ACT Math Quick Guide

Use this guide as a quick reference for the most important
ACT Math strategies and content.

Key Strategy #1: *Know Your Formulas*. The ACT has no formula sheet, and won't provide you with formulas in most questions. Know these and how to use them.

Geometry

Square and Rectangle Area: $A = s^2 \quad A = \ell w$
Parallelogram/Rhombus Area: $A = bh$

Triangle Area: $A = \frac{1}{2}bh \qquad A = \frac{1}{2}ab\sin C$

Trapezoid Area: $A = \frac{1}{2}(b_1 + b_2)h$

Circle Area & Circumference: $A = \pi r^2 \quad C = \pi d = 2\pi r$
Cubic/Rectangular Prism Volume: $V = \ell wh$
Cylinder Volume: $V = \pi r^2 h$

Trigonometry

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Helpful identities: $\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta = 1$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate Geometry

Standard form of a line: $y = mx + b \quad m = \text{slope},\ b = y\text{-intercept}$

Slope: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad ax^2 + bx + c = 0$

Equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$

Equation of an ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Key Strategy #2: *Sometimes, the best approach is to work backwards*. Start with the middle value if you're going to take this route.

Example

Allison is reading a 300-page book. She plans to read the same number of pages every day until she finishes the book. If she reads 20 more pages per day than she currently plans to read, it will take Allison 4 fewer days to complete the book. How many pages per day is Allison currently planning to read?

A. 10
B. 15
C. 20
D. 30
E. 50

Setting up an algebraic equation here is difficult, but using the answers is easy. Let's say you assume C is the correct choice. That means Allison currently plans to read 20 pages per day. The problem makes it clear that you need to determine how long it will take Allison to read the full book so that we can compare the timeframes. To do so, you can divide.

$$\frac{300 \text{ pages}}{20 \text{ pages per day}} = 15 \text{ days}$$

Then, you do what the problem tells you: increase the number of pages Allison will read by 20. This means she'll read 40 pages per day. Let's figure out how long this will take her.

$$\frac{300 \text{ pages}}{40 \text{ pages per day}} = 7.5 \text{ days}$$

Thus, it would take Allison 15 – 7.5 = 7.5 fewer days to read the book. But the problem said that if she increases the amount she plans to read by 20 pages per day, it should take her 4 fewer days. Thus, C is not the answer. Moreover, because the gap is too large, you know to increase the initial number of pages she plans to read per day. This means you should eliminate choices A and B, and go to choice D. When you perform the same calculation, you'll see Choice D is correct.

Key Strategy #3: *Know how to translate words to math.*

- Fixed-rate and variable equations

Example

To shovel snow from a driveway, Arthur charges an initial fee of $12, plus an $8 per-hour fee for every hour or fraction thereof that he shovels. Lolita charges an initial fee of $20, plus a $5 per-hour fee for every hour or fraction thereof that she shovels. For how much time would the two need to shovel a given house's driveway for Lolita's overall fee to be cheaper than Arthur's overall fee?

A. Between 0 and 1 hour
B. Between 1 and 2 hours
C. Between 2 and 3 hours
D. Between 3 and 4 hours
E. Between 4 and 5 hours

Here, each person charges what is known as a *fixed*, or constant, rate for simply showing up, and then a per-hour rate for shoveling. To deal with this, you must set up an expression that represents the total charge for each person. Since the fixed cost never changes, it will be part of each expression as-is. The per-hour cost must be multiplied by the number of hours, which you can call *h*, and added to the fixed rate. You can then set the two expressions equal to each other to find the "break even" point, when both fees are the same.

$$\text{Arthur: } 12 + 8h$$
$$\text{Lolita: } 20 + 5h$$

$$12 + 8h = 20 + 5h$$
$$3h = 8$$
$$h = \frac{8}{3} = 2.\overline{6}$$

This means that, between 2 and 3 hours, Arthur's and Lolita's charges are the same. Anything greater than this value will make Lolita's plan more cost effective, so Choice C is correct.

- Simple algebraic and percent-related phrases.

Example

When 40 percent of a number is increased by 20, the result is 4 less than the number. What is the number?

A. 16
B. 20
C. 24
D. 36
E. 40

You can guess and check here, but it's far more efficient to translate the words in the problem into an equation and then do simple algebra to solve. Here are some examples of key phrases and their mathematical equivalents.

"5 **more than** a number"	$x + 5$
"5 **less than** a number"	$x - 5$
"30 percent **of** a number"	$0.3x$
"increased by"	$+$
"decreased by"	$-$

The words in the problem thus translate to the following.

$$0.4x + 20 = x - 4$$

$$24 = 0.6x$$

$$x = \frac{24}{0.6} = 40$$

Choice E is correct.

Pro tip: to increase or decrease a number by a certain percent in a single step, simply multiply the original number by 1 ± the decimal equivalent of a percent. For example:

"20 percent more than 150" = $150(1 + 0.2) = 150(1.2) = 180$

"70 percent less than 400" = $400(1 - 0.7) = 400(0.3) = 120$

Key Strategy #4: *Recognize systems of equations and deal with them in the simplest way possible.*

- Many times, the ACT presents two linear equations that have one or multiple "solutions". This is a system.

Example

$$8x - \ \ 6y = 12$$
$$ax - 18y = 36$$

If the system of equations above has infinite solutions, what is the value of a ?

A. 24
B. 18
C. 9
D. 6
E. 3

Because a "solution" is where two lines intersect, "infinite solutions" is code language for "the same line" — they must intersect at every point. Thus, they must have the same equation. Notice the relationship between the coefficients of the two equations; it looks like the bottom is obtained by multiplying everything in the top by 3. This makes all the coefficients match up. Thus, simply multiply 8 by 3 to get 24 for the value of *a*. Choice A is correct.

Note that *parallel* lines will have *no solutions*, because they never intersect. Their equations might look like this.

$$2y + 5x \ \ = 10$$
$$4y + 10x = 25$$

At first, this looks like the example above, but observe closely. If you multiply the top equation by 2, you get $4y + 10x = \mathbf{20}$, *not* 25. Therefore, these cannot represent the same line. If you solve for *y*, this is what you'd get.

$$y = -\frac{5}{2}x + 5$$

$$y = -\frac{5}{2}x + 6.25$$

The lines have the same slope, but different *y*-intercepts, so they will never intersect.

- Other times, the system appears by translating words to math in a word problem.

Example

At a fall festival, you can purchase 1 cup of candy corn and 1 cup of apple cider for \$3.50. You can also purchase 2 cups of candy corn and 3 cups of apple cider for \$9.75. What is the price of one cup of apple cider?

A. \$0.75
B. \$1.50
C. \$2.25
D. \$2.50
E. \$2.75

Again, you can use guess-and-check here, but it's better to create a system of equations. To do so, you must choose variables for the price of 1 cup of candy corn and 1 cup of apple cider. Let's choose *c* and *a* respectively, and translate words to math to create equations:

$$\begin{aligned} c + \quad a &= 3.50 \\ 2c + 3a &= 9.75 \end{aligned}$$

There are two ways to handle this. Since you have to solve for *a*, a good way is to solve the first equation for *c*, and then to plug the equivalent expression into the second equation.

$$c + a = 3.50 \quad \text{becomes} \quad c = 3.50 - a$$

Now, plug in $(3.50 - a)$ for *c* in the second equation, and solve.

$$2c + 3a = 9.75 \quad \text{becomes} \quad 2(3.50 - a) + 3a = 9.75$$
$$7 - 2a + 3a = 9.75$$
$$a = 2.75$$

<u>Choice E</u> is correct.

Key Strategy #5: *Know your laws of exponents, cold.*

$$x^a x^b = x^{a+b} \qquad \frac{x^a}{x^b} = x^{a-b} \qquad (x^a)^b = x^{ab} \qquad x^{-a} = \frac{1}{x^a} \qquad x^0 = 1$$

Example

$$\frac{(2x^4y^3)^6}{4xy^{-4}}$$

When the expression above is simplified, what is the sum of the exponents on the resulting x and y terms?

A. 46
B. 45
C. 37
D. 32
E. 24

On the top, you can apply the "power to power" exponent rule (the third one listed above), which requires multiplying the exponents. Note that there is an implied "1" for the exponent on the coefficient 2. The expression thus becomes:

$$\frac{2^6x^{24}y^{18}}{4xy^{-4}}$$

Now, you can apply the division/subtraction rule (the second one listed above) to simplify like terms. The 24 on the numerator's *x*-term subtracts the implied 1 on the denominator's *x*-term, and the 18 on the numerator's *y*-term subtracts the –4 on the denominator's *y*-term. You don't need to worry about the coefficients, since the problem cares about the x and *y* exponents only. The expression becomes:

$$\frac{2^6x^{23}y^{22}}{4}$$

Since the final *x* power is 23, and the final *y* power is 22, the answer to the problem is 23 + 22 = 45, or Choice B.

Sometimes, knowing simple rules of exponents can make a seemingly difficult problem very easy.

Example

$$n^{a^2-25} = 1$$

For all nonzero n, the expression above is true for which values of a ?

A. 0 only
B. –5 only
C. 5 only
D. –5 and 5
E. –5, 0, and 5

If you know the law of exponents that states anything raised to the power 0 is equal to 1 (the last rule listed at the top of the previous page — this is true only if the base is not itself zero), then you're all set: you know that the exponent on n must equal 0. Thus, you can set the exponent expression equal to 0 and solve.

$$a^2 - 25 = 0$$
$$(a+5)(a-5) = 0$$
$$a = -5, \quad a = 5$$

Thus, Choice D is correct.

It is EXTREMELY IMPORTANT to know the following about exponents:

$$(x+y)^2 \text{ DOES NOT EQUAL } x^2 + y^2$$

Once there's a + or – sign, you cannot distribute the exponent. You must write out the expanded expression and multiply:

$$(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$$

Similarly,

$$\sqrt{x^2+y^2} \text{ DOES NOT EQUAL } x + y$$

Key Strategy #6: *When dealing with averages, ALWAYS write out the average formula first. This will help you stay organized.*

$$\text{Avg} = \frac{\text{Sum of \#s}}{\text{\# of \#s}}$$

Example

Cassandra must take 6 exams this semester, each of which will be graded from 0 to 100 and weighted equally to calculate her final course average. If the average of her scores on the first two exams of the semester is 89, what is the <u>minimum</u> average score she must earn on her last four exams in order to earn a 95 average for all exams this semester?

A. 92
B. 94
C. 95
D. 96
E. 98

A common mistake is to simply try averaging 89 and each answer choice until you come up with an average of 95. This doesn't work, because the four remaining tests will contribute more to the overall average than the first two. Instead of making that mistake, always start by writing what you know, using the formula as a template. You know the average of the first two exams is 89:

$$\text{Avg} = \frac{\text{Sum of \#s}}{\text{\# of \#s}} \qquad 89 = \frac{\text{Sum first 2 test scores}}{2}$$
$$178 = \text{Sum of first 2 test scores}$$

Now, create a new average formula for all 6 test scores. You know you want the average to be at least 95, and the sum of the 6 tests has to appear in the numerator. The numerator should be the 178 you already have for the first two tests, plus whatever the sum must be for the remaining 4 tests, which you can call *x*:

$$95 = \frac{178 + x}{6} \qquad 570 = 178 + x \qquad x = 392$$

That's the sum of the 4 remaining test scores, which you can divide by 4 to find their average.

$$\text{Average of final 4 exams} = \frac{392}{4} = 98$$

<u>Choice E</u> is correct.

Sometimes, the *kind* of average you're looking for is a bit different. This happens mostly with frequency-related questions.

Example

Total # of rainy days	# of months with this total
12	2
13	1
14	2
15	3
16	3
17	1

The table above shows the number of months for which it rained a certain number of days over a full year. To the nearest tenth, what is the average number of rainy days per month for the year?

A. 14.5
B. 14.6
C. 14.7
D. 14.8
E. 14.9

The key here is to pay attention to the language of the average statement: the problem wants the "average number of rainy days *per* month". Read the "per" as the division sign, put whatever is mentioned first on top, and whatever is mentioned after the "per" on the bottom:

$$\text{Average} = \frac{\text{Sum of \#s}}{\text{\# of \#s}} = \frac{\text{Sum of rainy days}}{\text{\# of months}}$$

Think: if 2 months had a total of 12 rainy days each, this represents 24 rainy days total. This means you must multiply across rows in the table to get the sum total of rainy days.

$$\text{Avg rainy days per month} = \frac{12(2) + 13(1) + 14(2) + 15(3) + 16(3) + 17(1)}{12}$$

≈ 14.6 rainy days per month. Choice B is correct.

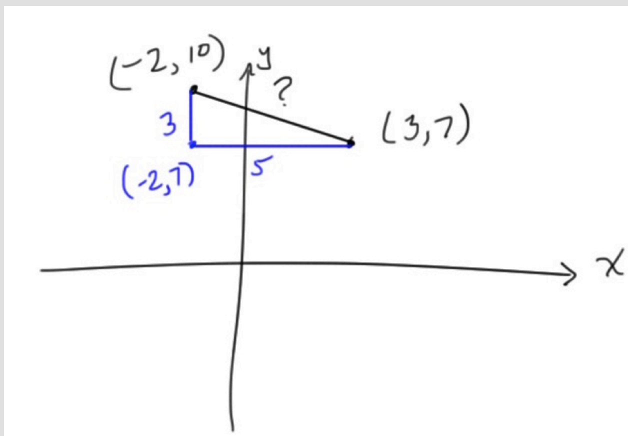
Key Strategy #7: *To find the distance between two points on the xy-plane, you don't need the distance formula!*

Find the distance between any two points by making a simple sketch (if one isn't provided) and dropping a third point to create a right triangle with the two given points. Find the measures of the legs, and then use the Pythagorean Theorem to determine the distance you're trying to find.

Example

A line segment is created by joining the points A (3, 7) and B (–2, 10) on the standard xy-plane. What is the distance between A and B in coordinate units?

A. $\sqrt{15}$
B. $\sqrt{10}$
C. 4
D. $\sqrt{34}$
E. 8



A sketch like the one above is perfect. Note the third point dropped is right under one of the points and directly across from the other. This allows you to determine the coordinates of the third point, and therefore the lengths of the legs as shown. Then, use the Pythagorean Theorem to find the length of *AB*.

$$a^2 + b^2 = c^2$$
$$3^2 + 5^2 = c^2$$
$$9 + 25 = c^2$$
$$\sqrt{34} = c$$

Choice D is correct.

Key Strategy #8: *You don't need the midpoint formula either.*

The midpoint "formula" is just telling you to calculate the average of the *x*- and *y*-coordinates separately, so do just that: take the average.

Example

The endpoints of a line segment are (–4, 12) and (3, –5). What is the midpoint of the segment?

A. (3.5, 8.5)
B. (–4.5, 7.5)
C. (–0.5, 3.5)
D. (0, 6)
E. (1, 4)

To find the midpoint of a segment given its endpoints, just average the midpoints. Average the *x*-coordinates first, and then the *y*-coordinates.

$$\frac{-4+3}{2} = -\frac{1}{2}$$

$$\frac{12+(-5)}{2} = \frac{7}{2}$$

Converted to decimals, these give you <u>Choice C</u>.

It's important to read carefully. Sometimes, you'll be given an endpoint and the midpoint of a segment, and will be asked to find the other endpoint. In this case, you can just count! This is because the midpoint is located exactly halfway between the endpoints with respect to both the *x*- and *y*-coordinates.

ALWAYS read carefully when dealing with midpoint problems to determine whether you're being given two endpoints, or an endpoint and a midpoint.

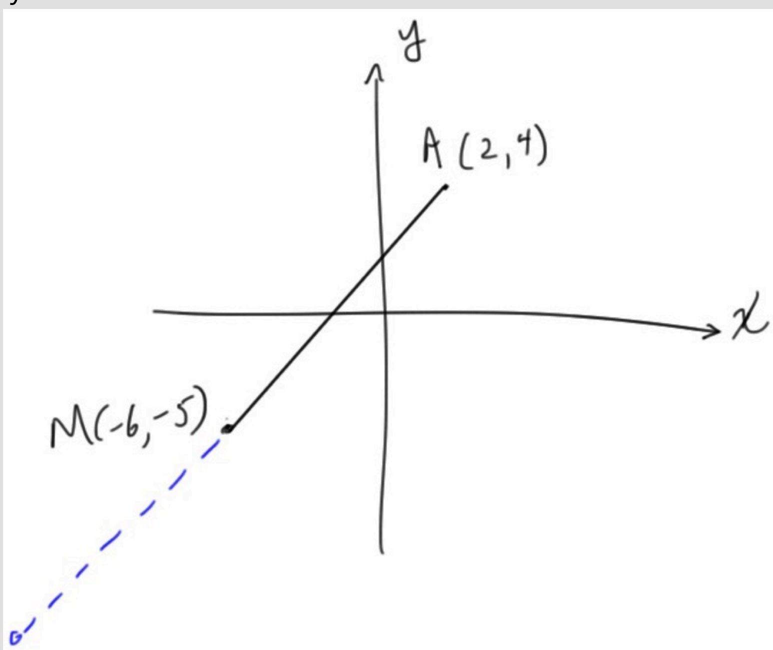
Sketches can also be immensely helpful in eliminating unreasonable answer choices with very little work

Example

Line segment AB has midpoint M. If the coordinates of A are (2, 4) and the coordinates of M are (–6, –5), what is the y-coordinate of point B ?

A. –14
B. –1
C. –0.5
D. 4.5
E. 9

Draw a simple sketch to get a rough idea of where things are. This may even allow you to eliminate one or multiple choices because they don't make sense visually.



Notice how the unknown *y*-value must be way down below the others. This immediately exposes the answer as Choice A, which gives the only *y*-value anywhere close to where point *B* would have to be located. If it wasn't visually obvious, notice that to move from the endpoint *A* to the midpoint *M*, the *y*-value had to decrease by 9 units (it went from 4 to –5). To get from the midpoint to point *B*, it must again decrease by 9 units. –5 – 9 = –14, which is Choice A.

Key Strategy #9: *Watch out for language specific to common coordinate geometry.*

Perpendicular lines have negative reciprocal slopes. This means their slopes are the negative "flips" of one another. Parallel lines have the same slope.

Example

Line k has equation $y = 3x – 5$ and is graphed in the standard xy-coordinate plane. Which of the following lines is perpendicular to k and crosses through the point (2, –1) ?

A. $6y + 2x = –2$
B. $3y + 3x = –3$
C. $2y – x = –5$
D. $y + x = –2$
E. $y – x = –8$

The line you're trying to find must have a slope that is the *negative reciprocal* of 3, which is the slope line *k*. This means the slope of the line to find is $-\frac{1}{3}$. Let's get all of the answers into *y* = *mx* + *b* form to check out the slopes.

A. $6y + 2x = –2$ ---> $y = -\frac{1}{3}x - \frac{1}{3}$
B. $3y + 3x = –3$---> $y = -x - 1$
C. $3y + x = –5$ ---> $y = -\frac{1}{3}x + \frac{5}{3}$
D. $y + x = –2$ ---> $y = -x - 2$
E. $y – x = –8$ ---> $y = x - 8$

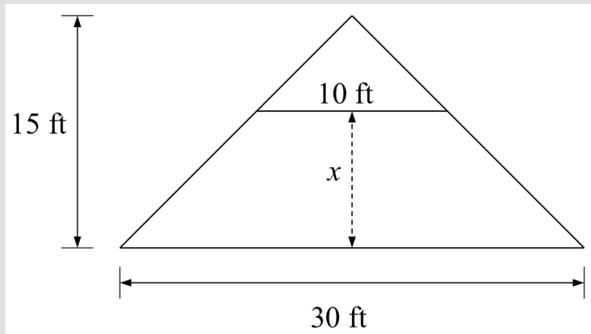
Immediately, all answers but choices A and C are out, because their slopes are not what you need. But how do you know which line passes through (2, –1)? Simply plug in 2 for *x* and –1 for *y* in each of the remaining choices, and see which one gets you a correct equation:

A. $y = -\frac{1}{3}x - \frac{1}{3}$ $\quad -1 = -\frac{1}{3}(2) - \frac{1}{3}$ ---> $-1 = -\frac{2}{3} - \frac{1}{3}$ ---> $-1 = -1$ Yes!
C. $y = -\frac{1}{3}x + \frac{5}{3}$ $\quad -1 = -\frac{1}{3}(2) + \frac{5}{3}$ ---> $-1 = -\frac{2}{3} + \frac{5}{3}$ ---> $-1 = 1$ NO!

Choice A is correct.

Key Strategy #10: *Recognize similar triangles, and know how to deal with similarity in other figures.*

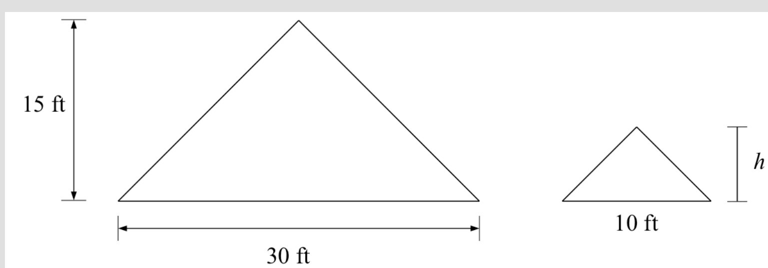
Example



The figure above shows an isosceles triangular roof with a 30-foot base and a 15-foot height. A 10-foot support crossbeam parallel to the base of the roof is to be installed a distance x feet from the base. What is the value of x ?

A. 5
B. 10
C. 15
D. 20
E. 25

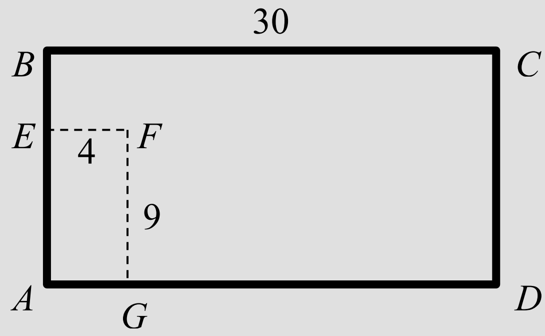
You can't solve directly for *x*, because it is not itself a side of any triangle. You can separate the triangles though, and set up a proportion to solve for the height of the small triangle (shown below as *h*). Then, you can subtract this from the total height to solve for *x*, which is the leftover height above the base of the roof.



$$\frac{15}{h} = \frac{30}{10} \qquad 30h = 150 \quad h = 5$$

Since *h* is 5 and the roof is 15 feet high in total, the support must be installed 15 – 5 = 10 feet above the base of the roof. Choice B is correct.

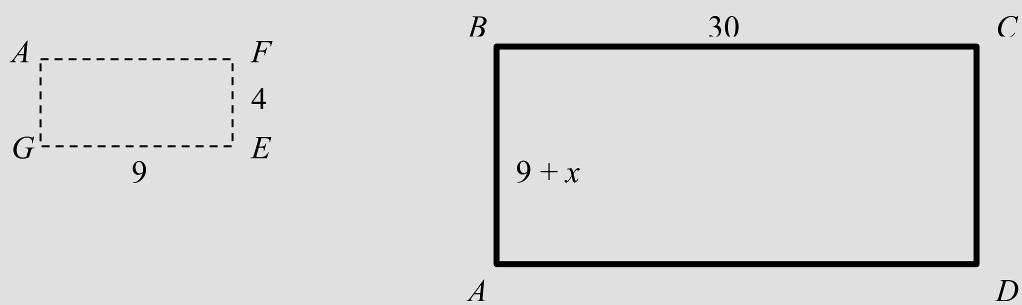
Example



The figure above provides certain dimensions, in feet, of the outline of a rectangular shed foundation (solid line) and the outline of a rectangular storage room with two of its walls against the sides of the foundation (dashed line). If *ABCD* is similar to *GAEF*, what is the area of the shed's foundation, in square feet? (Note: figure is not drawn to scale.)

A. 90
B. 210
C. 400
D. 1,500
E. 1,800

Notice the order in which the similarity statement is given. You're told *ABCD* is similar to *GAEF*, which means you should separate and re-draw the figures so that the corresponding sides are lined up with one another. This is done by rotating *GAEF* 90°, so that you can see that *GA*, which is 4 (it has to be the same as *EF*), corresponds to *AB*.



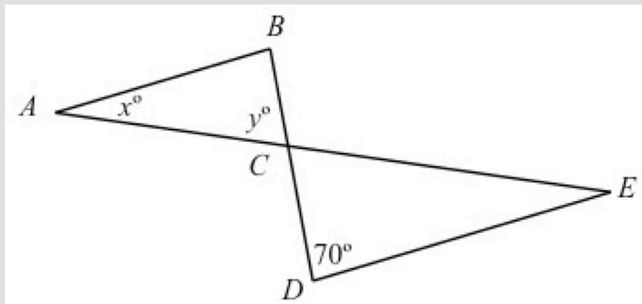
Notice that side *AB* was designated (9 + *x*) because you don't know the extra length between points *E* and *B* in the original figure, which you can refer to as *x*. The appropriate proportion for determining *AB* is therefore

$$\frac{4}{x+9} = \frac{9}{30}$$

Solving for *x* gets you $x = 4\frac{1}{3}$, which means side *AB* is $13\frac{1}{3}$. The area of *ABCD* is therefore $30 \times 13\frac{1}{3} = 400$. Choice C is correct.

Key Strategy #11: *When you have parallel lines, you may have to extend them to reveal simple angle relationships.*

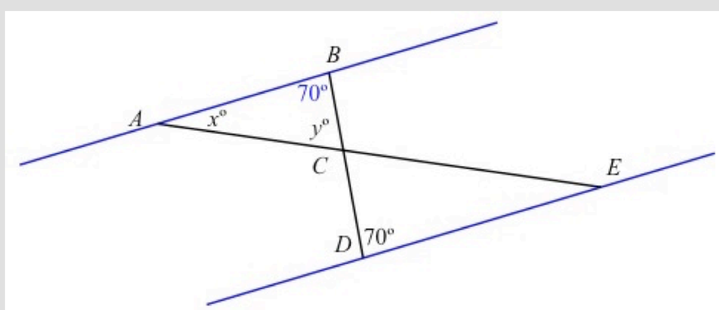
Example



In the figure above, *AB* and *DE* are parallel, and *C* lies on both *BD* and *AE*. What is the sum of *x* and *y* ?

A. 70º
B. 90º
C. 100º
D. 110º
E. 120º

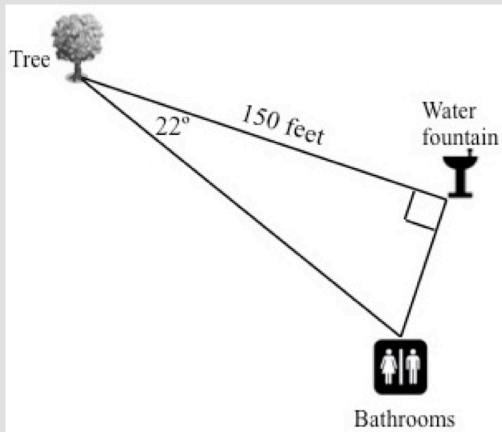
When you have parallel line segments, there are often rules you can take advantage of that are hiding in plain sight. A good strategy is to extend the parallel lines to make the figure look more like the typical "parallel lines cut by a transversal" figure you're used to seeing.



See? You can establish that angle *ABD* is 70° because it is an alternate interior angle with angle *CDE*. Furthermore, since *ABC* forms a triangle, its angles must sum to 180°. Thus, the sum of *x* and *y* is 110°. Choice D is correct.

Key Strategy #12: *When confronted with trigonometry, use the form of the answers to reveal which technique is necessary.*

Example



The figure above shows the orientations of a tree, a water fountain, and the bathrooms in a city park. Which of the following gives the distance, in feet between the tree and the bathrooms?

A. $150\sin(22°)$
B. $150\cos(22°)$
C. $150\tan(22°)$
D. $\frac{150}{\cos(22°)}$
E. $\frac{\sin(22°)}{150}$

The answer choices may make this look intimidating, but it couldn't be much easier. When you see a single trig function in each answer choice, it means you can use basic SOH-CAH-TOA trigonometry to determine the answer. Ask yourself: with respect to the angle provided, which sides of the triangle are involved in the problem? Answer: the 150, which is the adjacent side, and the side between the tree and the bathrooms that we're trying to find, which is the hypotenuse. This tells us to use cosine (Cosine = Adjacent over Hypotenuse), and thus eliminates choices other than B and D.

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}} \qquad \cos(22°) = \frac{150}{x} \qquad x\cos(22°) = 150 \qquad x = \frac{150}{\cos(22°)}$$

Choice D is correct.