{2^2 \times 5} \times \frac{5}{5} = \frac{3 \times 5}{2^2 \times 5^2} = \frac{15}{100}$; and $\frac{15}{100} = 0.15$.

b. **Write the number $\frac{10}{25}$ as a decimal. Describe your process.**

The fractional form is $\frac{10}{25} = \frac{2 \times 5}{5 \times 5}$; and, since $\frac{5}{5} = 1$, then $\frac{2 \times 5}{5 \times 5} = \frac{2}{5}$. The denominator lacks a factor of 2 to be a power of ten. To arrive at the decimal form I multiply the fractional form by $\frac{2}{2}$ to arrive at $\frac{2}{5} \times \frac{2}{2} = \frac{4}{10}$; and $\frac{4}{10} = 0.4$

c. **Write the number $\frac{8}{40}$ as a decimal. Describe your process.**

The fractional form is $\frac{8}{40} = \frac{2^3}{2^3 \times 5}$. There are factors of 2 in the numerator and denominator that will cancel. If I leave one factor of two in the denominator, it will be 10 (a power of ten). $\frac{2^3}{2^3 \times 5} = \frac{2}{2 \times 5} = \frac{2}{10}$; $\frac{2}{10} = 0.2$

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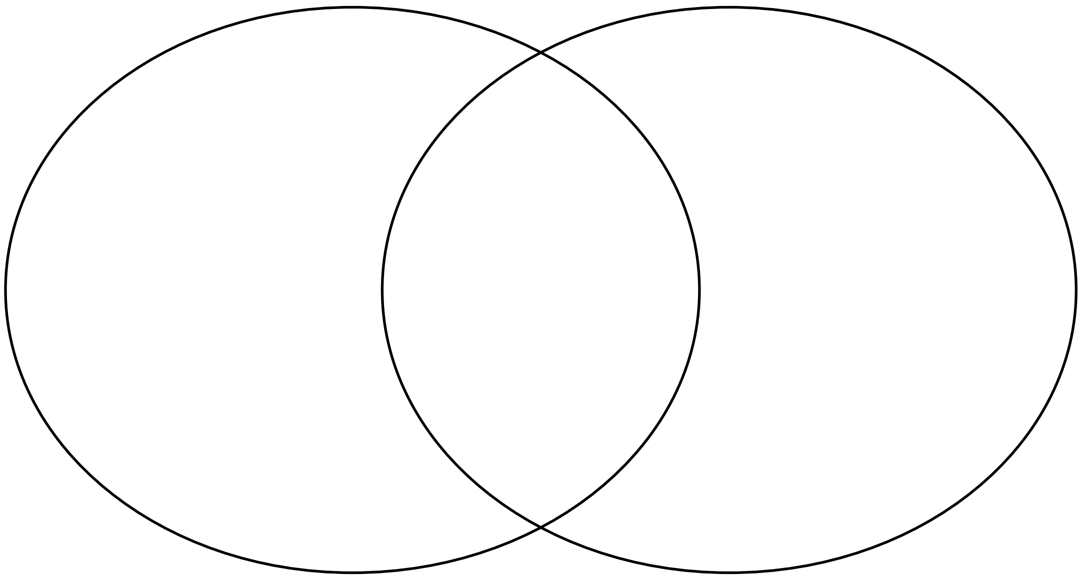
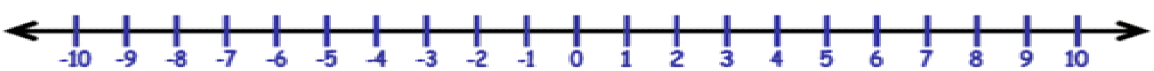
<u>Positive and Negative Numbers-</u>

Positive numbers **Negative numbers**



<u>Integers</u>: <u>Whole</u> numbers and their opposites. *or* Positive and negative <u>whole</u> numbers.

Ex. All of the numbers shown below are integers. The numbers increase as we move to the right, and decrease as we move to the left.



<u>Opposites:</u> Numbers that are the same distance from zero on the number line, but in different directions.

Ex. The opposite of 5 is -5.
The opposite of -3 is 3.

The opposite of -7 is _____.

The opposite of 14 is _____.

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<u>Absolute Value:</u> The distance a point is from zero on the number line. (The number of units between the given number and zero.)

Ex. |-2| = 2 |+5| = 5

The symbol for absolute value is two vertical lines around a number.
| -5| = 5

1) |-7| = _____

2) |+3| = _____

3) |-8| = _____

4) | 9| = ______

5) - |-4| = _____

6) - |2| = _____

Try these: |-2| + |-7| = __9__

2 + 7 = 9

Absolute value holds the same status as parenthesis, always do it first*!

7) |-12| - |-3| = _____

8) |+6| + |-3| = ______

9) |-8| + |-11| = ______

* If there is an operation within the absolute value notation, complete that **first**, then find the absolute value of the answer. *Ex.* |-8 + 3| = |-5| = 5

Try these:

1.) Using the term **<u>opposite</u>**, describe the relationship between 8 and -8.

____

____

2.) Using the term **<u>absolute value</u>**, describe the relationship between 8 and -8.

____

____

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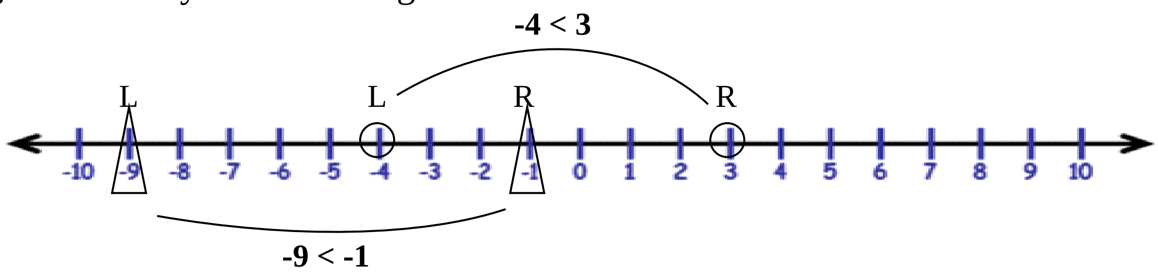
Comparing integers: Given two integers ***on the number line***, the number on the right will always have the larger value.



Try the following: Answer with <, >, or =. ***Show work for questions 6-13***

1) -7 _____ -12

2) -8 ______-3

3) 6 ______ 12

4) -5 ______ 0

5) 0 ______ -3

6) |-3| _____ 3

7) |-6| _____|-8|

8) 0 _____ |-5|

9) |-9| _____|7|

10) -3 ______(-3)

11) 4 ______-(-4)

12) -(-8)______-8

13) - |-6| ______-6

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Adding Integers:

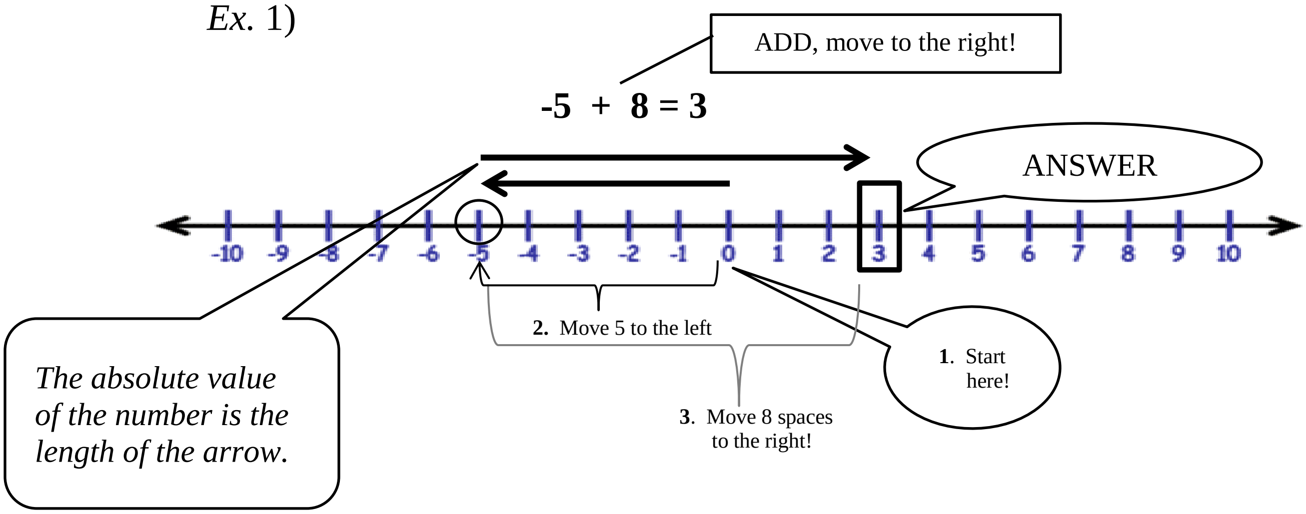
Adding integers using the number line:

Positive: Count up. Right
Negative: Count down. Left

1. The starting point is **<u>always zero</u>**.
2. Move to the first number.
3. To <u>ADD</u> numbers, we always move to the <u>RIGHT</u>, however, if you "bump" into a negative, you must turn around!!!
4. The second number represents the number of times you move.

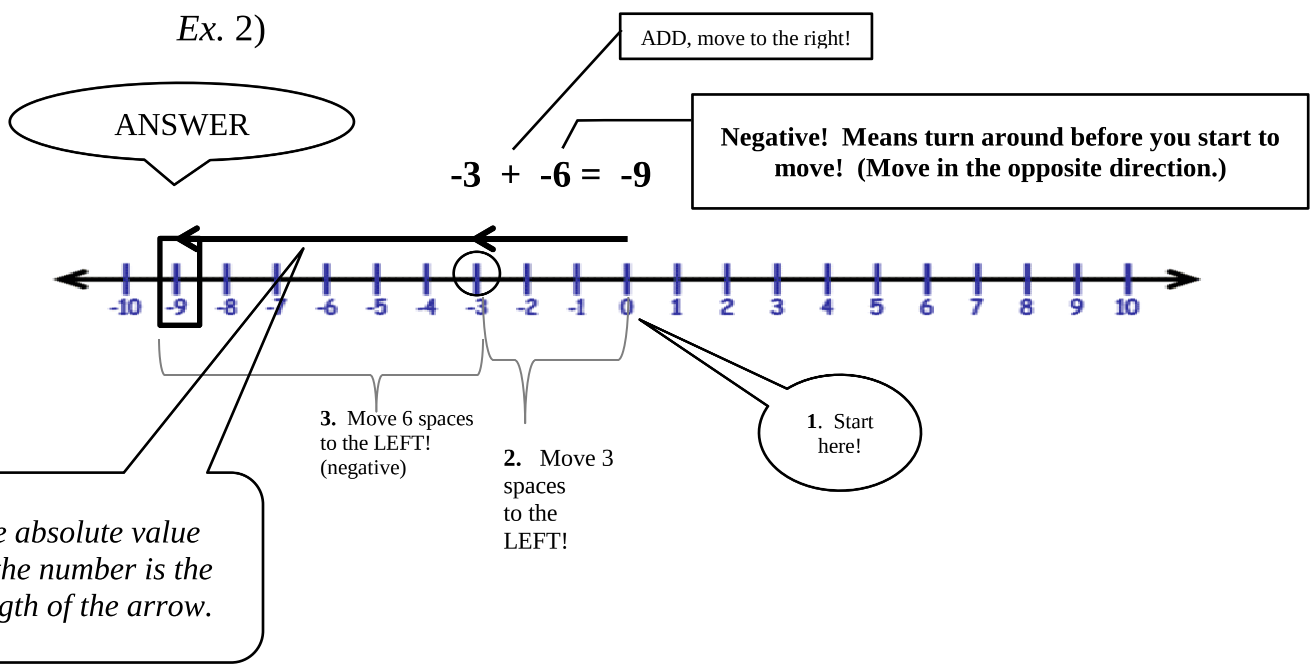
Ex. 1)

-5 + 8 = 3



Ex. 2)

-3 + -6 = -9



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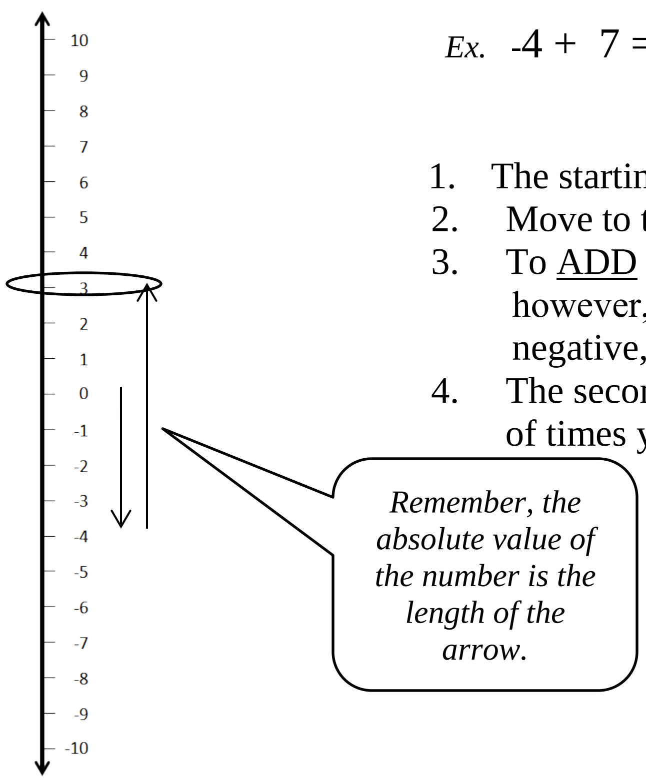
Vertical number lines:

Follow the same steps for vertical number lines:

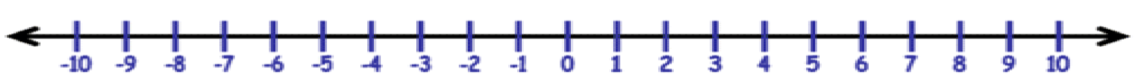


Ex. $-4 + 7 = 3$

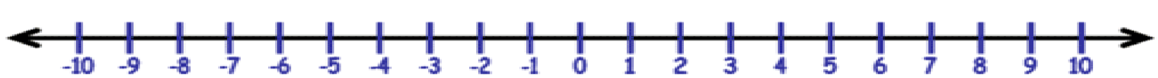
1. The starting point is **<u>always zero</u>**.
2. Move to the first number.
3. To <u>ADD</u> numbers, we always move UP, however, if you "bump" into a negative, you must turn around!!!
4. The second number represents the number of times you move.

Try these using the number line:

1.) $-4 + 10$



2.) $-2 + (-5)$



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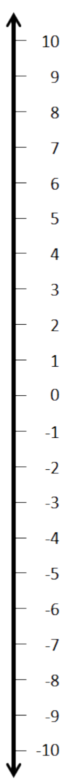
3.) 5 + (-9)

4.) -3 + (-5) + 4

5.) -5 + 12



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Adding integers:

1) Adding integers with **like** signs (two of the same sign).
   a.) Find the **sum** of the absolute values of the numbers. (Add the numbers.)
   b.) Copy the sign.

Ex. a. 7 + 2 = 9
b. -4 + -8 = -12
|-4| + |-8|
4 + 8 = -12

Same sign, find the sum!
Same sign- sum!

Try these:

1) -3 + -9 = ________
2) -2 + -4 = ________
3) 9 + 1 = ________

2) Adding integers with **unlike** signs (two **different** signs).
   a.) Find the **difference** between the absolute values of the numbers. (SUBTRACT)
   b. Copy the sign that belongs to the larger number.

Ex. a. -8 + 15 =
|-8| |15|
8 15 → The difference between 15 and 8 is 7.
7 → Since 15 is larger and positive, the answer is positive 7.

Different signs, find the difference.
Different signs- difference!

b. -12 + 3 =
|-12| |3|
12 3 → The difference between 12 and 3 is 9.
-9 → Since the 12 is larger and negative, the answer is negative 9.

Try these:

1) -5 + 4 = ______
2) 9 + (-6) = ______
3) 0 + (-4) = ______
4) -7 + 11 = ______
5) 2 + (-8) = ______

Sometimes there will be parenthesis around an integer(s). They simply separate the integer from the operation symbol.

Date____________________

The Opposite of the Sum is the Sum of its Opposites:

Let's say we have then numbers 7 and -2. The sum of 7 and -2 is 5 ($7 + -2 = 5$). The opposite of the sum of 5 is -5. The opposites of 7 and -2 are -7 and 2. their sum is -5 (-7 + 2 = -5). Now we can see that the opposite of the sum is equal to the sum of the opposites.

Example 1

Explain the meaning of "The opposite of a sum is the sum of its opposites." Use a specific math example.

Rational Number	Rational Number	Sum	Opposite of the Sum
7	**-2**	**5**	**-5**

Opposite Rational Number	Opposite Rational Number	Sum
-7	**2**	**-5**

It means that if you have a sum and want to take the opposite, for instance, $-(7 + (-2))$, you can rewrite it as the sum of each addend's opposite: $-7 + 2$.

M2L8Ex1

Try these:

1) -4 + (-7) = _______ Therefore, the opposite of (-4 + (-7)) = _____

opposite Rewrite Sum

-(-4 + (-7)) = ____4 + 7_________ ______

\- (-11)

$\boxed{}$

2) -(2 + -13) = ________________ _____

3) -(-7 + 15) = ________________ _____

4) -(-3 + (-9)) = ________________ _____

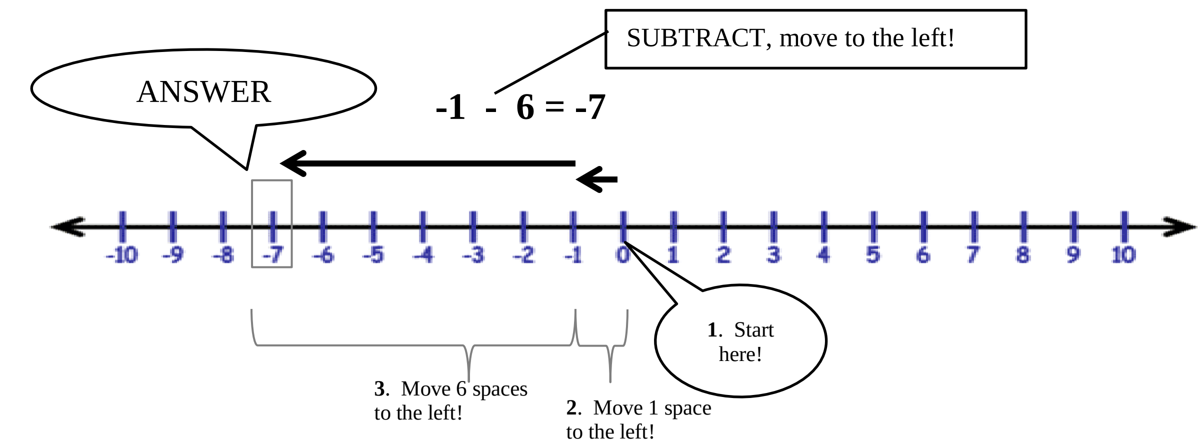
**This also demonstrates the Distributive Property which we will study more later!* ☺

Date___________________

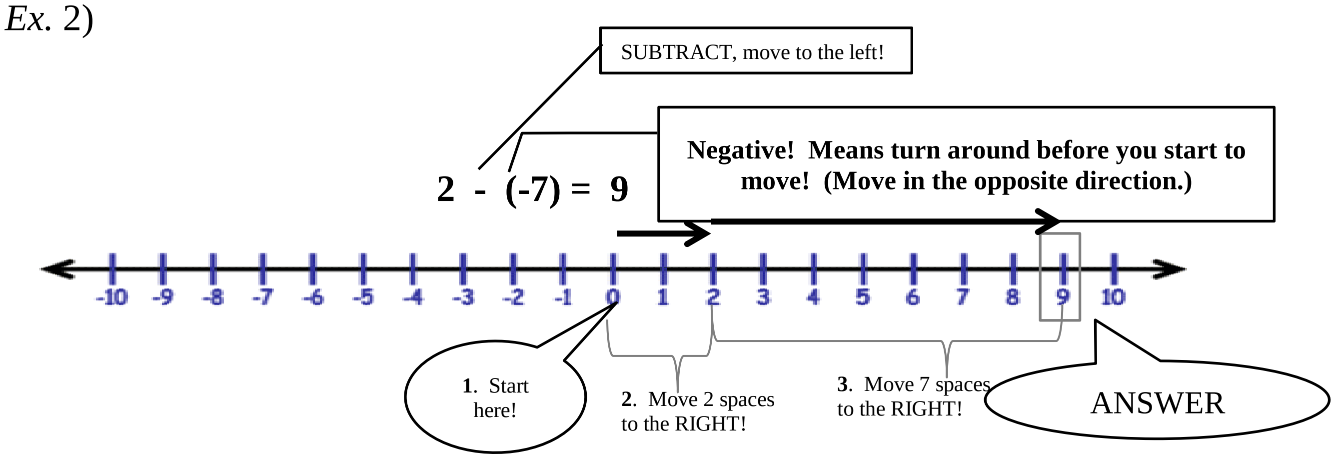
Subtracting Integers:

- **Subtracting integers using the number line:**
  1. The starting point is always zero.
  2. Move to the first number.
  3. To <u>SUBTRACT</u> numbers, we always move to the LEFT, however, if you "bump" into a negative, you must turn around!!!
  4. The second number represents the number of times you move.

Ex. 1)

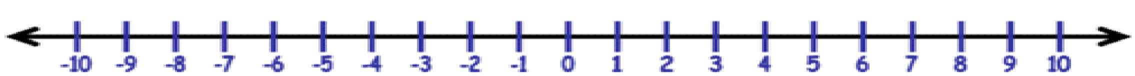


Ex. 2)



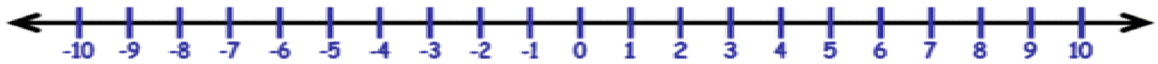
Try these using the number line:

1.) 4 - 10

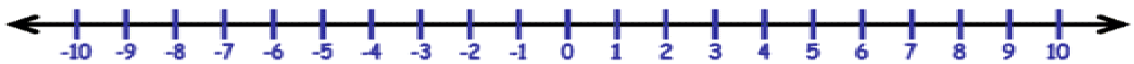


Date___________________

2.) -2 - (-5)



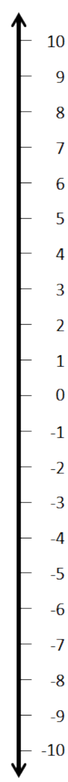
3.) -6 - 4



4.) 2 - (-4)

5.)

-3 - (-8)



Date___________________

Subtraction of integers:

Subtraction of integers can be thought of as adding the opposite.
Follow the steps below when subtracting integers.

1) Change the subtraction symbol to an addition symbol.

2) Use the opposite of the second number. (Change the sign of the second number.)

3) Follow the rules for addition.

Ex. 3 - (-5) =
↓ ↓ ↓
3 + (+5) = 8

Same signs, find the **sum** and copy the sign.
Same sign- sum!

Ex. -8 - 6 =
↓ ↓ ↓
-8 + (-6) = -14

Different signs, find the **difference** and use the sign of the larger number.
Different signs- difference!

Ex. 6 - 15 =
↓ ↓ ↓
6 + (-15) = -9

To help you remember, sing this little song:

♫ *Change subtraction to addition and the sign of the second.* ♪
(repeat)

Try the following:

Show work!

1) 7 - (-3) = _______
2) -2 - 4 = _______
3) 8 - 5 = _______
4) -9 - (-6) = _______
5) 6 - 12 = ________
6) 3 - 9 = _______

Date___________________

Multiplying Integers:

Multiplying integers:

1. Multiply the numbers.
2. Count the number of ***negative signs***.

- If there are an ***even*** number of <u>negative signs</u>, the answer will be ***positive***.

Ex. -2 · 4 · -7 = +56

There are **2** negative signs –
2 is an even number-
therefore, the answer is **positive**.

- If there are an ***odd*** number of <u>negative signs</u>, the answer will be ***negative***.

Ex. -3 · -4 · -5 = -60

There are **3** negative signs –
3 is an odd number-
therefore, the answer is **negative**.

*Remember the smiley face rule!

-3 · -1 · 4 · 1 · -6 · -2 = 144 ☺

-3 · -1 · -4 · 1 · -6 · -2 = -144 ☹

↑ (no negative to pair up!)

Try these:

1) -5 · -3 = _____
2) 7 · -2 = _____
3) -2 · -3 · -4 = _____
4) -3 · -2 · -1 · -2 = _____
5) 2 · |-3| · -4 = _____

Dividing integers: (Same as the multiplication rule!!)

1. Divide the numbers.
2. Count the negative signs.

- If there are an ***even*** number of <u>negative signs</u>, the answer will be ***positive***.
- If there are an ***odd*** number of <u>negative signs</u>, the answer will be ***negative***.

Date___________________

Sign rules with other rational numbers:

The sign rules that we learned with integers can be applied to all rational numbers.

For fractions, the negative sign can be in one of three places:

1) before the fraction *ex.* $-\frac{3}{5}$

2) in the numerator *ex.* $\frac{-3}{5}$

3) in the denominator *ex.* $\frac{3}{-5}$

Follow the division rule: A single negative sign indicates a negative fraction. *ex.* $\frac{8}{-9}$

Two negative signs indicate a positive fraction. *ex.* $\frac{-2}{-3}$

Three negative signs indicate a negative fraction. *ex.* $-\frac{-6}{-7}$

Ex. $-\frac{4}{5} \bullet \frac{2}{3} = -\frac{8}{15}$ A negative fraction multiplied by a positive fraction is a negative fraction.

Ex. -4.4 ÷ -0.4 = +11 A negative decimal divided by a negative decimal results in a positive quotient.

Ex. $-4\frac{2}{3} + 2\frac{1}{3} = -2\frac{1}{3}$ Addition of fractions with different signs; find the difference and take the sign of the number with the larger absolute value.

Try these:

1) -3.7 + 8.09 2) $-\frac{2}{3}$ - $-\frac{3}{5}$ 3) (-0.5)($\frac{-3}{4}$)

Date___________________

Mathematical Properties

- **Associative property**: keyword: *Grouping*
  - Associative property of addition: The grouping of the numbers does not change the sum. For any whole numbers a, b, and c; $(a+b)+c=a+(b+c)$

    Ex. $(5+6)+7=5+(6+7)$

  - Associative property of multiplication: The grouping of numbers does not change the product. For any whole numbers a, b, and c; $(a \bullet b) \bullet c = a \bullet (b \bullet c)$

    Ex. $(5 \bullet 6) \bullet 7 = 5 \bullet (6 \bullet 7)$

  The associative property does ***not*** *apply to subtraction and division.*

> To help remember the **associative property** – think of the word **associate**. Who do you associate with? That is your **group** of friends.

***You may abbreviate the associative property with **assoc**. – NO EXCEPTIONS

- **Commutative property**: keyword: *Order*
  - Commutative property of addition: The order of the numbers does not change the sum. For any whole numbers a, b, and c; $a+b+c=c+b+a$

    Ex. $5+6+7=7+6+5$

  - Commutative property of multiplication: The order of numbers does not change the product. For any whole numbers a, b, and c; $abc=cba$

    Ex. $5 \bullet 6 \bullet 7 = 7 \bullet 6 \bullet 5$

  *The commutative property does **not** apply to subtraction and division.

> To help remember the **commutative property** – think of the word **commute**. As a student you commute to school.
> First you travel from home to school (H → S),
> then you travel from school to home (S → H).

You may abbreviate the commutative property with **comm.**

Date____________________

- **<u>Distributive property</u>**: keyword: *Distribute*

  - <u>Distributive property of multiplication over addition:</u> For any numbers a, b, and c. $a(b + c) = (ab) + (ac)$

    Ex. $5(7 + 3) = (5 \cdot 7) + (5 \cdot 3)$
    $5(7 + 3) = 5(7) + 5(3)$

  - <u>Distributive property of multiplication over subtraction*:</u> For any numbers a, b, and c. $a(b - c) = (a \times b) - (a \times c)$

    Ex. $5(7 - 3) = (5 \cdot 7) - (5 \cdot 3)$
    $5(7 – 3) = 5(7) - 5(3)$

* The distributive property is the only property that works with subtraction!!!

Try these examples:

Ex. 1) 3(8 + 1) = (   ) + (   )

Ex. 2) 7(5 – 2) = (   ) - (   )

Ex. 3) 10(3 + 4) =

Ex. 4) ½ (6 – 2) =

Ex. 5) __ (__ + __) = (5 • 3) + (5 • 2)

Ex. 6) __(   ) = (7 • 5) - (7 • 2)

They might also look like this: (3 + 4)8 it is the same thing!

To help you remember the **distributive** property, think of Mrs. Cavallaro **distributing** papers to each person in the class.

Mrs.C (Sam, Alex, Jake, David)

*You may abbreviate the distributive property with **<u>dist.</u>**

Date___________________

Using the properties of operations:

Using the properties operations can make some problems much simpler to solve mentally. The examples below demonstrate how using the properties alone or together can help you solve the problems.

$-13\frac{5}{7}+6-\frac{2}{7}$

$-13\frac{5}{7}+6+\left(-\frac{2}{7}\right)$	*Subtracting a number is the same as adding its inverse.*
$-13+\left(-\frac{5}{7}\right)+6+\left(-\frac{2}{7}\right)$	*The opposite of a sum is the sum of its opposite.* (Distributing the negative to each part of the term.)
$-13+\left(-\frac{5}{7}\right)+\left(-\frac{2}{7}\right)+6$	*Commutative Property of Addition*
$-13+(-1)+6$	*Associative Property of Addition*
$-14+6$	
-8	

M2L8

Exercise 1

Represent the following expression with a single rational number.

$-2\frac{2}{5}+3\frac{1}{4}-\frac{3}{5}$

Two Possible Methods:

$-2\frac{2}{5}+3\frac{1}{4}+\left(-\frac{3}{5}\right)$

$-2\frac{8}{20}+3\frac{5}{20}-\frac{12}{20}$

$\frac{17}{20}-\frac{12}{20}$

$\frac{5}{20}$ *or* $\frac{1}{4}$

OR

Commutative Property:

$-2\frac{2}{5}+3\frac{1}{4}+\left(-\frac{3}{5}\right)$

$-2\frac{2}{5}+\left(-\frac{3}{5}\right)+3\frac{1}{4}$

$-2\frac{5}{5}+3\frac{1}{4}$

$-3+3\frac{1}{4}$

$=\frac{1}{4}$

M2L8

Examples 2 and 3

Represent each of the following expressions as one rational number. Show your steps.

2. $4\frac{4}{7}-\left(4\frac{4}{7}-10\right)$

$4\frac{4}{7}-\left(4\frac{4}{7}+(-10)\right)$	***Why? Subtracting a number is the same as adding its inverse.***
$4\frac{4}{7}+\left(-4\frac{4}{7}+10\right)$	***Why? The opposite of a sum is the sum of its opposites.***
$\left(4\frac{4}{7}+\left(-4\frac{4}{7}\right)\right)+10$	***Why? The Associative Property of Addition.***
$0+10$	***Why? A number plus its opposite equals zero.***
10	

M2L9

Date____________________

3. $5+\left(-4\frac{4}{7}\right)$

$5+\left[-\left(4+\frac{4}{7}\right)\right]$ *Why? The mixed number $4\frac{4}{7}$ is equivalent to $4+\frac{4}{7}$.*

$5+\left(-4+\left(-\frac{4}{7}\right)\right)$ *Why? The opposite of a sum is the sum of its opposites.*

$(5+(-4))+\left(-\frac{4}{7}\right)$ *Why? Associative Property of Addition.*

$1+\left(-\frac{4}{7}\right)$ *Why? $5+(-4)=1$.*

$\frac{7}{7}+\left(-\frac{4}{7}\right)$ *Why? $\frac{7}{7}=1$.*

$\frac{3}{7}$

M2L9

Example 1: Using the Commutative and Associative Properties to Efficiently Multiply Rational Numbers

a. **Evaluate the expression below:**

$$-6\times 2\times(-2)\times(-5)\times(-3)$$

$\underbrace{-6\times 2}\times(-2)\times(-5)\times(-3)$

$\underbrace{-12\times(-2)}\times(-5)\times(-3)$

$\underbrace{24\times(-5)}\times(-3)$

$\underbrace{-120\times(-3)}$

360

$-6\times 2\times\underbrace{(-2)\times(-5)}\times(-3)$

$-6\times 2\times 10\times(-3)$ *Associative Property of Multiplication*

$-6\times\underbrace{2\times(-3)}\times 10$ *Commutative Property of Multiplication*

$\underbrace{-6\times(-6)}\times 10$ *Associative Property of Multiplication*

$\underbrace{36\times 10}$

360

M2L16

Try these:

1. **Jamal is completing a math problem and represents the expression $-5\frac{5}{7}+8-3\frac{2}{7}$ with a single rational number as shown in the steps below. Justify each of Jamal's steps.**

$-5\frac{5}{7}+8+(-3\frac{2}{7})$

$-5\frac{5}{7}+\left(-3\frac{2}{7}\right)+8$

$-5+\left(-\frac{5}{7}\right)+(-3)+\left(-\frac{2}{7}\right)+8$

$-5+\left(-\frac{5}{7}\right)+\left(-\frac{2}{7}\right)+(-3)+8$

$-5+(-1)+(-3)+8$

$-6+(-3)+8$

$(-9)+8$

-1

Date____________________

Answer the following. Justify your steps!

a. $-5.2 - (-3.1) + 5.2$

b. $3\frac{1}{6} + 20.3 - \left(-5\frac{5}{6}\right)$

c. $\frac{16}{20} - (-1.8) - \frac{4}{5}$

M2L9

d. $\mathbf{1 \div \frac{2}{3} \times (-8) \times 3 \div \left(-\frac{1}{2}\right)}$

M2L16

Date___________________

- **<u>Identity property</u>**:

<u>Additive Identity</u>: The sum of a number and zero is the original number.

Therefore, the additive identity is always **zero**.

Ex. $23 + 0 = 23$

Ex. $a + 0 = a$

or

Ex. $0 + 3 = 3$

<u>Multiplicative Identity</u>: The product of a number and one is the original number.

Therefore, the multiplicative identity is always **one.**

Ex. $4 \bullet 1 = 4$

Ex. $\frac{2}{7} \bullet 1 = \frac{2}{7}$

Ex. $a \bullet 1 = a$

or

Ex. $1 \bullet 5 = 5$

Date___________________

- **Inverse property**:

Additive Inverse: The sum of a number and its additive inverse is zero.

Therefore, the additive inverse is the opposite of the original number.

Keyword: *Opposite*

Ex. $1 + -1 = 0$

Ex. $5 + -5 = 0$

Ex. $-6 + 6 = 0$

Ex. $a + -a = 0$

Multiplicative Inverse: The product of a number and its multiplicative inverse is one.

Therefore, the multiplicative inverse is the reciprocal of the original number.

Keyword: *Reciprocal*

Ex. $3 \bullet \frac{1}{3} = 1$

Ex. $56 \bullet \frac{1}{56} = 1$

Ex. $\frac{7}{8} \bullet \frac{8}{7} = 1$

Ex. $a \bullet \frac{1}{a} = 1$

Date____________________

<u>Order of Operations:</u> The universally accepted order in which mathematical operations are completed.

First: Grouping symbols / Parentheses ( ) \ Brackets [ ] } in order from L to R or from the inside out.

Second: Exponents and Roots – in order from L to R

Third: Multiplication and Division – in order from L to R
(multiplication does NOT always come first)

Fourth: Addition and Subtraction – in order from L to R
(addition does NOT always come first)

*The expressions: "“GEMDAS” may be helpful in remembering the order, however, it can also be misleading.

Ex) 5 + 6 x 7
5 + 42
47

Ex) $(3^2 + 2^4) - \sqrt{25}$
$(9 + 16) - \sqrt{25}$
$25 - \sqrt{25}$
$25 - 5$
20

When solving problems – following procedure and recording your steps is very important. I expect you to follow the format illustrated above. As you complete each step, record the answer **below the part that was completed. Copy the items that are left to do. Continue until all steps are complete. Remember, only one “level” may be done at a time. The only item on the last line should be the answer. You don’t need to go back to the top to write it.

Date___________________

Exponents (Powers):

<u>Base</u>: The number being multiplied together. It is the larger sized number.

<u>Exponent</u> (power): The number that indicates how many times the base is used as a factor. It is the smaller sized number and is raised in the air.

Ex) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

** Any base to the power of 1 is that number.
*****Any non-zero number to the power of zero is ***<u>one</u>***.

Powers of Ten:

If the base is ten, the exponent indicates the number of zeros to follow the one.

Ex) $10^2 = 10 \times 10 = 1\underline{00}$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 1\underline{00{,}000}$

$10^5 = 100{,}000$ *divide by 10*
$10^4 = 10{,}000$
$10^3 = 1{,}000$
$10^2 = 100$
$10^1 = 10$
$\boxed{10^0 = 1}$
$10^{-1} = \frac{1}{10} = .1$
$10^{-2} = \frac{1}{100} = .01$

Negative exponents do not create negative answers.

**Negative exponents tell you to use the reciprocal of the value.

Ex) $2^{-5} \text{-----------------}\rightarrow \frac{1}{2^5} \text{-------------}\rightarrow \frac{1}{32}$

Date___________________

Exponents w/ fractions:

- When a fraction is raised to an exponent; multiply the base together the number of times indicated by the exponent.

Ex. $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{8}$

If the exponent is **negative,** remember to use the **reciprocal** of the base.

Remove negative

Ex. $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3}{2} \bullet \frac{3}{2} \bullet \frac{3}{2} = \frac{27}{8} = 3\frac{3}{8}$

Reciprocal

Exponents w/ decimals:

- When a decimal is raised to an exponent; multiply the base together the number of times indicated by the exponent.

Ex. $(.4)^3 = (.4)\ (.4)\ (.4) = .064$ *Remember to move the decimal point!!*

If the exponent is **negative**, convert the decimal to a fraction and use the reciprocal!!!

Exponents w/ Integers:

- When an integer is raised to an exponent we must pay careful attention to the placement of the negative sign.

Ex.

$(-2)^4$
$(-2)\ (-2)\ (-2)\ (-2)$
$+16$

When the negative sign is held to the number with parenthesis, the base is negative. Use the negative sign with each number.

$-\ 2^4$
$-\ 2 \bullet 2 \bullet 2 \bullet 2$
-16

When the negative is **not** held to the number, it implies that the result of the exponent is negative. *Remember, order of operations- exponents before subtraction!*

Date___________________

Negative base!

Ex. $(-4)^3 = (-4)(-4)(-4) = -64$

$-4^3 = -4 \bullet 4 \bullet 4 = -64$

Sometimes you will get the same answer.

Positive base!

Try these:

1) $\left(\frac{1}{3}\right)^4$

2) $\left(\frac{2}{5}\right)^2$

3) $\left(\frac{3}{4}\right)^{-2}$

4) $(.2)^3$

5) $(.06)^2$

6) -5^2

7) $(-5)^2$

8) $(-3)^2$

9) -3^2

10) $-(-3)^2$

11) Explain how you know if the base is positive or negative:

____

____

____

____

Date____________________

<u>Square Root</u>: One of two equal factors of a number.

Ex. $\sqrt{25} = 5$ because $5 \times 5 = 25$

When the square root of a number is a whole number, the original number is called a **<u>perfect square</u>**. See the table below:

The square root symbol is call the radical.

$$\sqrt{100}$$

The number in the radical is called the radicand.

Perfect squares		Square root
1	$\sqrt{1}$	1
4	$\sqrt{4}$	2
9	$\sqrt{9}$	3
16	$\sqrt{16}$	4
25	$\sqrt{25}$	5
36	$\sqrt{36}$	6
49	$\sqrt{49}$	7
64	$\sqrt{64}$	8
81	$\sqrt{81}$	9
100	$\sqrt{100}$	10
121	$\sqrt{121}$	11
144	$\sqrt{144}$	12
169	$\sqrt{169}$	13
196	$\sqrt{196}$	14
225	$\sqrt{225}$	15

MEMORIZE

To find the square root of a number that is not a perfect square – use a calculator or a square root table. The table is on page 527 in your math text book.

To use the table:

1) Find the number for which you need a square root in column N. *
2) Follow the row across to the column labeled $\sqrt{N}$ – this is the square root for the given number. Note that the square roots are rounded to the nearest thousandth.

* The table is split in half, for numbers larger than fifty, look half-way across the table.

Find the following square roots:

$\sqrt{23} =$ $\sqrt{57} =$ $\sqrt{98} =$

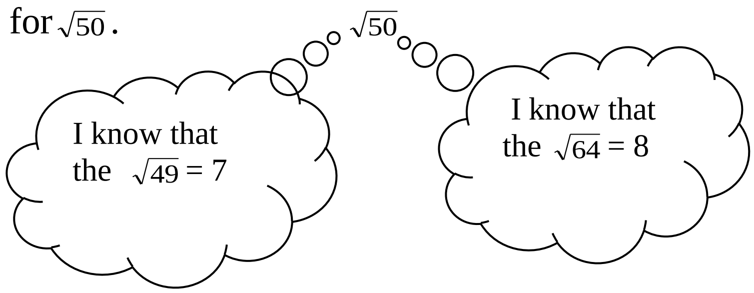
Date___________________

Estimating square roots:

It is possible to estimate the square root by determining the **<u>two consecutive whole numbers</u>** that the square of a non-perfect square falls between.

It is possible to determine the two consecutive whole numbers that the square of a non-perfect square falls between by finding the closest but smaller perfect square and the closest but larger perfect square.

If we are looking for $\sqrt{50}$.



Since 50 is between 49 and 64, the $\sqrt{50}$ is between

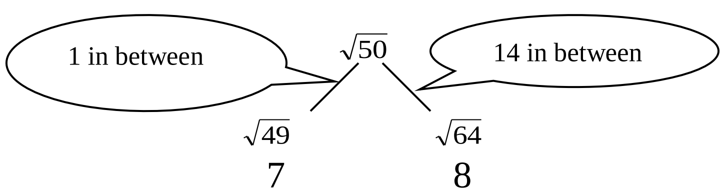
$\sqrt{49}$ and $\sqrt{64}$

7 and **8**

therefore, the $\sqrt{50}$ is between **7** and **8**.

Take it one step further:

Since 50 is only one number away from 49 and 14 away from 64, the decimal will be closer to 7.0.



The $\sqrt{50}$ is closer to 7.0 than 7.9. Possibly: 7.05 - 7.1

Exact (from the table): 7.071

Let's try these: Determine the two whole numbers that the square root lies between.

1. $\sqrt{12}$
2. $\sqrt{30}$
3. $\sqrt{55}$
4. $\sqrt{75}$
5. $\sqrt{90}$
6. $\sqrt{120}$

Date___________________

Complete the following examples. **Be sure to show all of the work that is written on the board.**

1) $\frac{1}{2} \bullet 8\frac{2}{5} - 3^2 + 4^0$

6) $1.2(11 - \sqrt{25})$

2) $\sqrt{100} - [(-2)^3 \div -2^2]$

7) $\frac{1}{3}(12 + 3^2 \bullet 5^0)$

3) $2\left(\frac{5}{8} - \frac{1}{2}\right) + 2^2$

$\boldsymbol{t = 12 \quad w = -6 \quad y = \frac{1}{4} \quad z = \frac{2}{3}}$

8) $ty - \frac{1}{2}wz$

4) $\left[5 \div \frac{1}{2} + \left(2 \div \frac{1}{3}\right)\right] \div \sqrt{4}$

9) $3z(y + w)$

5) $97 - 75 \div 5 \bullet \sqrt{9} + 68$

10) $ty + yz \div t$

Date___________________

Expressions and Equations (Algebra):

Term: One of the addends of an algebraic expression.

Variable: A symbol, such as a letter, that represents a number.
A placeholder for a number.
Ex. x, y, z

Coefficient: The numerical part of the term, usually before the variable.

x, z and t are variables

Ex. $4x$, $-7z$, $\frac{1}{2}t$

4 is the coefficient

$\frac{1}{2}$ is the coefficient

Constant: A fixed value. A number by itself. A number without a variable.

Polynomial: A monomial or the sum of two or more monomials whose exponents are positive. *Ex.* $4x^8$, $7x + 5y + 11$, $3x^2 + 5x - 12$

Monomial: A polynomial with one term; it is a number, a variable or the product of a number (coefficient) and one or more variables.
Ex. -5, x, $8n$, abc

Binomial: A polynomial with two terms. *Ex.* $4a + 9$

Trinomial: A polynomial with three terms. *Ex.* $9x^2 + 7x – 4$

Numerical Expression: A number or any combination of the sums, differences, products or divisions of numbers that evaluates to a number.

Value of a Numerical Expression: The number found by evaluating the expression.

Expression: A numerical expression or the result of replacing some (or all) of the numbers in a numerical expression with variables.

Equivalent Expressions: Two or more expressions that evaluate to the same number for every substitution of numbers into all the letters.

Equation: A mathematical sentence stating that two expressions are equal.

Date___________________

Evaluating expressions:

To evaluate an expression given values for the variable;

1) Copy the original expression.
2) Re-write the expression substituting the given values.
3) Evaluate the expression.

Ex. 1) $x + 4$ for $x = 10$

$10 + 4$

14

Ex. 2) $3x + y$ for $x = 5$ and $y = 2$

$3(5) + 2$

$15 + 2$

17

Ex. 3) $4m – 8$ for $m = -3$

$4(-3) – 8$

$-12 – 8$

-20

Class practice:

Ex. 1) $x^2 – 3$ for $x = 4$

Ex. 2) $xy + z$ for $x = 2$, $y = 7$, $z = -4$

Ex. 3) $2a – b$ for $a = 8$ and $b = -3$

Date___________________

Combining like terms:

Terms: A number, variable or the product of a number and a variable.

Like terms: Terms that have the same variable parts.

Circle the correct answer.

1.	$4x$ and $-4x$	**Like**	**Unlike**
2.	$5c$ and 5	**Like**	**Unlike**
3.	$12xy$ and $9yx$	**Like**	**Unlike**
4.	$17xx$ and $-12x^2$	**Like**	**Unlike**
5.	8 and -4	**Like**	**Unlike**

Match the like terms.

6.	$6k$	m
7.	$12nn$	12
8.	$3st$	$4xxz$
9.	$8m$	$10x$
10.	-9	$-42k$
11.	x	$15n^2$
12.	$3x^2z$	$5ts$

Give the equivalent expression by combining the like terms.

13. $7n + 2n$

14. $12d - 5d + 2d$

15. $5g + 3y + 8g$

16. $3c + 4f + 7f + 9c$

17. $16a - 4a + 3 + 9$

18. $7s + 4t + 11z$

19. $2r + 6w + 6r$

20. $2k + 9d - 11d$

21. $8p + 7k + 4p$

22. $12f + 7 - 3f - 12$

Date___________________

Equivalent expressions using the Distributive Property:
(Expanding expressions using the distributive property)

1. Start by distributing the outer term across the terms that are on the inside of the parenthesis.
2. Sometimes you will be able to combine like terms.

$5(2x - 3)$
$10x - 15$

Ex. 1) $4(6x + 11)$
$24x + 44$

Notice that the number being distributed is a negative. We must follow the rules for integer multiplication.

Ex. 2) $-3(2x - 4)$
$-6x + 12$

Ex. 3) $\frac{1}{2}(-6x - 12)$
$-3x - 6$

Ex. 4) $-\frac{2}{3}(\frac{1}{2}x + \frac{4}{5})$

Remember to reduce if possible.

$-\frac{2}{6}x - \frac{8}{15} = -\frac{1}{3}x - \frac{8}{15}$

Ex. 5) $5(2x + 8 + 4x)$
$5(6x + 8)$
$30x + 40$

Follow order of operations and combine like terms in the parenthesis.

OR

$5(2x + 8 + 4x)$
$10x + 40 + 20x$
$30x + 40$

Remember, we write the variables in alphabetical order and coefficient first!

Ex. 6) $a(x - 2)$
$ax - 2a$

Date___________________

Adding and subtracting polynomials:

Addition and subtraction of polynomials may be completed horizontally or vertically.

ADD

Ex. Find the sum of $-4x + 11$ and $9x - 2$.

$(-4x + 11) + (9x - 2)$ — **1) It is important to group the expressions so that the terms stay together.**

$-4x + 11 + 9x - 2$ — **2) Distribute to clear the parenthesis**
(Since there is a +1 in front of each set of parenthesis, no changes are made!)

$5x + 9$ — **3) Combine the like terms**

OR

$$\begin{array}{r} -4x + 11 \\ \underline{+\ 9x\ -\ 2} \\ 5x\ +\ 9 \end{array}$$

1) Write the expressions vertically aligning the like terms!!!!
2) Follow the rules of integers!

Ex. Give the resulting expression when $-7x + 12$ is subtracted from $8x - 4$.

$(8x - 4) - (-7x + 12)$ — **1) Re-write, grouping the expressions. Order is important!!**

$8x - 4 + 7x - 12$ — **2) Distribute!**

$15x - 16$ — **3) Combine the like terms.**

There is a +1 in front of the first set of parenthesis, but a -1 in front of the second set.

OR

1) Write the expression vertically. **BE CAREFUL-THE ORDER IS IMPORTANT!**

$$\begin{array}{r} 8x - 4 \\ \underline{-\ (-7x + 12)} \\ 15x - 16 \end{array}$$

Remember to distribute the subtraction sign.

Date____________________

Equivalent expressions by factoring the expression:

To factor an expression, we think of the distributive property in reverse. Is there a common factor in each of the terms in the expression? If so, we can factor it out.

Here's how:

1) Find the GCF (greatest common factor) for the terms in the expression.
2) Move this factor to the outside of a set of parenthesis.
3) With-in the parenthesis, list each term after factoring out the GCF.
4) Check by applying the distributive property.

Ex. 1)

$4x + 10$ — 1) The GCF of 4 and 10 is 2.

$2(\quad)$ — 2) Place the GCF outside of the parenthesis.

$4x \div 2 = 2x$

$10 \div 2 = 5$

$\boxed{2(2x + 5)}$ — 3) Divide **each term** by the GCF and place the quotients in the parenthesis.

Check:

$2(2x + 5)$

$4x + 10$ √

4) To check your answer; use the distributive property. The result should be the original expression.

Ex. 2)

$-6x - 18$

The GCF of -6 and -18 is -6.

$-6x \div -6 = +1x$

$-18 \div -6 = +3$

$-6(+1x + 3)$

$\boxed{-6(x + 3)}$

Remember to check:

$-6(x + 3)$

$-6x - 18$ √

Ex. 3)

$\frac{5}{8}x - \frac{1}{4}$

The GCF of $\frac{5}{8}$ and $-\frac{1}{4}$ is $\frac{1}{8}$.

$\frac{1}{8}(5x - 2)$

Remember to make common denominators to help you find the common factors:

$\frac{1}{4} = \frac{2}{8}$

Date____________________

Solving 1-Step Equations:

Follow the following steps to solve 1-step equations.

1) Identify the variable. Circle it.
2) Determine the operation that is being performed on the variable.
3) Perform the inverse operation on **both** sides of the equation.

Ex. A

$$\begin{aligned} x + \cancel{8} &= 12 \\ -8 \quad & \quad -8 \\ \hline x \quad &= 4 \end{aligned}$$

1) Identify the variable. (x) Circle it.
2) Determine the operation being performed on the variable. (*addition of 8*)
3) Perform the inverse operation. (*subtraction of 8)*

Ex. B

$$\begin{aligned} x - \cancel{3} &= 7 \\ +3 \quad & \quad +3 \\ \hline x \quad &= 10 \end{aligned}$$

1) Identify the variable. (x) Circle it.
2) Determine the operation being performed on the variable. (*subtraction of 3*)
3) Perform the inverse operation. (*addition of 3*)

Ex. C

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{24}{4}$$
$$x = 6$$

1) Identify the variable. (x) Circle it.
2) Determine the operation being performed on the variable. (***multiplication** of 4*)
3) Perform the inverse operation. (*divide by 4*)

Ex. D

$$\cancel{5} \cdot \frac{x}{\cancel{5}} = 3 \cdot 5$$
$$x = 15$$

1) Identify the variable. (x) Circle it.
2) Determine the operation being performed on the variable. (***division** by 5*)
3) Perform the inverse operation. (*multiply by 5*)

These steps can be used regardless of the type of numbers involved. They work for whole numbers, integers, decimals or fractions.

Ex. E

$$\frac{\cancel{-3}x}{\cancel{-3}} = \frac{-15}{-3}$$
$$x = 5$$

Ex. F

$$\begin{aligned} x + \cancel{.75} &= 2.00 \\ -\ .75 \quad & \quad -.75 \\ \hline x \quad &= 1.25 \end{aligned}$$

Date___________________

Solving 2-step Equations:

Follow the following steps to solve 2-step equations.

1) Identify the variable. Circle it.
2) Determine the operations that are being performed on the variable.
3) Perform the inverse operation on **both** sides of the equation in **reverse order of operations order**. *Use the bulls-eye to help you!
4) Check your work. *Start with the original equation!

Ex. A

$$\begin{aligned} 4x + \mathbf{8} &= \mathbf{20} \\ -8 \quad & \;\; -8 \\ \frac{\cancel{4}x}{\cancel{4}} &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

1) Identify the variable. (x) Circle it.
2) Determine the operations being performed on the variable. (*multiplication of 4 and addition of 8*)
3) Perform the inverse operations. (*subtraction of 8 and division by 4*)

ck: **$4x + 8 = 20$**

$4(3) + 8 = 20$

$12 + 8 = 20$

$20 = 20$

√

1) Write the original.
2) Substitute.
3) Solve.

Ex. B

$$\begin{aligned} 3x - \mathbf{5} &= \mathbf{16} \\ +5 \quad & \;\; +5 \\ \frac{\cancel{3}x}{\cancel{3}} &= \frac{21}{3} \\ x &= 7 \end{aligned}$$

1) Identify the variable. (x) Circle it.
2) Determine the operations being performed on the variable. (*multiplication of 3 and subtraction of 5*)
3) Perform the inverse operations. (*addition of 5 and division by 3*)

ck: **$3x - 5 = 16$**

$3(7) - 5 = 16$

$21 - 5 = 16$

$16 = 16$

√

1) Write the original.
2) Substitute.
3) Solve.

Date____________________

Ex. C $\dfrac{\mathbf{x}}{\mathbf{5}} \mathbf{+ 2 = 4}$

$$\underline{-2 \quad -2}$$

$$(5)\,\dfrac{\cancel{x}}{\cancel{5}} = 2\,(5)$$

$$x = 10$$

1) Identify the variable. (*x*) Circle it.
2) Determine the operations being performed on the variable. (*division by 5 and addition of 2*)
3) Perform the inverse operations. (*subtract 2 and divide by 5*)

ck: $\dfrac{\mathbf{x}}{\mathbf{5}} \mathbf{+ 2 = 4}$ — 1) Write the original.

$\dfrac{(10)}{5} + 2 = 4$ — 2) Substitute.

$2 + 2 = 4$ — 3) Solve.

$4 = 4$

√

Ex. D $\dfrac{\mathbf{x}}{\mathbf{2}} \mathbf{- 5 = 5}$

$$\underline{+5 \quad +5}$$

$$(2)\,\dfrac{\cancel{x}}{\cancel{2}} = 10\,(2)$$

$$x = 20$$

1) Identify the variable. (*x*) Circle it.
2) Determine the operations being performed on the variable. (*division by 2 and subtraction of 5*)
3) Perform the inverse operations. (*addition of 5 and multiplication by 5*)

ck: $\dfrac{\mathbf{x}}{\mathbf{2}} \mathbf{- 5 = 5}$ — 1) Write the original.

$\dfrac{(20)}{2} - 5 = 5$ — 2) Substitute.

$10 - 5 = 5$ — 3) Solve.

$5 = 5$

√

Try these:

1) $\mathbf{6x + 2 = 14}$

2) $\dfrac{\mathbf{x}}{\mathbf{2}} \mathbf{- 5 = 3}$

These steps can be used regardless of the type of numbers involved. They work for whole numbers, integers, decimals or fractions.

Date____________________

Solving equations with combining like terms:

1. $4n + 7n = 44$

2. $21n - 12n = -18$

3. $3n + 2n + 2 = 37$

4. $23h - 3h - 10 = 50$

5. $\frac{1}{4}k + \frac{2}{3}k - 14 = 13$

6. $7w + 3\frac{4}{5} + 2w - 1\frac{2}{3} = 38$

7. $2c + 5c + c + 9 = 65$

8. $12 + 13q - 7q = 54$

Date____________________

Expanding polynomials using the distributive property to solve an equation:

$3(4x + 2) = 42$

$12x + 6 = 42$

$\underline{\quad - 6 \quad -6}$

$\dfrac{12x}{12} = \dfrac{36}{12}$

$x = 3$

1. Distribute the outer term across the terms inside the parenthesis.
2. Solve the 2-step equation.
3. Check. Start with the original equation.

ck: $3(4x + 2) = 42$

$3(4 \bullet 3 + 2)$

$3(12 + 2)$

$3(14)$

$42 = 42$

Ex. 1) $2(3x + 4) = 68$

Ex. 2) $-\frac{1}{4}(-2x + 3) = 6$

Ex. 3) $-\frac{2}{3}(\frac{3}{8}x - \frac{1}{4}) = -\frac{1}{6}$

Date___________________

Writing and solving equations using tape diagrams:

John and Ag are summarizing some of the expenses of their family vacation for themselves and their three children, Louie, Missy, and Bonnie. Create a model to determine how much each item will cost, using all of the given information. Then, answer the questions that follow.

Expenses:

Car & insurance fees $400	Airfare & insurance fees $875	Motel & tax $400
Baseball Game and hats: $103.83	Movies for one day $75	Soda & Pizza $37.95
	Sandals & T-Shirts $120	

7M2L17

Date___________________

Scenario 1

During one rainy day on the vacation, the entire family decided to go watch a matinee movie in the morning and a drive-in movie in the evening. The price for a matinee movie in the morning is different than the cost of a drive-in movie in the evening. The tickets for the matinee morning movie cost $6 each. How much did each person spend that day on movie tickets if the ticket cost for each family member was the same? What was the cost for a ticket for the drive-in movie in the evening?

Tape Diagram

$$\overbrace{\boxed{e+6}\boxed{e+6}\boxed{e+6}\boxed{e+6}\boxed{e+6}}^{75}$$

John *Ag* *Louie* *Missy* *Bonnie*

$5e$ $75 - 30 = 45$

$5(6) = 30$ $45 \div 5 = 9$

The total each person spent on movies in one day was $15. The evening drive-in movie costs $9 each.

Algebraic Equation & Solution

Morning matinee movie: $6 each

Evening Drive-In Movie: e each

$5(e+6) = 75$ *OR* $5(e+6) = 75$

$5e + 30 = 75$

$\left(\frac{1}{5}\right)5(e+6) = 75\left(\frac{1}{5}\right)$

$5e + 30 - 30 = 75 - 30$

$e + 6 = 15$

$5e + 0 = 45$

$e + 6 - 6 = 15 - 6$

$\left(\frac{1}{5}\right)5e = 45\left(\frac{1}{5}\right)$

$e = 9$

$1e = 9$

$e = 9$

Date___________________

Scenario 2

For dinner one night, the family went to the local pizza parlor. The cost of a soda was $3. If each member of the family had a soda and one slice of pizza, how much did one slice of pizza cost?

Tape Diagram

$$\overbrace{\boxed{p+3}\boxed{p+3}\boxed{p+3}\boxed{p+3}\boxed{p+3}}^{37.95}$$

John	Ag	Louie	Missy	Bonnie

Algebraic Equation & Solution

One Soda: $3

Slice of Pizza: p *dollars*

$5(p+3) = 37.95$ *OR* $5(p+3) = 37.95$

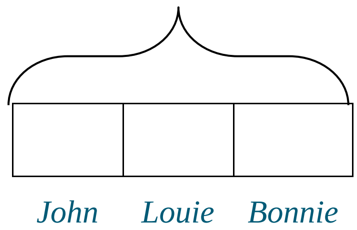
One slice of pizza costs $4.59.

Date___________________

Scenario 3

One night, John, Louie and Bonnie went to the see the local baseball team play a game. They each bought a ticket to see the game and a hat that cost $10 each. How much was each ticket to enter the ballpark?

Tape Diagram



Algebraic Equation & Solution

Ticket: t *dollars*

Hat: $10

$3(t + 10) = 103.83$ *OR* $3(t + 10) = 103.83$

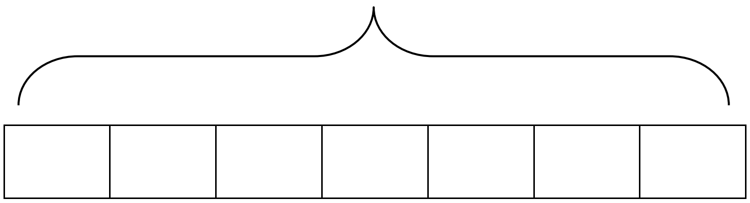
One ticket costs

Date____________________

Scenario 4

While John, Louie and Bonnie went to see the baseball game, Ag and Missy went shopping. They bought a t-shirt for each member of the family and bought two pairs of sandals that cost $10 each. How much was each T-shirt?

Tape Diagram



Algebraic Equation & Solution

T-Shirt: t dollars

Sandals: $2 \times \$10 = \$20$

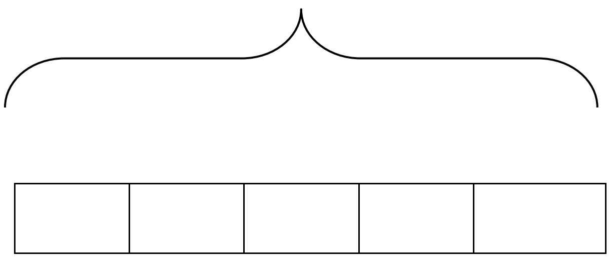
One t-shirt costs

Date___________________

Scenario 5

The family was going to fly in an airplane to their vacation destination. Each person needs to have their own ticket for the plane, and also pay $25 in insurance fees per person. What was the cost of one ticket?

Tape Diagram



Algebraic Equation & Solution

One ticket: t dollars

Insurance: $25 *per person*

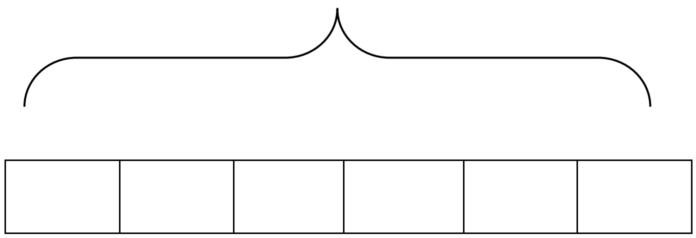
One ticket costs

Date___________________

Scenario 6

While on vacation, the family rented a car to get them to all the places they wanted to see for five days. The car costs a certain amount each day, plus a one-time insurance fee of $50. How much was the daily cost of the car (not including the insurance fees)?

Tape Diagram



Algebraic Equation & Solution

Daily fee: d dollars

Insurance fee: $50

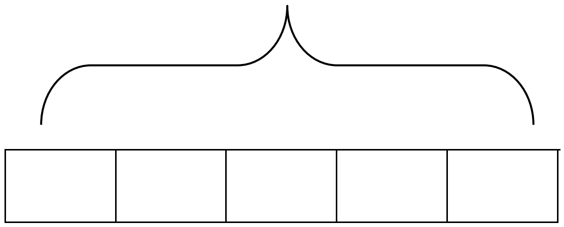
One day costs

Scenario 7

Date___________________

The family decided to stay in a motel for 4 nights. The motel charges a nightly fee plus $60 in state taxes. What is the nightly charge with no taxes included?

Tape Diagram



Algebraic Equation & Solution

One night costs

Date___________________

Complete the table below.

Cost of Evening Movie	
Cost of 1 Slice of Pizza	
Cost of the admission ticket to the baseball game	
Cost of 1 T-Shirt	
Cost of 1 Airplane Ticket	
Daily Cost for Car Rental	
Nightly charge for Motel	

Using the results, determine the cost of:

2. A slice of pizza, 1 plane ticket, 2 nights in the motel, and 1 evening movie

3. One t-shirt, 1 ticket to the baseball game, 1 day of the rental car

Write and solve an algebraic equation:

4. The cost of a babysitting service on a cruise is $10 for the first hour, and $12 for each additional hour. If the total cost of babysitting baby Aaron was $58, how many hours was Aaron at the sitter?

Date___________________

Solving an equation with variables on both sides:

$$\begin{array}{rcl} 4x + 2 &=& 2x + 8 \\ \underline{-2x} & & \underline{-2x} \\ 2x + 2 &=& + 8 \\ \underline{-2} & & \underline{-2} \\ \frac{2x}{2} &=& \frac{6}{2} \\ x &=& 3 \end{array}$$

1) Move the variable term with the smaller coefficient to the opposite side by performing the inverse operation.
2) Solve the 1-step or 2-step equation that is formed.
3) Check. Start with the <u>original</u> equation.

ck:
$$\begin{array}{rcl} 4x + 2 &=& 2x + 8 \\ 4(3) + 2 &=& 2(3) + 8 \\ 12 + 2 &=& 6 + 8 \\ 14 &=& 14 \end{array}$$

Ex. 1) $7x - 4 = 4x + 2$

Ex. 2) $6x + 4 = 8x + 6$

Ex. 3) $-2x - 2 = 3x - 12$

Ex. 4) $\frac{1}{2}x - \frac{1}{4} = \frac{1}{4}x + 1\frac{1}{2}$

Date____________________

Solving multi-step equations:

When solving a multi-step equation, you may need to use some or all of the procedures learned. ☺

First- Simplify one side, then the other. You must simplify each side completely before starting to move the variable. As you are simplifying, you may need to combine like terms, distribute or both. Don't forget about order of operations. *Remember, when you are working on a single side, use the operations as shown!*

Next – Continue solving the equation by moving the variable terms so that they are on the same side. *Remember, when you move a term to the opposite side, you must use the opposite operation!*

Finally – Solve the remaining 1 or 2 step equation!!!

Don't forget to check! You must start with the* ***original*!!!!*

Ex. $-2(3y - 6 - 2y) = 5(5 + y + -9) + y$

Left side:

combine: $-2(y - 6) =$

distribute: $-2y + 12 =$

Right side: combine: $= 5(y - 4) + y$

distribute: $= 5y - 20 + y$

combine: $= 6y - 20$

$$-2y + 12 = 6y - 20$$

Move the term with the smaller coefficient.

$$\underline{+2y \qquad + 2y}$$

$$12 = 8y - 20$$

Solve the remaining equation:

$$+\underline{20 \qquad +20}$$

$$\frac{32}{8} = \frac{8y}{8}$$

$$4 = y$$

$$y = 4$$

Check:

$-2(3y - 6 - 2y) = 5(5 + y + -9) + y$

$-2(3 \cdot 4 - 6 -+ 2 \cdot 4) = 5(5 + 4 + -9) + 4$

$-2(12 - 6 - 8) = 5(0) + 4$

$-2(-2) = 0 + 4$

$4 = 4$

√

☺

Date___________________

Solving Inequalities:

Inequality: The comparison of two expressions that uses one of the following symbols <, >, ≤, or ≥.

- < - less than: the number before the symbol is smaller than the number after the symbol
- > - greater than: the number before the symbol is larger than the number after the symbol
- ≤ - less than **or equal to**: the number before the symbol is smaller than **or equal to** the number after the symbol
- ≥ - greater than **or equal to**: the number before the symbol is larger than **or equal to** the number after the symbol

You can solve inequalities by following steps that are similar to those used when solving equations. However, the solution to an inequality is the set of all numbers that make the inequality true.

Ex)

$$\begin{array}{rcr} x + 8 & > & 14 \\ \underline{\quad - 8} & & \underline{-8} \\ x & > & 6 \end{array}$$

*answer: all numbers larger than 6
(6.5, 7, 7½ , 9, 100, etc.)

Ex)

$$\frac{9x}{9} \geq \frac{18}{9}$$

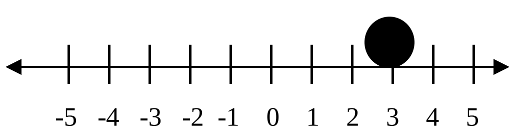
$$x \geq 2$$

*answer: all numbers larger than or equal to 2
(**2**, 3, 5½, 9.8, 99, etc.)

**If you divide or multiply by a negative coefficient, you must reverse the inequality symbol.*

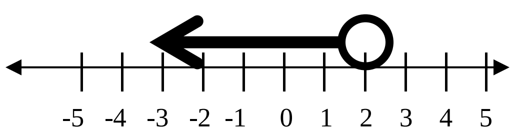
Graphing Equalities and Inequalities:

$x = 3$



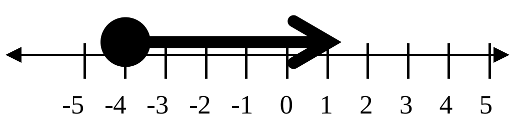
Single point on value
(The value is equal.)

$n < 2$



Open point with arrow
(Greater than or less than.)

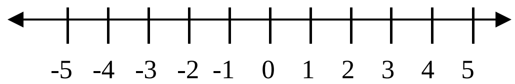
$a \geq - 4$



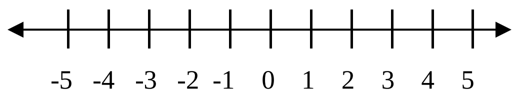
Closed point with arrow
(Greater than or equal to or less than or equal to)

Try these:

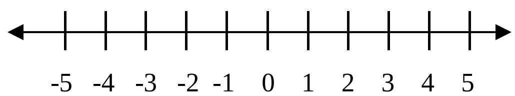
1) $x + 2 = 6$



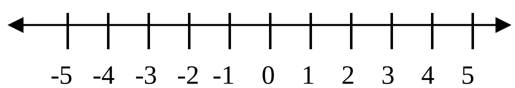
2) $x - 1 > 3$



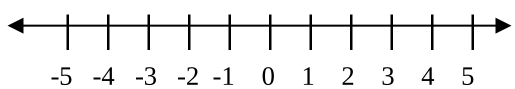
3) $x + 2 < 6$



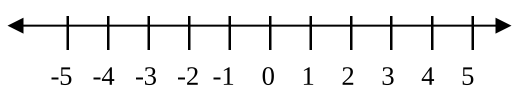
4) $3x \geq 6$



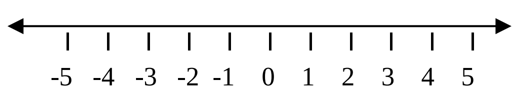
5) $x + 4 \leq 7$



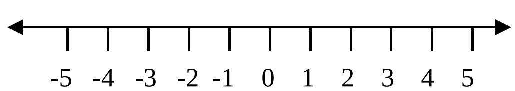
6) $x + 2 < -2$



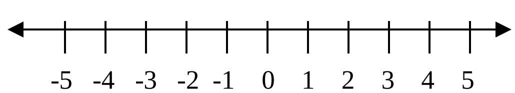
7) $\frac{x}{2} \geq -1$



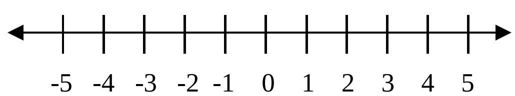
8) $x - 3 > 0$



9) $-2 < u \leq 3$



10) $0 < f \leq 5$



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Ratios, Proportional Relationships and Percent

<u>Ratio</u>: An ordered pair of non-negative numbers, which are not both zero. The comparison of two numbers by division.

Ratios can be denoted (written) in three ways:

1) fraction: 1/15
2) colon: 1 : 15
3) to: 1 to 15

All three ways are read 1 to 15.

Ratios are always given in lowest terms, but are not converted to mixed numbers.

Numbers are in lowest terms if they are relatively prime.
(Relatively prime: the greatest common factor is one.)

Example 2:

	Number of boys	Number of girls	Ratio of boys to girls	Ratio of girls to boys
Class 1	8	11		
Class 2	10	11		
Class 3	14	10		
Class 4	10	12		
Total				

*When possible, the quantities of the ratio must be converted to the same unit of measure.

Ex. If you were asked to compare feet and inches, you would need to convert them to the same unit. It would be preferable to change the feet into inches to avoid having a fraction or decimal within the ratio.

Ex. 1) 2 feet : 18 inches → 24 inches to 18 inches
Ex. 2) 2 days : 1 week → 2 days to 7 days

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Common conversions:

12 inches in a foot	16 ounces in a pound
3 feet in a yard	8 ounces in a cup
36 inches in a yard	2 cups in a pint
5280 feet in a mile	16 ounces in a pint
1760 yards in a mile	2 pints in a quart
	4 quarts in a gallon

Equivalent Ratios: Two ratios that have the same value.

Rate: A ratio comparing two different quantities.

Ex. 125 words in 2 minutes

$\frac{\text{125 words}}{\text{2 minutes}}$ or 125 words : 2 minutes

Unit Rate: A rate with a denominator of 1. *Divide the numerator by the denominator.*

Ex. We can travel 96 miles using 3 gallons of gasoline.

$\frac{\text{96 miles}}{\text{3 gallons}} = \frac{\text{32 miles}}{\text{1 gallon}} \rightarrow$ 32 miles per gallon or 32 mpg

To find the unit rate: Divide the numerator by the denominator. The quotient becomes the new numerator, the new denominator is 1.

Examples: Find the unit rate.

Ex.) $2\frac{1}{2}$ miles in $\frac{1}{2}$ hr

Remember, to divide fractions, we **KEEP – CHANGE – FLIP!**

$2\frac{1}{2} \div \frac{1}{2}$

$\frac{5}{2} \times \frac{2}{1}$

$\frac{10}{2} = 5$ 5 miles per hour or 5 mph

Date___________________

Key Terms from Grade 6 Ratios and Unit Rates

A ratio is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A: B$ to indicate the order of the numbers: the number A is first and the number B is second.

Two ratios $A: B$ and $C: D$ are **equivalent ratios** if there is a positive number, c, such that $C = cA$ and $D = cB$.

A ratio of two quantities, such as 5 miles per 2 hours, can be written as another quantity called a **rate**.

The numerical part of the rate is called the **unit rate** and is simply the value of the ratio, in this case 2.5. This means that in 1 hour the car travels 2.5 miles. The **unit** for the rate is miles/hour, read miles per hour.

Lesson Summary

Unit Rate is often a useful means for comparing ratios and their associated rates when measured in different units. The unit rate allows us to compare varying sizes of quantities by examining the number of units of one quantity per 1 unit of the second quantity. This value of the ratio is the unit rate.

Unit Price: The price per unit of an item.
**Always divide the price by the quantity.*

Ex. 1) 15 cookies cost \$1.80

$\frac{\$0.12}{1 \text{ cookie}}$ or 12¢ per cookie

Ex. 2) \$1.75 for 3 muffins → \$0.583333333

\$0.58 or 58¢ per muffin

(1.75 ÷ 3)

Remember, to round money to 2 decimal places.

Exercise 1: Which is the Better Buy?

Value-Mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for \$7.97 whereas a 12-pack of the same brand cost for \$ 4.77. Which is the better buy? How do you know?

Date___________________

Proportional Relationships:

Proportion: A statement that two ratios are equal. *Ex:* $\frac{10}{160} = \frac{9}{144}$

Proportional: (Proportional to) Measures in one quantity are proportional to measures of a second quantity if there is a positive number, *k*, so that for every measure *x* of the first quantity, the corresponding quantity *y* is given by *kx*. The equation $y = kx$ models this relationship.

> *Two quantities are proportional if there is a positive constant value (k) being multiplied by the x value to give the corresponding y value.*

Proportional relationship: A proportional relationship is one in which the measures of one quantity are proportional to the measures of the second quantity.

Solving a Proportion:

To solve a proportion: Use cross multiplication.

1. Find the cross products.
2. Set the two cross products equal to each other.
3. Solve for n.

Ex. $\frac{3}{5} = \frac{n}{15}$

$3 \cdot 15 = 5 \cdot n$

$\frac{45}{5} = \frac{5n}{5}$

$9 = n$

$n = 9$

Find the value of n.

1) $\frac{7}{12} = \frac{n}{30}$

2) $\frac{12}{n} = \frac{5}{4}$

3) $\frac{1}{8} = \frac{n}{24}$

Date____________________

Determining if two ratios form a proportion (proportional relationship).

Method 1: To determine if two ratios form a proportion, cross multiply.

- If the product of the crosses is equal, it is a proportion.
- If the product of the crosses is **not** equal, it is **not** a proportion.

Ex.)

$36 = 36$ $\qquad$ $10 \neq 9$

$\frac{3}{4} = \frac{9}{12}$ $\qquad$ $\frac{1}{3} = \frac{3}{10}$

PROPORTION $\qquad$ **NOT A PROPORTION**

Method 2: To find a proportional relationship from a table:

Calculate the unit rate for each set of data.

- If the ratios created from the data in a table are equivalent, the relationship **is proportional**.

Ex. <u>Barry's Balloons</u>

Number of balloons	10	18	24
Cost	$1.50	$2.70	$3.60

Calculate the unit rate for each set of data.

$\frac{1.50}{10} = 0.15$ $\quad$ $\frac{2.70}{18} = 0.15$ $\quad$ $\frac{3.60}{24} = 0.15$

Since the unit rate for all three balloon amounts is equal, there is a proportional relationship. The constant of proportionality is 0.15.

Date____________________

- If the ratios created from the data in a table are **<u>not</u>** equivalent, the relationship **is <u>NOT</u> proportional**.

Ex. <u>Balloons by Bart</u>

Number of balloons	10	18	24
Cost	\$1.50	\$2.50	\$3.20

Calculate the unit rate for each set of data.

$\frac{1.50}{10} = 0.15$ $\frac{2.50}{18} \approx 0.14$ $\frac{3.20}{24} \approx 0.13$

Since the unit rate for the three balloon amounts is **not** equal, there is **not** a proportional relationship.

Proportional Relationships

Classwork

Example 1: Pay by the Ounce Frozen Yogurt!

A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

Weight (ounces)	12.5	10	5	8
Cost (\$)	5	4	2	3.20

Unit rate: (cost per ounce) _______________ Constant of proportionality.______________

Weight (ounces)	0	12.5	10	5	8	15	18	$22\frac{1}{2}$
Cost (\$)		5	4	2	3.20			

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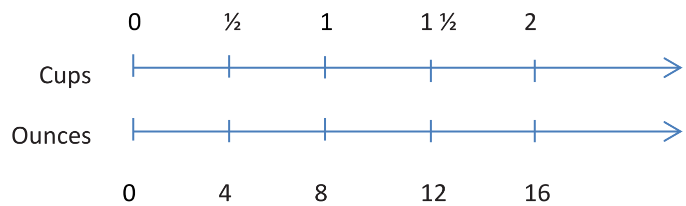
Example 2: A Cooking Cheat Sheet!

In the back of a recipe book, a diagram provides easy conversions to use while cooking.

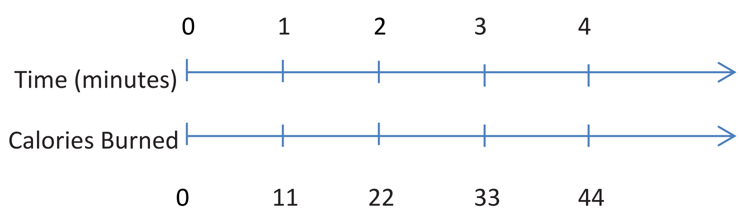


Unit rate: (ounces per cup) ________________

Practice Exercise 1

During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting how many Calories (on average) would be burned by completing the activity.

Calories burned while Jumping Rope



a. Is the number of Calories burned proportional to time? How do you know?

b. If Jose jumped rope for 6.5 minutes, how many calories would he expect to burn?

Example 3: Summer Job

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To check his assumption, he decided to make a table. He entered his total money earned at the end of week 1 and his total money earned at the end of Week 4.

Week	0	1	2	3	4	5	6	7	8
Total Earnings		$28			$112				

a. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

b. Will Alex have enough money at the end of the summer? Explain.

Date___________________

Lesson Summary:

Measures in one quantity **are proportional to** measures of a second quantity if there is a positive number k so that for every measure x of the first quantity, the corresponding quantity y is given by kx. The equation $y = kx$ models this relationship.

A **proportional relationship** is one in which the measures of one quantity are proportional to the measures of the second quantity.

In the example given below, the distance *is proportional to* time since each measure of distance, y, can be calculated by multiplying each corresponding time, t, by the same value, 10. This table illustrates a *proportional relationship* between time, t, and distance, y.

Time (hrs), t	0	1	2	3
Distance (km), y	0	10	20	30

Identifying Proportional and Non-Proportional Relationships in Tables

Classwork

You have been hired by your neighbors to babysit their children on Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

Hours Worked	Pay
1	
2	
3	
4	
4 ½	
5	
6	
6.5	

Based on the table above, is pay proportional to hours worked? How do you know?

Date___________________

Examples 1–4

For Examples 1–3, determine if y is proportional to x. Justify your answer.

d. The table below represents the amount of snow fall in 5 counties (in inches) to hours of a recent winter storm.

x Time (hrs)	y Snowfall (In)
2	10
6	12
8	16
2.5	5
7	14

e. The table below shows the relationship between cost of renting a movie to the number of days on rent.

x Number of Days	y Cost
6	2
9	3
24	8
3	1

f. The table below shows the relationship between the amount of candy (pounds) bought and the total cost.

x Pounds	y Cost
5	10
4	8
6	12
8	16
10	20

g. Randy is planning to drive from New Jersey to Florida. Randy recorded the distance traveled and the total number of gallons used every time he stopped for gas.

Assume miles driven is proportional to Gallons Consumed in order to complete the table.

Gallons Consumed	2	4		8	10	12
Miles Driven	54		189	216		

Lesson Summary:

One quantity is proportional to a second if a constant (number) exists such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.

Steps to determine if two quantities in a table are proportional to each other:

a. For each given measure of Quantity A and Quantity B, find the value of $\frac{B}{A}$.

b. If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities are proportional to each other.

Date____________________

Example: Which Team Will Win the Race?

You have decided to run in a long distance race. There are two teams that you can join. Team A runs at a constant rate of 2.5 miles per hour. Team B runs 4 miles the first hour and then 2 miles per hour after that.

Task: Create a table for each team showing the distances that would be run for times of 1, 2, 3, 4, ,5 and 6 hours.

Team A	
Time (hrs)	Distance (miles)

Team B	
Time (hrs)	Distance (miles)

a. For which team is distance proportional to time? Explain your reasoning.

b. Explain how you know distance for the other team is not proportional to time.

c. If the race were 2.5 miles long, which team would win? Explain.

If the race were 3.5 miles long, which team would win? Explain.

If the race were 4.5 miles long, which team would win? Explain.

d. For what length race would it be better to be on Team B than Team A? Explain

e. Using this relationship, if the members on the team ran for 10 hours, how far would each member run on each team?

f. Will there always be a winning team, no matter what the length of the course? Why or why not?

g. If the race is 12 miles long, which team should you choose to be on if you wish to win? Why would you choose this team?

h. How much sooner would you finish on that team compared to the other team?

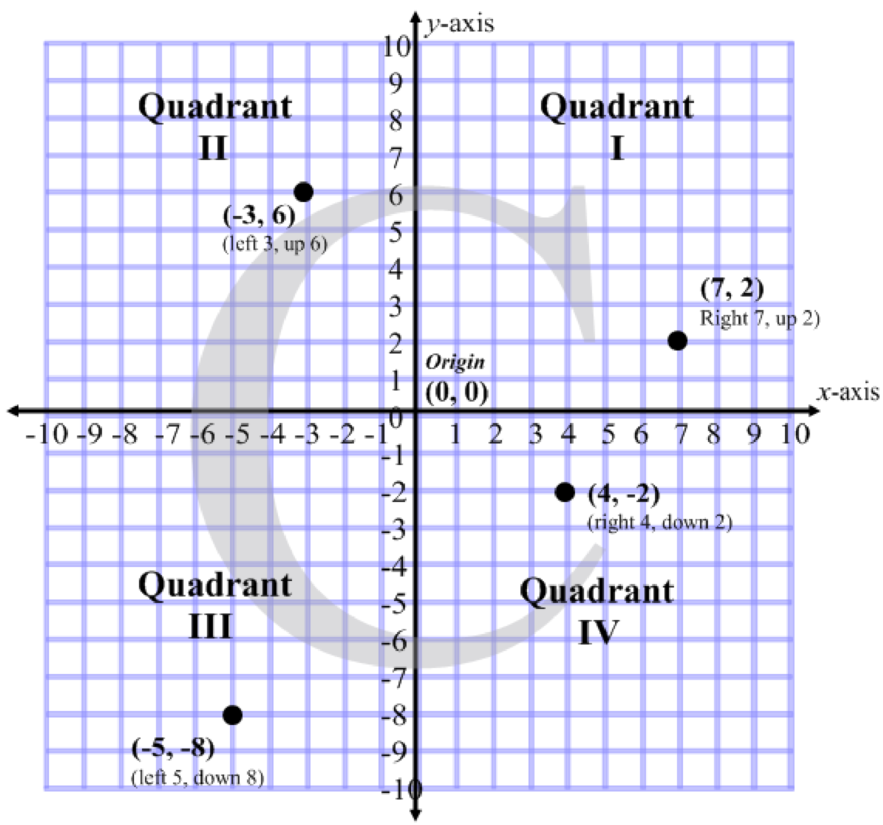
Date___________________

Coordinate Graphing Review

Quadrant: The *x* and *y* axes divide the coordinate plane into four sections called quadrants. They are labeled: I, II, III, IV.

Information is given to you in ordered pairs. Ex. (4, -2)

The first number in the parenthesis is the "x" value and is placed on the horizontal axis. The second number is the "y" value and is placed along the vertical axis.



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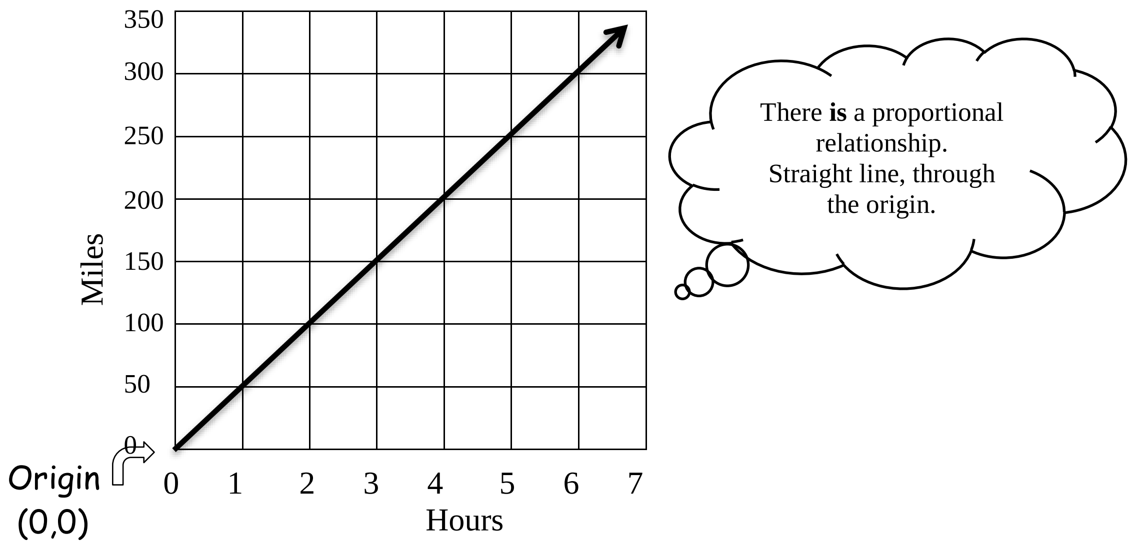
Graphing proportional relationships:

- Plot the data points on a coordinate plane.

If the line plotted is a straight line passing through the **origin**, there is a proportional relationship.

Ex.

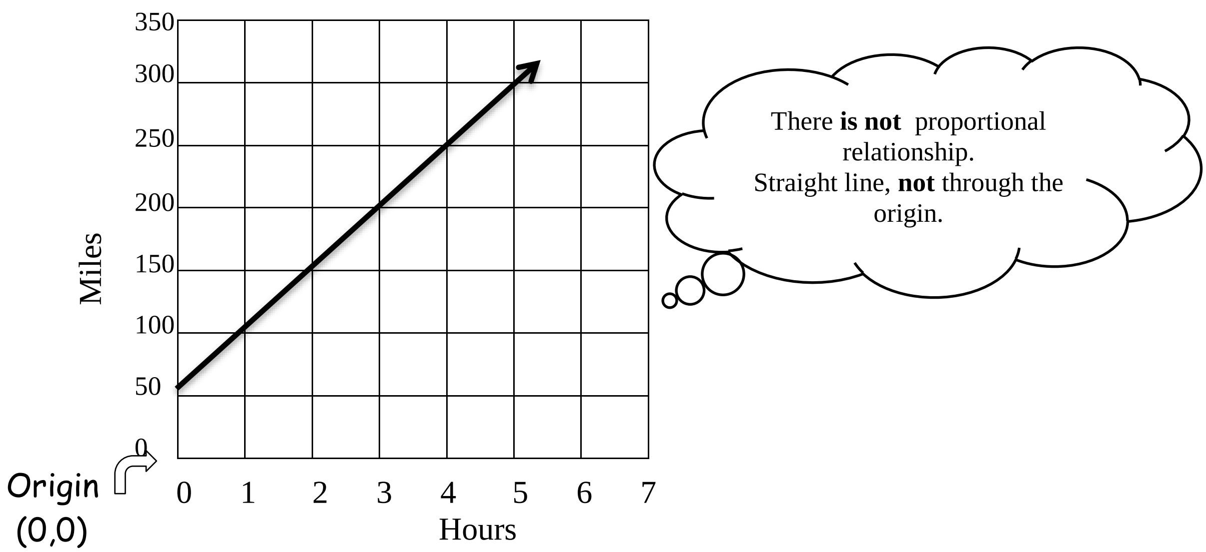
Miles	50	100	150
Hours	1	2	3



Ex. 2)

This team gets a 50 mile head start because they will need to stop and charge their car.

Miles	50	100	150
Hours	0	1	2



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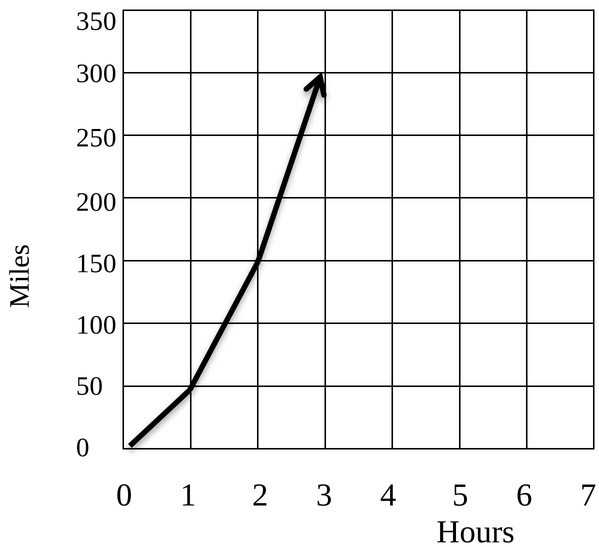
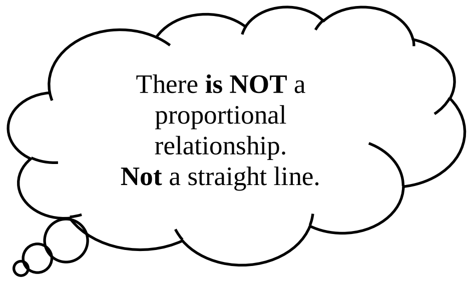
Ex. 3)

This team drives at an inconsistent rate of speed.

Miles	50	150	300
Hours	1	2	3





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Identifying Proportional and Non-Proportional Relationships in Graphs

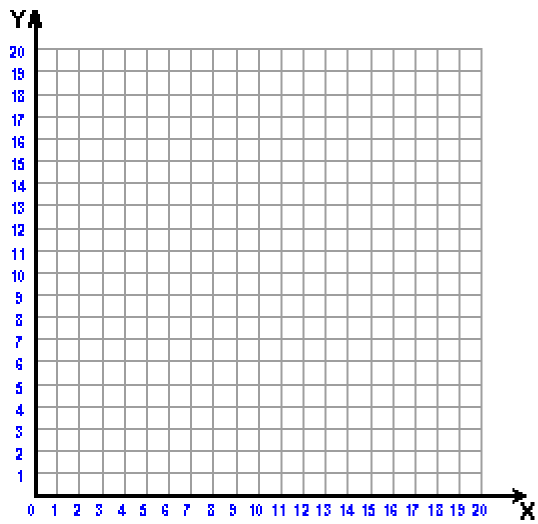
Classwork

Opening Exercise

Isaiah sold candy bars to help raise money for his scouting troop. The table shows the amount of candy he sold to the money he received.

Example 1: From a Table to Graph

x Candy Bars Sold	y Money Received ($)
2	3
4	5
8	9
12	12



Is the amount of candy bars sold proportional to the money Isaiah received? How do you know?

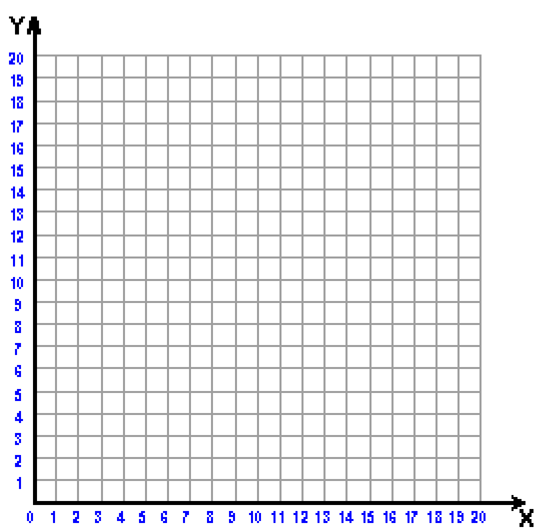
____

____

Example 2

Create a table that shows a proportional.

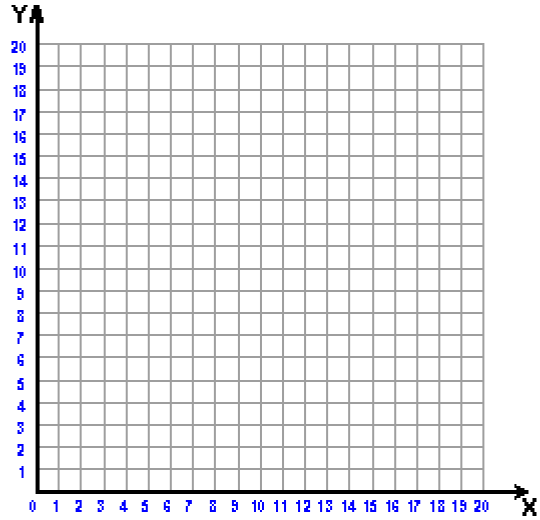
x	y
2	3



Date____________________

Example 3

x	y
0	6
3	9
6	12
9	15
12	18



Lesson Summary:

When two proportional quantities are graphed on a coordinate plane, the points lie on a straight line that passes through the origin.

<u>Identifying the Constant of Proportionality:</u>

- The **<u>Constant of Proportionality</u>** is another name for the **<u>unit rate</u>**.

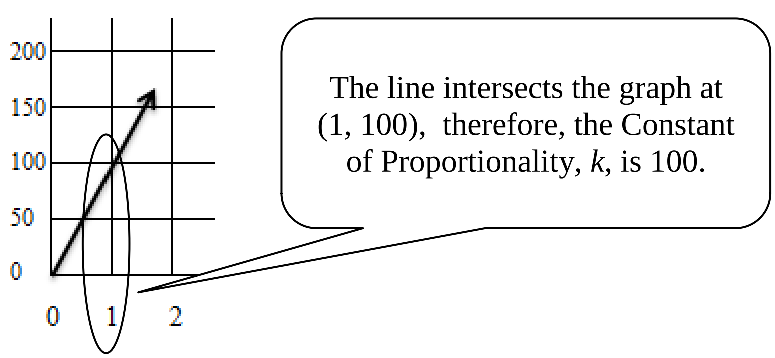
  There are many ways to determine the constant of proportionality:

  - **From an equation**: In the equation $y = kx$, the k is the Constant of Proportionality.

  - **From a table**: In a table, the unit rate is the Constant of Proportionality

  - **From a graph or diagram**: The "*y*" value that corresponds to the "*x*" value of 1 is the Constant of Proportionality. $k = \frac{y}{x}$

    Ex.



Date___________________

Lesson 7: Unit Rate as the Constant of Proportionality

Classwork

Example 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 216 deer in a 24 square acre plot of the forest. Do conservationists need to be worried?

a. What is the population density of deer per square mile?

Table:

The Unit Rate of deer per 1 square mile is _______.

Constant of Proportionality:

Meaning of Constant of Proportionality in this problem:

b. Use the unit rate of deer per square mile to determine how many deer are there for every 207 square miles.

c. Use the unit rate to determine the number of square miles in which you would find 486 deer?

Date___________________

Vocabulary:

A **constant** specifies a unique number.

A **variable** is a letter that represents a number.

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the **constant of proportionality**. It is the value that describes the multiplicative relationship between two quantities, x and y. The (x, y) pairs represent all the pairs of values that make the equation true.

Note: In a given situation, it would be reasonable to assign any variable as a placeholder for the given quantities. For example, a set of ordered pairs (t, d) would be all the points that satisfy the equation $d = rt$, where r is the positive constant, or the constant of proportionality. This value for r specifies a unique number for the given situation.

Example 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needed 3 cookies for each of the 96 students in 7^{th} grade. Unfortunately, he needed the cookies for an event at school on the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets.

d. Is the number of cookies proportional to the number of sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies needed.

Table:

The Unit Rate is ___________.

Constant of Proportionality:

Meaning of Constant of Proportionality in this problem:

e. It took 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 pm, when will they finish baking the cookies?

Example 3: French Class Cooking

Suzette and Margo want to prepare crepes for all of the students in their French class. A recipe makes 20 crepes with a certain amount of flour, milk, and 2 eggs. The girls know that they already have plenty of flour and milk but need to determine the number of eggs needed to make 50 crepes because they are not sure they have enough eggs for the recipe.

f. Considering the amount of eggs necessary to make the crepes, what is the constant of proportionality?

g. What does the constant or proportionality mean in the context of this problem?

h. How many eggs will be needed for 50 crepes?

Date___________________

Lesson #9 – Representing Proportional Relationships with Equations

- Defining the variable(s):

  When writing an equation to represent a proportional relationship it is often helpful to change the variable from the usual *x* and *y*, to variables that are more reflective of the problem. We often use the first letter of the word that the variable is representing. When identifying these variables, we write let statements. Let statements assign meaning to the variable.

  Ex. Let *m* = the number of miles we drove
  Let *g* = the number of gallons used

- Determining the independent and dependent variables:

  Often, several different equations can be written for a single problem. The determining factors are the independent and dependent variables.

  The **independent variable** represents the information you know.
  The **dependent variable** represents the information you are looking for or the question.

- Determining the constant of proportionality:

  The constant of proportionality or unit rate is determined by the question. The order of the values in the question should be followed. The value that you are finding should be in the numerator.

Ex.
Bethany and her mom bake cookies each holiday season to sell at the school holiday fair. If Bethany can make 150 cookies in 2 hours, write an equation that will allow her to calculate the time it will take her to bake a given number of cookies.

Let *c* = the number of cookies
Let *h* = the number of hours

c will be the independent variable, because in this question, we know the number of cookies.

h will be the dependent variable because we are trying to find the amount of hours needed to make the cookies.

The question ask how long it will take her to bake any number of cookies, so constant of proportionality will be hours to cookies.

Constant of proportionality is: $\frac{2}{150} = \frac{1}{75}$

Remember:

Dependent variable $y = kx$ Independent variable

So we should write the equation $h = \frac{1}{75}c$ to find the number of hours needed to make any amount of cookies.

Ex. We need 400 cookies: $h = \frac{1}{75}c$

$h = \frac{1}{75}(400)$

$h = \frac{400}{75}$

$h = 5.\overline{3}$ hours or 5 hours and 20 minutes.

Using the same story, the question might be how many cookies could she bake in 7 hours?

h = independent variable (we know it is 7 hours)
c = dependent variable (what we are looking for)

constant of proportionality: $\frac{150}{2} = \frac{75}{1}$

$c = \frac{75}{1}h$

ex. $c = \frac{75}{1}h$

$c = \frac{75}{1}(7)$

$c = \frac{525}{1}$

$c = 525$ cookies

Date____________________

Example 1: Do We have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother's Gas Record

Gallons	Miles driven
8	*224*
10	*280*
4	*112*

i. Find the constant of proportionality and explain what it represents in this situation.

j. Write an equation that will relate the miles driven to the number of gallons of gas.

k. Knowing that there is a half gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

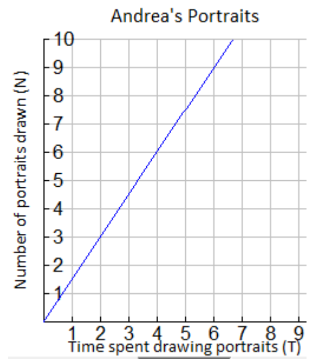
l. Using the equation found in part k, determine how far your mother can travel on 18 gallons of gas.

m. Using the equation found in part *k*, determine how many gallons of gas would be needed to travel 750 miles.

Date___________________

Example 2: Andrea's Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits of tourists). People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours needed to draw the portraits.



n. Write several ordered pairs from the graph and explain what each coordinate pair means in the context of this graph.

o. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

p. Determine the constant of proportionality and explain what it means in this situation.

Lesson Summary:

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the *constant of proportionality*.

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Using Proportions to calculate Percent:

(part of 100)

To solve any type of percent question use the proportion: $\dfrac{\%}{100} = \dfrac{\text{is (part)}}{\text{of (whole)}}$

1) Determine the parts of the equation based on the question.

2) Substitute the value in the appropriate place. (Use a variable for the unknown.)

3) Cross multiply and divide to find the missing value.

Ex. 1. What percent of 150 is 30?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

$$\frac{x}{100} = \frac{30}{150}$$

$$100 \cdot 30 = 150 \cdot x$$

$$\frac{3000}{150} = \frac{150x}{150}$$

$$20\% = x$$

$$x = 20\%$$

Remember, when multiplying by 100, add two zeros.

Only label the answer with a % if you are finding the percent!

Ex. 2. 25% of what number is 18?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

$$\frac{25}{100} = \frac{18}{x}$$

$$100 \cdot 18 = 25 \cdot x$$

$$\frac{1800}{25} = \frac{25x}{25}$$

$$72 = x$$

$$x = 72$$

Remember, when multiplying by 100, add two zeros.

Date____________________

Ex. 3. 30 % of 180 is what?

$$\frac{\%}{100} = \frac{is}{of}$$

Remember, to divide by 100, move the decimal point two places to the left.

$$\frac{30}{100} = \frac{x}{180}$$

$$30 \cdot 180 = 100 \cdot x$$

$$\frac{5400}{100} = \frac{100x}{100}$$

$$54 = x$$

$$x = 54$$

Try these:

1) What is 15% of 95?

2) What percent of 50 is12?

3) 32% of what number is 15?

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Converting between fraction – decimals - percents

$\frac{2}{5} \longrightarrow$ divide: $5\overset{.4}{\overline{)2.0}} \longrightarrow 0.4 \longrightarrow$ x 100 $\longrightarrow$ 40%

(move the decimal 2 places to the right)

$\frac{6}{10} = \frac{3}{5} \longleftarrow$ “say it” $\longleftarrow$ 0.6 $\longleftarrow$ ÷ 100 $\longleftarrow$ 60%

(move the decimal 2 places to the left)

$\frac{5}{8} \longrightarrow$ proportion: $\frac{5}{8} = \frac{x}{100} \longrightarrow$ 62.5%

$\frac{54}{100} = \frac{27}{50} \longleftarrow$ percents are always over 100 $\longleftarrow$ 54%

Converting Fractions to Decimals to Percents

Fractions to Decimals: To convert a fraction to a decimal:

a) create an equivalent fraction with a denominator of a power of ten and then write the decimal in the correct place value.

or

b) divide the numerator by the denominator.

Decimals to Fractions: To convert decimals to fractions:

place the decimal over the appropriate power of ten denominator.

* If it is a repeating decimal, use the process for repeating decimals. Let n = the repeating decimal

Let 10n or 100n or 1000n = (depending on the number of repeating digits.)

Subtract n from the 10n or 100n etc.

Divide to find the value of the variable.

Reduce.

Date___________________

Decimals to Percents: To convert a decimal to a percent move the decimal two places to the right. (You are really multiplying by 100.) Follow the number with the percent symbol.

Percents to Decimals: To convert a percent to a decimal, move the decimal two places to the left. (You are really dividing by 100.)

Fractions to Percents: Percent means per cent (per 100), if possible, create an equivalent fraction whose denominator is 100, this will give you the percent. Follow your answer with the percent symbol.

Percents to Fractions: To convert a percent to a fraction, place the percent over 100 and reduce.

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Changing a repeating decimal into a fraction:

To change a repeating decimal to a fraction, follow the steps below:

Ex. 1) Change $.\overline{7}$ to a fraction:

Step 1: Create a let statement stating that the value of *n* is equal to the repeating decimal.

Let $n = .\overline{7}$ (.77777…)
Let $10n = 7.\overline{7}$

Step 2: Create a let statement that multiplies the variable by the power of ten needed to move the decimal point past one set of repeating values.

$$\begin{array}{r} 10n = 7.\overline{7} \\ -\quad n = .\overline{7} \\ \hline 9n = 7 \end{array}$$

Step 3: Subtract the let statements. This should result in a whole number on the right side of the =.

$$\frac{9n}{9} = \frac{7}{9}$$

Step 4: Solve the algebraic equation by dividing through by the coefficient.

$$n = \frac{7}{9}$$

Step 5: State your answer clearly in fraction form!!!

Let's see another:

Ex. 2) Change $.\overline{36}$ to a fraction.

We needed to multiply the variable by 100 because we needed to move the decimal past two repeating numbers.

Let $n = .\overline{36}$ (.363636…)
Let $100n = 36.\overline{36}$

We subtract because we want to eliminate the repeating part of the decimal.

$$\begin{array}{r} 100n = 36.\overline{36} \\ -\quad n = .\overline{36} \\ \hline 99n = 36 \end{array}$$

$$\frac{99n}{99} = \frac{36}{99}$$

Remember to divide by 99 this time!

$$n = \frac{36}{99} = \frac{4}{11}$$

You may need to reduce! ☺

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Changing a repeating decimal ***<u>with an intruder</u>*** into a fraction:

An intruder is a number that breaks in **before** the pattern begins. An example would be:

$.25555555\ldots$ or $.2\overline{5}$ (2 is the intruder before the pattern of repeated 5s)

The process is very similar!

Step 1: Create a let statement stating that the value of *n* is equal to the repeating decimal.

1) Create three let statements:
   a) For the original - Let $n = .2\overline{5}$
   b) For moving the decimal past the intruder. Let $10n = 2.\overline{5}$
   c) For moving the decimal past the intruder and a set of repeating numbers Let $100n = 25.\overline{5}$

Step 2: Create a let statement that multiplies the variable by the power of ten needed to move the decimal point past the intruder.

Step 3: Create a let statement that multiplies the variable by the power of ten needed to move the decimal point past the intruder *and* one set of repeating values.

2) Subtract to remove the repeating decimal!

$$\begin{array}{r} 100n = 25.\overline{5} \\ -\ 10n = 2.\overline{5} \\ \hline 90n = 23 \end{array}$$

3) Solve by dividing by the coefficient.

$$\frac{90n}{90} = \frac{23}{90}$$

$$n = \frac{23}{90}$$

Ex. 2) Change $.34\overline{567}$ to a fraction.

Let $n = .34\overline{567}$ (.34567567567…)
Let $100n = 34.\overline{567}$
Let $100{,}000n = 34567.\overline{567}$

$$\begin{array}{r} 100{,}000n = 34567.\overline{567} \\ -\ 100n = 34.\overline{567} \\ \hline 99{,}900n = 34533 \end{array}$$

$$\frac{99{,}900n}{99{,}900} = \frac{34533}{99{,}900}$$

$$n = \frac{34{,}533}{99{,}900} = \frac{1{,}279}{3{,}700}$$ ☺

Date____________________

Sale - Discount - Markdown:

Sale, discount, and markdown are all used to indicate that the price of a specific item will be reduced making the item cheaper.

Original price - markdown (or discount) = sale price

The markdown is calculated by multiplying the markdown percent by the original price.

The sale price is calculated by subtracting the markdown from the original price.

Ex. Regular price for comic book: $4.95
Markdown percent: 20%
Sale price: ?

Markdown = markdown percent x original price
20% x $4.95
0.20 (4.95)
$0.99

Sale price = original - markdown
$4.95 - $0.99
$3.96

Alternate method:
Find the percent being paid by subtracting the percent off from 100. Then multiply the percent being paid by the price.

4.95 x (100% – 20%)
4.95 x 80%
4.95 (0.80)
$3.96

- To find the percent of markdown: (amount of discount)
  Divide the difference between the regular price and the sale price by the regular price, then multiply by 100 to make it a percent.

$$\textbf{Percent markdown} = \left[\frac{\textbf{regular price – sale price}}{\textbf{regular price}}\right] \textbf{x 100}$$

Ex. Reg. price of a Blu-ray: $20
Markdown: ?
Sale price: $15

$$\% \text{ markdown} = \left[\frac{20 - 15}{20}\right] \text{x } 100$$
0.25 x 100
25%

Date___________________

- To find the <u>regular price</u>: *let x = regular price*

  Sale price = regular price - % markdown • regular price

  Sale price = x - % markdown • x

  Substitute and solve!

Ex. Regular price of a jacket : ?
Markdown: 35%
Selling price: $42.00

Let x = regular price of a jacket

Sale price = reg. price - % markdown • reg. price
Sale price = x - % markdown • x

$$\$42.00 = x - 35\%x$$
$$\$42.00 = 1x - 0.35x$$
$$\frac{\$42.00}{.65} = \frac{.65x}{.65}$$
$$\$64.62 = x$$
$$x = \$64.62$$

Try these:

1) You decide to buy a new snowboard at the end of the season to take advantage of the upcoming sales. The board originally costs $450, but will be on sale for 35% off. Calculate the sale price.

2) Mike picks out a new ski jacket that originally sold for $199. The jacket was marked down to $129.99. What percent discount is that?

3) Sydney paid $375 for a snowboard. She purchased it with a markdown of 20%. What was the original price?

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<u>Markup</u>: The amount added to the cost (cost price or wholesale price) to determine the selling price.

Markup = percent of markup x cost price

- To find the <u>selling price</u>:
  Find the mark up and add it to the cost.

Selling price = markup + cost

Ex. Cost of a comic book: \$4.95
Markup: 20%
Selling price: ?

Selling price = % (cost) + cost
20% (\$4.95) + \$4.95
0.20 (4.95) + \$4.95
\$0.99 + \$4.95
\$5.94

Alternate method: Add 100% to the markup percent, then multiply by the cost.

- To find the <u>percent of mark up</u>:
  Divide the difference between the cost and the selling price by the cost

$$\textbf{Percent markup} = \left[\frac{\textbf{selling price} - \textbf{cost}}{\textbf{cost}}\right] \textbf{x 100}$$

Ex. Cost of a CD: \$15.50
Markup: ?
Selling price: \$20.15

$$\text{\% markup} = \left[\frac{20.15 - 15.50}{15.50}\right] \text{x } 100$$

0.3 x 100 = **30%**

- To <u>find the cost</u>: *let x = the cost*
  Selling price = cost + % markup • cost
  Selling price = x + % markup • x
  Substitute and solve!

Ex. Cost of an tire: ?
Markup: 65%
Selling price: \$42.00

Let x = cost of the tire

Selling price = cost + % markup • cost Selling price = x + % markup • x

\$42.00 = x + 65%x

\$42.00 = 1 x + 0.65x

$$\frac{\$42.00}{1.65} = \frac{1.65x}{1.65}$$

\$25.45 = x

x = \$25.45

Date___________________

Sales tax:

Sales tax is <u>added</u> to a lot of the purchases we make. The percent varies by state and county. Ulster county is currently 8%.

<u>To calculate the amount of tax on a purchase</u>:

1.) multiply the price of the purchase by the tax rate
2.) **Add** it to the purchase price.

Ex. Taxable purchases total: \$79.56
Tax rate: 8%

$\$79.56 \bullet 0.08 = \$6.36$

$$\begin{array}{rl} & \$6.36 \text{ tax} \\ + & \underline{\$79.56} \text{ purchases total} \\ & \$85.92 \end{array}$$

Alternate method:

Multiply the purchases total by 108% or 1.08

Why it works:

$$108\%p = 100\%\, p + 8\%p$$
$$1.08p = \underbrace{1.00p}_{1 \,\bullet\, \text{purchases total} \,=\, \text{purchase total}} + \underbrace{0.08p}_{0.08 \,\bullet\, \text{purchase total} \,=\, \text{tax}}$$

Try these:

1) Taxable groceries: \$59.87
   Tax rate: 8%

2) New skateboard: \$229.95
   Tax rate: 8.5%

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Relative Error & Percent Error

Relative Error: The ratio of the **absolute error** to the size of the measurement.

Absolute error is the difference between an approximate number and the true number that it approximates.

To find the relative error:

1) Find the absolute error by finding the difference between the estimated value and the actual value.
2) Divide the absolute error by the actual value.

*To find the **absolute error**, find the difference between the estimated value and the actual value. Subtract the lower from the higher.*

$$\text{relative error} = \left(\frac{\text{absolute error}}{\text{actual value}}\right)$$

Percent Error:

***Multiply the relative error by 100 to find the percent error.**

Ex. Sally estimated that she had sent 95 text messages last week. Her account said that she only sent 93. What is the percent error of her estimate?

$$\text{percent error} = 100\left(\frac{\text{absolute error}}{\text{actual value}}\right)$$

$$100\left(\frac{95-93}{93}\right)$$

$$100\left(\frac{2}{93}\right)$$

$$100(0.0215)$$

$$2.15\%$$

Ex. Mike estimates the length of his ice hockey stick is 4 ft (48 inches). The actual length is 52 inches. What is the relative error? Percent error?

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Measures of Central Tendencies: (Section 1-4)

<u>Mean</u>: (Average) The sum of the data divided by the number of data items.

<u>Median</u>: The middle number in a set of data that is written in order from least to greatest. If there is no middle number, find the mean (average) of the two middle numbers.

<u>Mode</u>: The data item that occurs most often. There may be more than one mode. There is no mode when all the data items occur the same number of times.

<u>Range</u>: The difference between the greatest and least values in a set of data.

<u>Outlier</u>: A data item that is far apart from the rest of the data items. An outlier affects the mean more than the median or mode.

Class example: The class quiz grades were as follows. Find the mean, median, mode and range.

87, 89, 93, 81, 72, 95, 91, 93, 82, 86, 91, 89, 93, 73, 68

*To find the **mean**: 1) Add the numbers*
2) Divide the sum by the number of numbers

87+89+93+81+72+95+91+93+82+86+91+89+93+73+68 = 1283

1283 ÷ 15 = 85.5333…

*To find the **median**: 1) Write the numbers in order from least to greatest.*
2) Identify the middle number.

~~68~~, ~~72~~, ~~73~~, ~~81~~, ~~82~~, ~~86~~, ~~87~~ **89**, ~~89~~, ~~91~~, ~~91~~, ~~93~~, ~~93~~, ~~93~~, ~~95~~

*If there are two middle numbers, find the ***mean*** (average) of the two.

Ex. $1, 2, 2, 3, \underbrace{4, 5}, 6, 7, 8, 8$

4.5 (4 + 5 = 9, 9 ÷ 2 = 4.5)

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*To find the **mode**: 1) Write the numbers in order from **least to greatest.***
*2) Identify the number that appears most often.**

68, 72, 73, 81, 82, 86, 87 89, 89, 91, 91, $\underbrace{93, 93, 93,}_{\mathbf{93}}$ 95

If no number appears more often, write **NO MODE! **Do not write 0**.*

*To find the **range**: 1) Find the difference between the highest and the lowest numbers.*

95 – 68 = **27**

*Look for any **outliers**:* In the above data set there were **no** outliers. All of the data items are spaced evenly apart.

Class Practice - Ex 1: Find the mean, median, mode and range for the following data set.

7, 6, 10, 14, 8, 3, 2, 11, 5, 12, 78, 15

mean:_______

median:_____

mode:_______

range:_______

outlier:______

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Class Practice - Ex 2: Find the mean, median, mode and range for the following data set.

23, 76, 45, 32, 201, 76, 23, 57, 68

mean:_______

median:_____

mode:_______

range:_______

outlier:______

Class practice- Ex 3: Sue's grades are 89, 97, 86, and 94. What does she need to get on her next quiz in order to have a 93 average?

Determining the best measure of central tendency:

The best measure of central tendency depends on what you are trying to show as well as the type of data you have.

Mean	Median	Mode
• **is** affected by outliers or skewed data • can be used with numerical data • works well for linear data; height, mass, voltage	• less affected by outliers or skewed data • can be used when data is on a ranking scale; number of stars given	• not affected by an outlier • can be used with non-numeric data • often used for groups, classes or surveys • can be more than 1 mode

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Random Samples and Surveys

__Population:__ A population is a collection of objects or group of people about whom information is gathered.

__Random Sample:__ A small group that is representative of a larger population. Each member of a population must have an equal chance of being chosen for the sample.

__Biased Question:__ A question is biased if it makes assumptions about the person being questioned or if it makes one answer seem better that the other. An unfair question.

Example of a biased question: Do you think that the extreme violence of Saturday morning cartoons affects, impressionable children?

This is a biased question because it assumes that the cartoons contain violence and the words young and impressionable may influence the response.

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Probability

Probability: The likelihood that an event will occur. Probability is represented by a number between 0 and 1. When probability falls between these numbers, it can be expressed in several ways: as a fraction, a decimal or a percent.

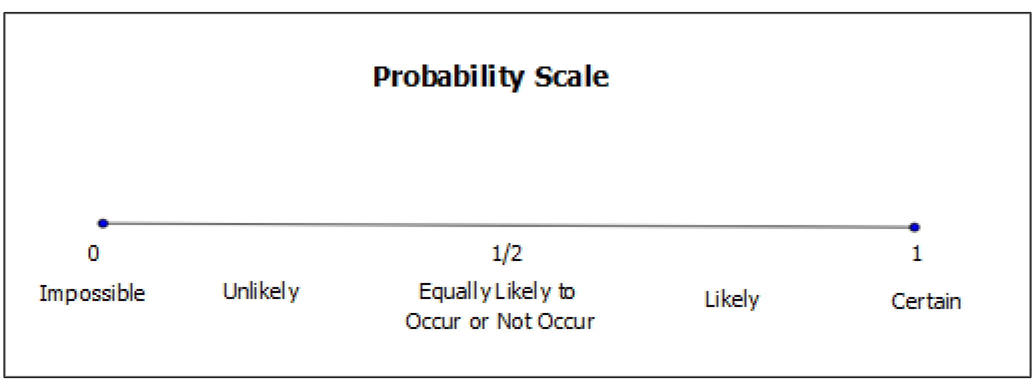
Event: One or more results (**outcomes**) of an experiment.

Compound Event: The combination of two or more single events.

Certain event: An event that is certain to happen in an experiment.
The probability of a certain event is 1.
Ex. Getting a head or a tail when a coin is tossed.

Impossible event: An event that cannot occur in an experiment.
The probability of an impossible event is 0.
Ex. Rolling a 7 on a standard number cube. (*A die*.)

Equally likely events: Events are said to be equally likely if the probabilities of the events are equal.



Complement: The complement for an event is all of the other possible events for a situation. The probability of an event plus the probability of its complement equals 1.

Example: If the event is 5 on a regular die, the complement is 1, 2, 3, 4, and 6.

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<u>Theoretical Probability:</u> The probability that is **calculated** by using the ratio:

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

Ex. A coin is flipped. There are a Total of two possible outcomes, a head or a tail. There is 1 head and 1 tail.

$P(\text{head}) = \frac{1}{2}$

$P(\text{tail}) = \frac{1}{2}$

the sum equals 1 whole.

<u>Experimental Probability:</u> The probability **found** by repeating an experiment many times and using the ratio:

$$P(\text{event}) = \frac{\text{Number of times an event happens}}{\text{Number of times the experiment is done}}$$

Ex. An experiment is done. A coin is flipped 50 times, 23 of those are heads.

$P(\text{head}) = \frac{23}{50}$

$P(\text{tails}) = \frac{27}{50}$

the sum equals 1 whole.

<u>Sample space:</u> The set of all possible outcomes (results).

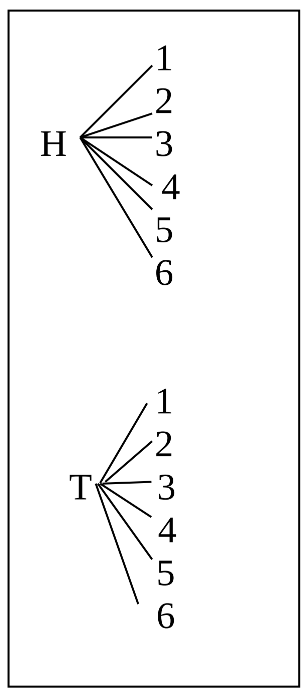
Example: The sample space for a coin is H, T.
The sample space for a die is 1, 2, 3, 4, 5, 6.
The sample space for tossing a coin and rolling a die is:

List:

H1, H2, H3, H4, H5, H6
T1, T2, T3, T4, T5, T6

or

Tree diagram:



Chart/Table:

	H	T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

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Counting Principle: The number of possible outcomes of a compound event equals the product of the number of outcomes in each individual event.

Ex. 1) Coin • Number Cube

Coin		Number Cube		
2	•	6	=	12 total possible outcomes
possible outcomes		possible outcomes		

Ex. 2) For a new house we can choose from 12 vinyl siding colors, 5 styles of windows and 8 front door styles. How many different houses could we design?

Siding		Windows		Doors		
12	•	5	•	8	=	480 different houses!

Try these:

1) How many possible outcomes are there if we flip 3 coins?

2) How many possible outcomes are there if we spin an equally divided 4-part spinner and a standard number cube?

3) How many possible meals could I order from a menu that has 5 entrees, 10 side dishes, 4 dessert choices and 6 different beverages?

Date___________________

<u>Independent events:</u> Two events are independent if the outcome of the first does not affect the outcome of a second.

Example: You choose a marble from a bag. Before choosing again you replace the original marble. The probability for the events remains the same for each trial.

<u>Probability of Independent Events:</u> P(A and B) = P(A) x P(B)

Example: You have a bag of 8 marbles. Three red and five blue. The probability of picking blue, then red (with replacement) would be:

$$\text{P(blue, red)} = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$$

<u>Dependent events:</u> Two events are dependent if the outcome of the first event affects the outcome of the second event.

Example: You choose a marble from a bag. You **do not replace** the marble before you choose again. The probability changes for each trial is different.

<u>Probability of Dependent Events:</u> P(A, then B) = P(A) x P(B after A)

Example: You have a bag of 8 marbles. Three red and five blue. The probability of picking blue, then red (**without replacement**) would be:

$$\text{P(blue, then red)} = \frac{5}{8} \times \frac{3}{\boxed{7}} = \frac{15}{56}$$

7 because we removed a marble for the first draw and did not replace it!

Date____________________

<u>Permutations:</u> An arrangement in which order is important.

There are 6 permutations for the word CAT. They are:

CAT	ACT	TCA
CTA	ATC	TAC

To calculate the number of permutations, use the *counting principle.*

There are 3 different letters that make the word cat. When they are arranged there are three positions to be filled.

$$\underline{\quad 3 \quad} \cdot \underline{\quad 2 \quad} \cdot \underline{\quad 1 \quad} = \text{ 6 permutations}$$

There are **three** letters that can be in the first

There are **two** letters left to fill the second

There is only **one** letter left to fill the last postion.

This can also be represented by a factorial, (n!), where n is the number of positions to fill.

Ex. MOUSE: 5 letters (positions) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ permutations.
MATH: 4 letters $\rightarrow$ $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations.

Some permutations do not use all available letters (objects):

Ex. Using the work MOUSE, find the number of **3 letter** permutations.

MOUSE has 5 available letters, but we are only using 3 of them.

$$\underline{\quad 5 \quad} \cdot \underline{\quad 4 \quad} \cdot \underline{\quad 3 \quad} = \text{ 60 3-letter permutations}$$

There are **five** letters that can be in the first

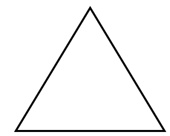
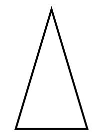
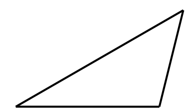
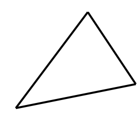
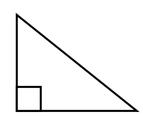
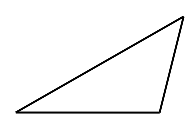
There are **four** letters left to fill the second

There are **three** letters left to fill the last position.

Date____________________

Geometry – 2D

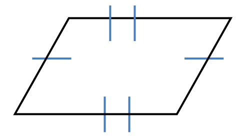
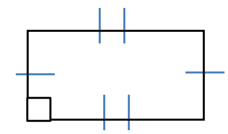
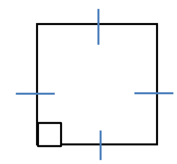
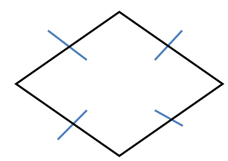
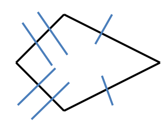
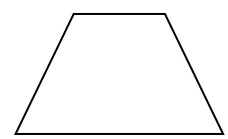
Polygons: Closed figures with sides that are straight line segments.

Triangle- A three sided closed figure.	**Equilateral:** A triangle with three congruent sides and **three** congruent angles. Each angle has a measure of 60°.	
	Isosceles: A triangle with two congruent sides. The angles opposite the congruent sides are congruent.	
	Scalene: A triangle with **no** congruent sides. This means that there are no congruent angles.	
	Acute: A triangle whose largest angle is an acute angle. (All angles are acute.)	
	Right: A triangle whose largest angle is a right angle.	
	Obtuse: A triangle whose largest angle is an obtuse angle.	

- **The sum of the measures of angles of a triangle is 180°.**
- **The perimeter of a triangle is found by adding all sides.**
- **The area of a triangle is found using:** $A_{\triangle} = \frac{1}{2} bh$

b = base length h = the height (altitude) of the triangle

Date____________________

Quadrilateral- A four sided closed figure.	**Parallelogram:** A quadrilateral with two pair of parallel and congruent sides. Opposite sides are parallel and congruent. $\mathbf{A}_{\square} = \boldsymbol{bh}$ b = base, h = height	
	Rectangle: A quadrilateral with two pair of parallel, congruent sides **and** four right angles. $\mathbf{A}_{\square} = \boldsymbol{bh}$ b = base, h = height	
	Square: A quadrilateral with four congruent sides **and** four right angles. $\mathbf{A}_{\square} = \boldsymbol{bh}$ b = base, h = height	
	Rhombus: A quadrilateral with four congruent sides. $\mathbf{A}_{\square} = \boldsymbol{bh}$ b = base, h = height	
	Kite: A quadrilateral with two pair of adjacent congruent sides. $\mathbf{A}_{\square} = \boldsymbol{bh}$ b = base, h = height	
	Trapezoid: A quadrilateral with one pair of parallel sides. $\mathbf{A} = \frac{1}{2}(\boldsymbol{b_1} + \boldsymbol{b_2})\boldsymbol{h}$ $\boldsymbol{b_1, b_2}$ **are the bases**	

The sum of the interior angles in a quadrilateral is 360º.

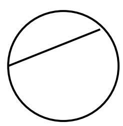
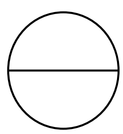
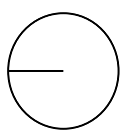
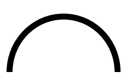
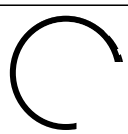
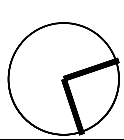
Pentagon	A **five** sided figure.
Hexagon	A **six** sided figure.
Heptagon	A **seven** sided figure.
Octagon	An **eight** sided figure.
Nonagon	A **nine** sided figure.
Decagon	A **ten** sided figure.

A **regular polygon** has all sides congruent and all angles congruent.

Date____________________

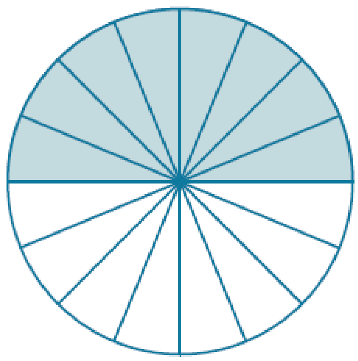
CIRCLES:

Circumference of a circle = πd or $2\pi r$
Area of a circle = πr^2

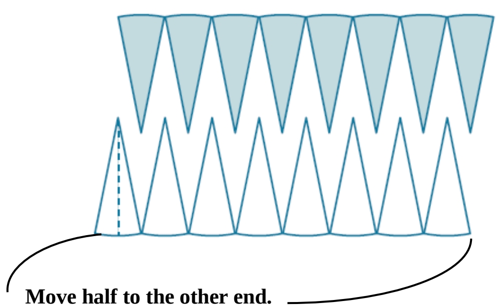
	Circles: The set of all points in a plane that are an equal distance from a given point, the **center**.
	Circumference: The distance around the circle.
	**Chord:** A line segment connecting any two points on the circle. Since the chord is a line segment it is notated in the same way; two capital letters with a line above the letters.
	**Diameter:** A chord that passes through the center of the circle. The diameter is also named with two capital letters with a line above them.
	**Radius:** A line segment extending from the center to any point on the circle. The radius is ½ of the diameter. The plural form of radius is radii. The radius is also named with two capital letters with a line above them.
	**Arc:** Part of a circle.
	**Semicircle:** Half of a circle. Notated by three capital letters with a long arc symbol above them.
	**Major arc:** An arc that is more than half of a circle. Also notated by three letters and a long arc above them.
	**Minor arc:** An arc that is less than half of the circle. Notated by two letters and a short arc above them.
	**Central angle:** An angle whose vertex is at the center of the circle and whose sides are radii.

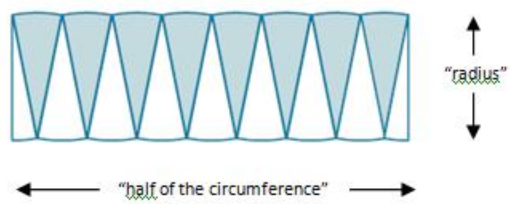
Derivation of the Area of a Circle Formula:

To find the formula for the area of a circle, cut a circle into **16** equal pieces:

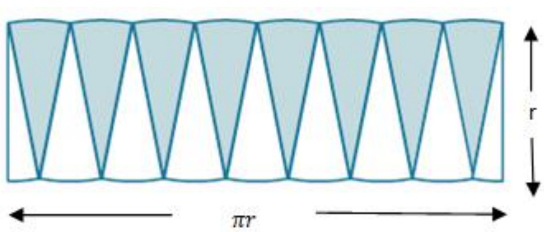


Arrange the triangular wedges by alternating the “triangle” directions and sliding them together to make a “parallelogram.” Cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to the other end of the parallelogram in order to create an approximate “rectangle.”





The circumference is $\boldsymbol{2\pi r}$, where the radius is “$\boldsymbol{r}$.” Therefore, half of the circumference is $\boldsymbol{\pi r}$.



Therefore, using the area formula for a rectangle (A = *lw*), the area formula for a circle is **A =** $\boldsymbol{\pi r^2}$

Date____________________

Practice Set:

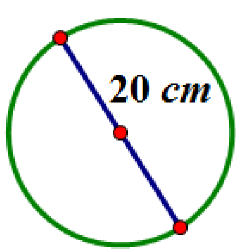
Ex. 1) A circle has a radius of 2 cm.
Find the area of the circular region.

2.) A circle has a diameter of 14 cm.
Find the exact area of the circular region.

What is the circumference of the circle?

3.) The area of a circle is 314 square inches.
Find the radius of the circle.

4.) Joan determined that the area of the circle below is $400\pi\ cm^2$. Melinda says that Joan's solution is incorrect; she believes that the area is $100\pi\ cm^2$. Who is correct and why?



Date____________________

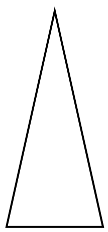
Determining the interior angles of a polygon:

All triangles have an interior angle sum of ________. We can use this information to determine the sum of the interior angles of polygons.

1) Starting at a single vertex. Divide a polygon into triangles. Each of these triangles has an interior angle sum of 180º. The lines you draw may not cross.

2) Multiply the number of triangles by 180º to determine the sum of the interior angles in the polygon.

A

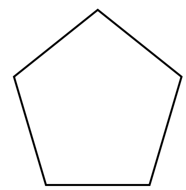


3 sides

B

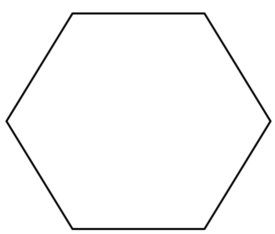
4 sides

C



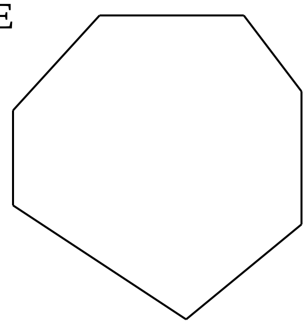
5 sides

D



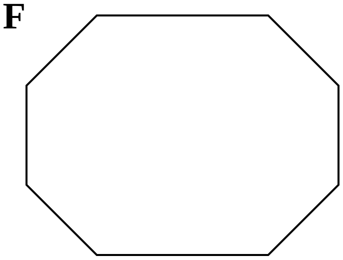
6 sides

E



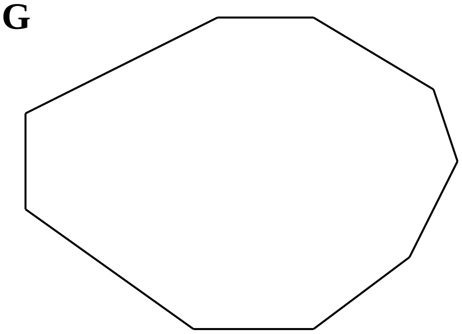
7 sides

F



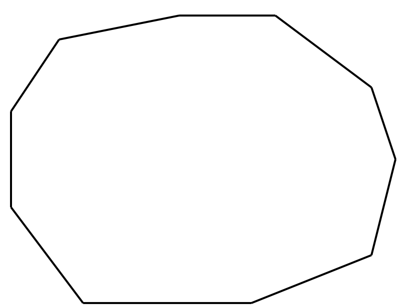
8 sides

G



9 sides

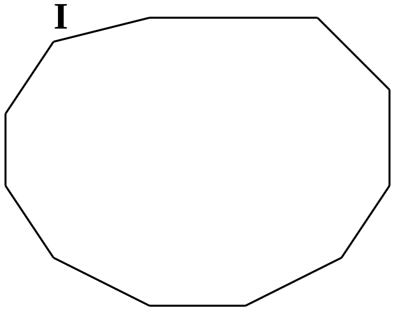
H



10 sides

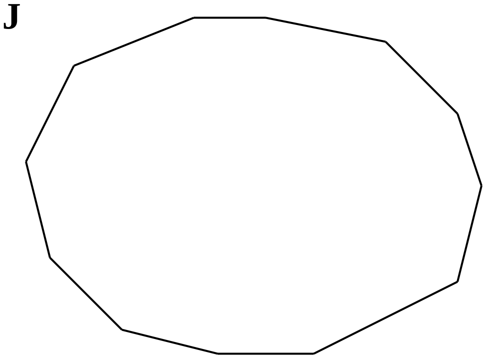
Date____________________

I



11 sides

J



12 sides

Complete the chart:

Number of sides	**Number of triangles**	**Number of interior degrees**
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

1) Can you find a relationship between the number of sides and the number of triangles?_______________________________________

2) Write an expression to find the sum of the interior angles in any polygon. Let *n* equal the number of sides.___________________

Finish the table below:

Number of sides	**Number of triangles**	**Number of interior degrees**
13		
14		
15		
18		
20		

Date___________________

Geometry – 3D

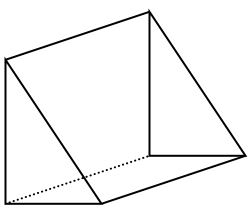
Surface Area

<u>Polyhedron</u>: A three-dimensional figure in which all the surfaces are polygons.

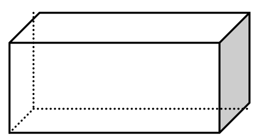
<u>Prism</u>: A three dimensional figure with two parallel and congruent polygonal faces, called bases. The other faces are rectangles. A prism is named for the shape of its base.

<u>Surface Area</u>: The sum of the areas of the faces of a polyhedron.

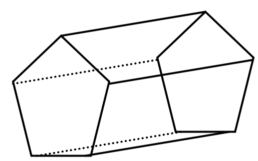
Triangular Prism



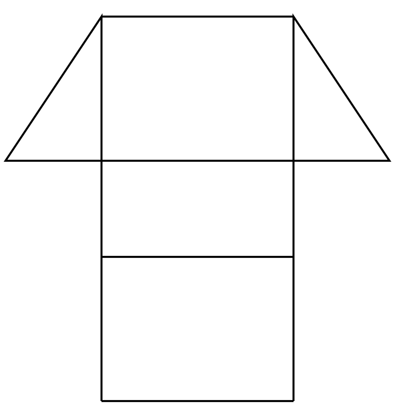
Rectangular Prism



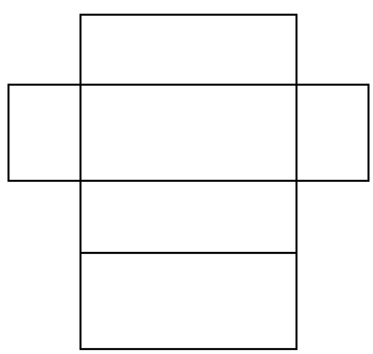
Pentagonal Prism



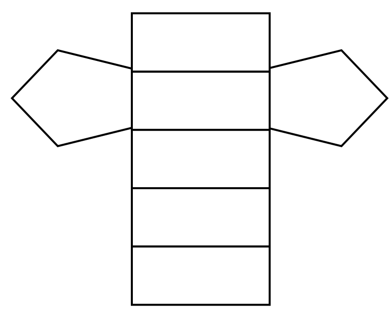
Triangular Prism Net



Rectangular Prism Net



Pentagonal Prism Net



To calculate the surface area:

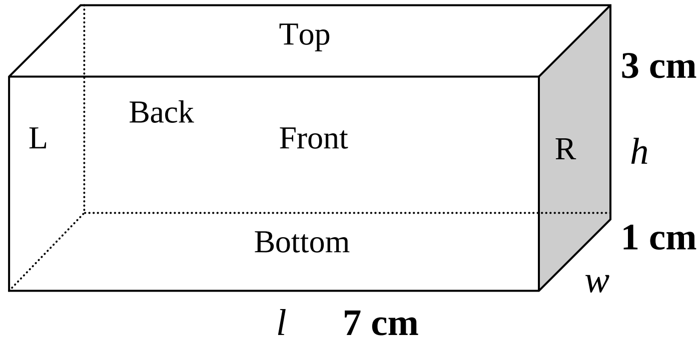
1) Calculate the area of each face.
2) Add the area together.

OR

USE THE APPROPRIATE FORMULA

Date___________________

Class Practice:



A(front) =

A(back) =

A(top) =

A(bottom) =

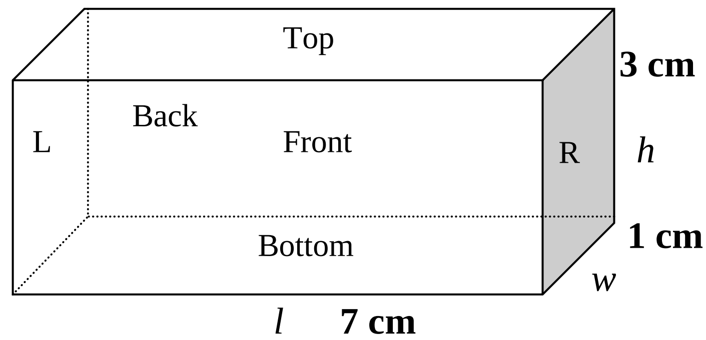
A(left) =

A(right) =

Total _________________________

Date____________________

Now try using the formula:

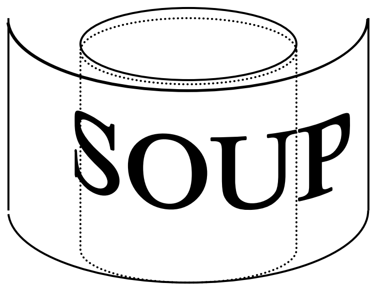


1) Write the formula.
2) Substitute carefully.
3) Calculate and label.

Date___________________

Surface Area of a Cylinder:

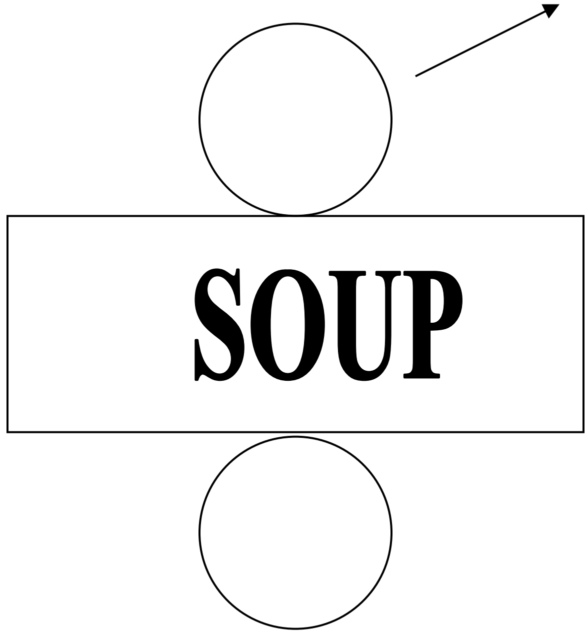
Before you can calculate the surface area of a cylinder, you need to identify the shapes that create it. Think of a can of soup. The top and bottom are circles, but what about the side? Imagine peeling off the label. If you recycle at home you may have already done this. When the label is removed we can see that it is a rectangle. That means that the side of the cylinder is really a rectangle!



Top and Bottom: Circles

$A = \pi r^2$

We want to multiply this formula by 2 because we have a top and a bottom. This gives us $2\pi r^2$.



Side: Rectangle

$A = l \cdot h$

Notice that the length of the rectangle is equal to the circumference of the circle. Therefore, we substitute the circumference formula for the length, giving us $2\pi rh$.

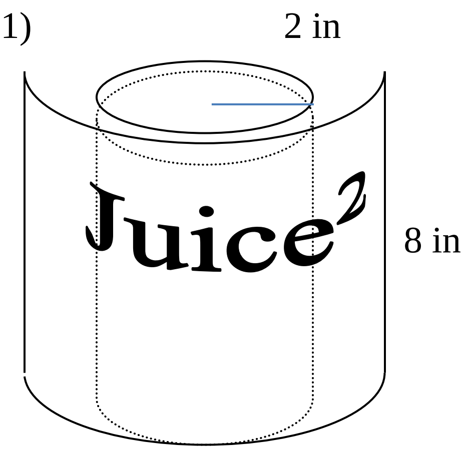
Since the surface area is the total of the individual surface areas of all of the sides, combine the two formulas to get:

Surface Area of a Cylinder = $2\pi rh + 2\pi r^2$

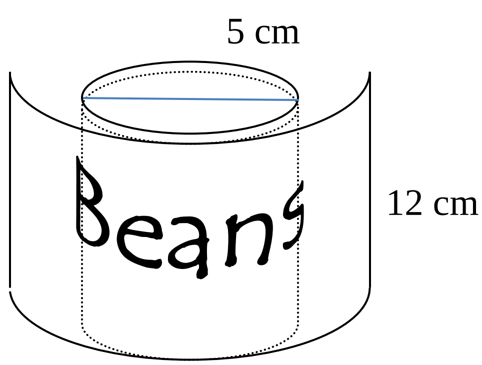
Date___________________

Class Practice:

1)



2)



Date___________________

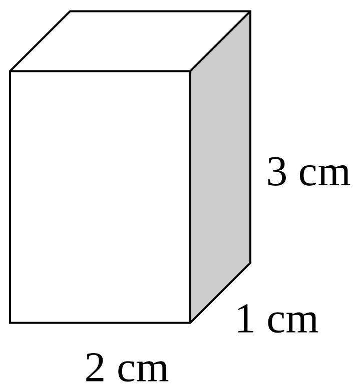
Volume:

<u>Volume</u>: The volume of the 3-d figure is the number of cubic units needed to fill the space inside the figure.

To find the volume:

1) Refer to the formula sheet to determine the correct formula based on the figure you are using.
2) Substitute the values carefully.
3) Calculate the volume and label with units cubed. Ex. in^3

Ex.



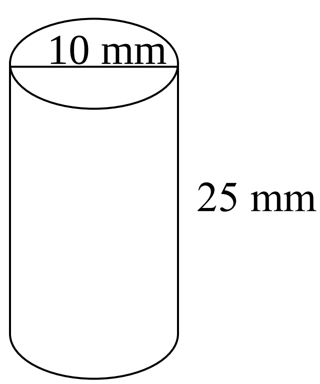
Volume (Rectangular Prism) = lwh

$V = l \ \bullet \ w \ \bullet \ h$

$\quad 2 \ \bullet \ 1 \ \bullet \ 3$

Volume = $6\ \text{cm}^3$

Ex.



Volume (Cylinder) = $\pi r^2 h$

$\text{V} = \pi \ \bullet \ r^2 \ \bullet \ h$

$\pi \ \bullet \ 5^2 \ \bullet \ 25$

$\pi \ \bullet \ 25 \ \bullet \ 25$

$\pi \ \bullet \ 625$

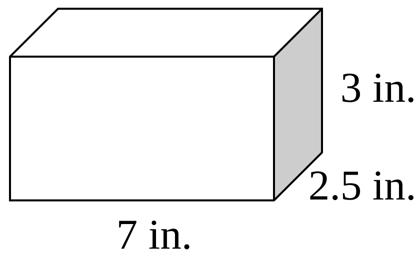
Volume = $1963.4937\ \text{mm}^3$

BE SURE TO USE THE RADIUS

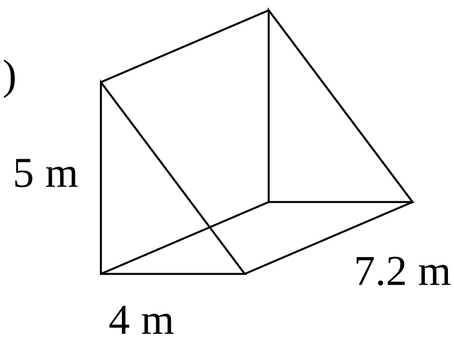
Date____________________

Class Practice:

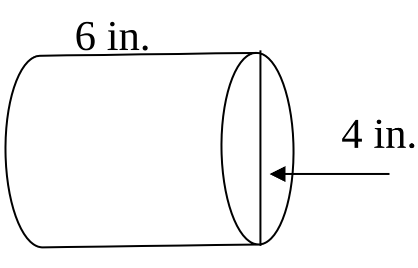
1)



2)



3)

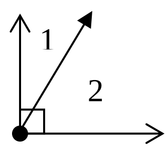


Date___________________

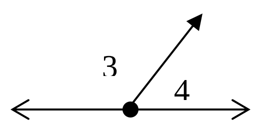
Angle Relationships:

- **Adjacent angles**: Angles that have a common side and a common vertex. There is no degree measurement for adjacent angles.

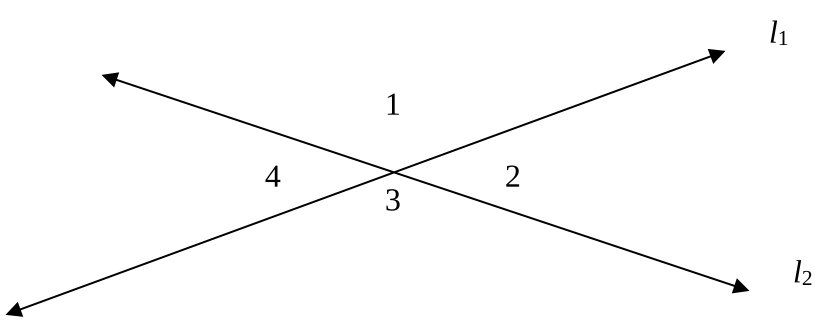
- **Complementary angles:** Two angles whose sum is 90°. The angles do not have to be adjacent. $\angle 1$ and $\angle 2$ are complementary angles.



- **Supplementary angles:** Two angles whose sum is 180°. The angles do not have to be adjacent. $\angle 3$ and $\angle 4$ are supplementary angles.



- **Vertical angles:** Opposite angles. Vertical angles are formed by the intersection of two straight lines. Vertical angles must share a vertex, but do not share a side. $\angle 1$ and $\angle 3$ are vertical angels. $\angle 2$ and $\angle 4$ are also vertical angles. **Vertical angles have equal measures.**



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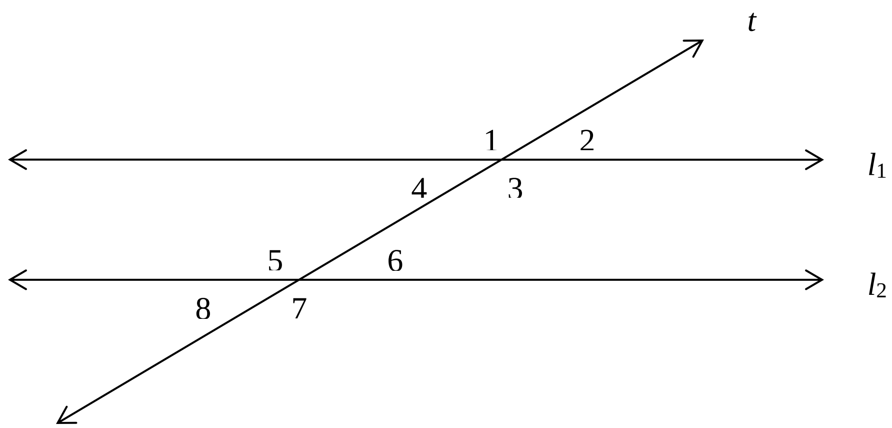
- **Transversal:** The oblique line that intersects two lines at separate places.



Given that l_1 and l_2 are parallel.

- **Corresponding angles:** Angles that hold the same position with respect to the transversal. $\angle 1$ and $\angle 5$ are corresponding angles. They are both above the lines and to the left of the transversal. $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$ are also corresponding angles. **Corresponding angles have equal measures.**

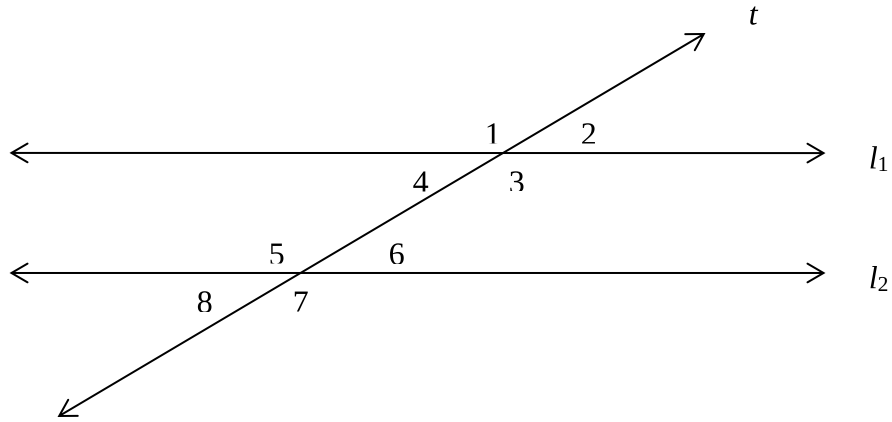
- **Alternate interior angles:** Angles that are between the lines and on opposite sides of the transversal. $\angle 3$ and $\angle 5$ are alternate interior angles. $\angle 4$ and $\angle 6$ are also alternate interior angles. **Alternate interior angles have the same measure.**

- **Alternate exterior angles:** Angles that are on the outsides of the lines and on opposite sides of the transversal. $\angle 1$ and $\angle 7$ are alternate exterior angles. $\angle 2$ and $\angle 8$ are also alternate exterior angles. **Alternate exterior angles have the same measure.**

Date____________________

Identifying the angles in parallel lines cut by a transversal.

All of these angle relationships can be used to find the missing angles in the diagram below.   Given that lines 1 and 2 are parallel and that $\angle 2$ is 30°.



$\angle 1 = 150°$ because it is *supplementary* to $\angle 2$.

$\angle 3 = 150°$ because it is *supplementary* to $\angle 2$ **or**
it is *vertical* to $\angle 1$.

$\angle 4 = 30°$ because it is *supplementary* to $\angle 3$ **or**
it is *supplementary* to $\angle 1$ **or**
it is *vertical* to $\angle 2$

$\angle 5 = 150°$ because it is *corresponding* to $\angle 1$

$\angle 6 = 30°$ because it is *corresponding* to $\angle 2$ **or**
it is *supplementary* to $\angle 5$

$\angle 7 = 150°$ because it is *corresponding* to $\angle 3$ **or**
it is *supplementary* to $\angle 6$ **or**
it is *vertical* to $\angle 5$

$\angle 8 = 30°$ because it is *corresponding* to $\angle 4$ **or**
it is *supplementary* to $\angle 7$ **or**
it is *supplementary* to $\angle 5$ **or**
it is *vertical* to $\angle 6$.