INTERNATIONAL SCHOOL OF ZUG AND LUZERN

One International Experience – Three Campuses

A Whole-School Approach to Calculation in the Primary School

A Guide for Parents and Teachers

Calculation at ISZL
A Whole-School Approach

Foreword

People who are numerate are confident in using numbers in a variety of situations, both in school and later on in life. They understand the number system and are able to use a range of skills and strategies to solve problems. At ISZL we aim to continue to raise standards in mathematics through putting a greater emphasis on the development of numeracy. In school this means children will learn a range of mental calculation strategies and written methods to give them a greater understanding of the number system and a more flexible approach to solving mathematical problems.

When faced with a problem or calculation, we want children to say to themselves, “Can I do this in my head?” Mental calculation needs to be a first resort, although some more complex calculations will require a pencil and paper method. When working mentally, children will be taught to make informal jottings to help them communicate their thinking and remember the method they are using.

At ISZL we advocate that the teaching of formal written calculations, set out vertically, be delayed until approximately the beginning of Grade 3 (obviously depending upon ability). This is because children need to be confident and competent with a range of mental calculation methods which they fully understand so that they can manipulate numbers with ease. We will ensure that the children are happy in their understanding of the mental methods (with jottings) before they move on to more formal written methods. As a bridge between mental calculation strategies and formal, compact algorithms, children will also be taught informal, expanded vertical methods which rely on previously learnt mental methods.

This guide is intended to help support teachers in the progression of calculation within school and to inform parents about the methods being taught and how children are expected to record them.

Some of the methods explained in this booklet will be familiar to you whilst others may not be and we would hope that anyone with any questions would come and discuss them with their class teacher. We will always be available to help.

Neil McCallion, November 2011

Number Facts – rapid recall and deriving related facts

In addition to using mental calculation strategies to solve problems, we need children to be able to recall and derive quickly a range of number facts such as their multiplication tables. We have specific learner outcomes for each year group but it is important to emphasise that not all children will work at their grade level with every aspect of mathematics; children will be taught appropriate to their needs.

Pre-school and Pre-K – The Early Years

Early years classes will develop understanding of mathematics through the various areas of learning within learning engagements such as games, rhymes, songs, stories, construction, imaginative play, outdoor play, cooking, shopping, music, art, exploring patterns and number in our environment and of course daily routines.

Many routines and activities at home also have potential for developing children's mathematical understanding. A question or a comment that is offered during play activities or daily routines can help children see the mathematics they are using and challenge them to think mathematically.

The following ideas are starting points for families to dip into, experiment with and build on:

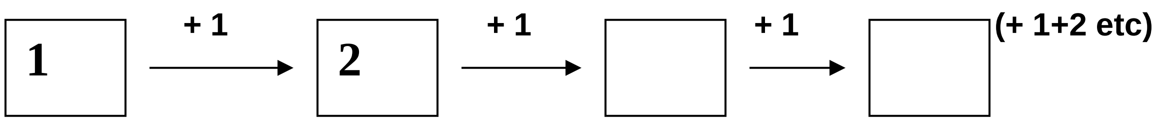
Theme	Questions/Ideas	Mathematics
Money and Shopping	How much do you think will be in here? Let's count it and see…	Prediction based on experience
	What's this coin? What's this one? Which is worth more? So why is this one smaller?	Recognition of coins
	What coin do you think we've got most of? *Make piles of coins (same value) Children will often think the tallest tower has the larger amount of coins as they will disregard the thickness of the coins*	Prediction
	When counting, ask your child to watch carefully, and only count when a coin is picked up or placed down. This helps children match a number name to an object when counting	Counting and one-to-one matching
	How much is this? Too much? Too little? How much change do we need?	Calculation and solving problems
	Collect small amounts of money in a piggy bank, tin, or play till. Children can keep a record of how many of each coin they have	Recognition of coins Counting Checking
The Order of the Day	*What do you do first when you get to school?* (eg. Taking coats off, going to the toilet, drawing/painting, washing hands, looking at books) *Can you be more detailed?* (eg. Open the door, hang my bag up, go to the toilet, wash my hands, sit in the book corner…)	Order Sequence of events
	Link above to any home routines…*In a few minutes we have to do something else. What do you think we're going to do next? Are you sure? How do you know? What happens after that?*	Prediction based on experience Justification
Dates, the Calendar and the Weather	*What day of the week is it? What day was it yesterday? What day will it be tomorrow? How many days until….? How many days in a week? How did you know? Are there always seven days in a week, four weeks in a month, twelve months in a year?* *What's the number in the date today? Try writing it in the air, on your hand, on the carpet, on someone's back…*	The structure of days, weeks and months, passage of time Writing numerals
	Draw a simple chart so that your child can record the weather over a seven day period. A piece of paper with the days of the week written along the bottom or down the left hand side can be used. Your child can then draw a picture or pictures next to each day to show what the weather is like,e.g. sun, cloud	Using a chart Using symbols

Birthdays	*How old are you?* *How old were you last year?* *How old will you be on your next birthday?*	Passage of time
	How old is your sister/brother/friend? *Can you ever be older/younger than them? Why?*	Mathematical reasoning
	Count the number of candles on a cake during a birthday party. Ask the children to clap the same number of times	Counting
News Time	Show your child an item such as a doll, toy car or an interesting picture. After a short time hide the item and ask your child some simple questions about what they might have noticed, e.g. *what colour were the eyes? How many wheels were there? What number was on the bus? What was under the table?* Some children find this quite difficult. As a lead into the above activity, it is sometimes better to talk to your child about the item before hiding it from view.	Counting Observing and remembering numbers, shapes and colours Using mathematical language Searching for properties
Getting Changed	*How long does it take you to get dressed for school?* *How long does it take you to get ready for bed?*	Timing (in minutes or counting
Helping at Home	Let your child help you to prepare for a meal or a party. *How many cups? How many plates? How many sandwiches if everyone has two?* Mixing soft drinks e.g. one part squash to 5 parts water	Calculating and checking
	Sorting the washing into colours and whites *How many of each? How many scoops of washing powder/how many tablets?*	Separate a given number of objects into groups Counting
	Ask your child to help you put away the weekly shopping. What goes in the fridge, which shelf? Where does the washing powder go? What about the bread? Can the items be stored neatly?	Mathematical language (position, size, shape) Counting
In the Street	Encourage your child to look for and say numbers around them, e.g. on houses, shops, buses, car number plates, elevators, speed signs Look for situations where you can encourage your child to count, e.g. Up and down steps, the number of red cars, the number of dogs you see on your trip Encourage your child to look for, and name, shapes they see around them e.g. road signs, on buildings	Recognise and name numerals Counting Recognising and naming shapes
Games	Games can be both fun and educational. For example, hopscotch, skittles or snakes and ladders provide opportunities for children to count and calculate	

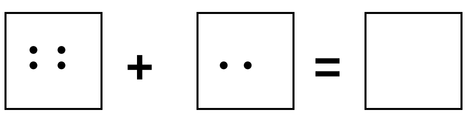
First steps to recording calculation

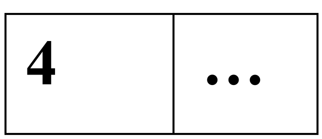
Being able to record the numerals 0 to 9 is not an indication of mathematical ability. As with developing writing generally, children's early mark making should be encouraged and it is important to recognise that what may look like a scribble or a series of random dots may well have meaning for the child. Quantities and simple calculation can be recorded pictorially, using a range of mark-making tools, e.g. paint, finger paint, crayons, printing blocks, objects stuck on to paper, dots, towers of bricks, beads threaded on string, circles and lines in sand or finger marks in clay. The important realisation form a mathematical point of view is that children very often begin to think about quantities and numbers *before* they have developed writing skills.

Teachers **do** need to model simple addition and subtraction using formal notation, including use of operation and equals symbols. Children should be encouraged to use their own pictorial recording to represent quantities and the results of simple calculations.



$4 + 2 = \square$





$4 + \square = \square$

Take away two

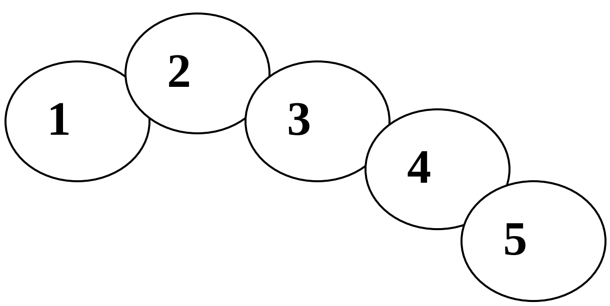
4	Take away 2 →	

Using number tracks

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

5 + 1 = ☐

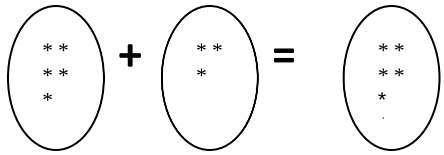
Adding one



Addition

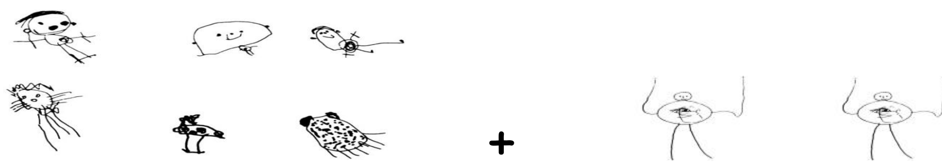
- Using dots to represent numbers

5 + 3 = 8



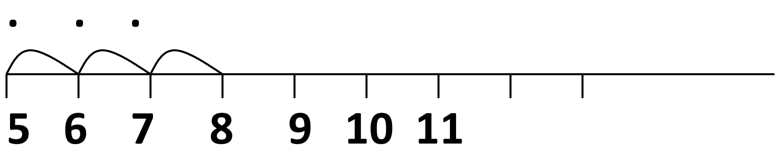
- Drawing the objects as two sets

6 + 2 = 8

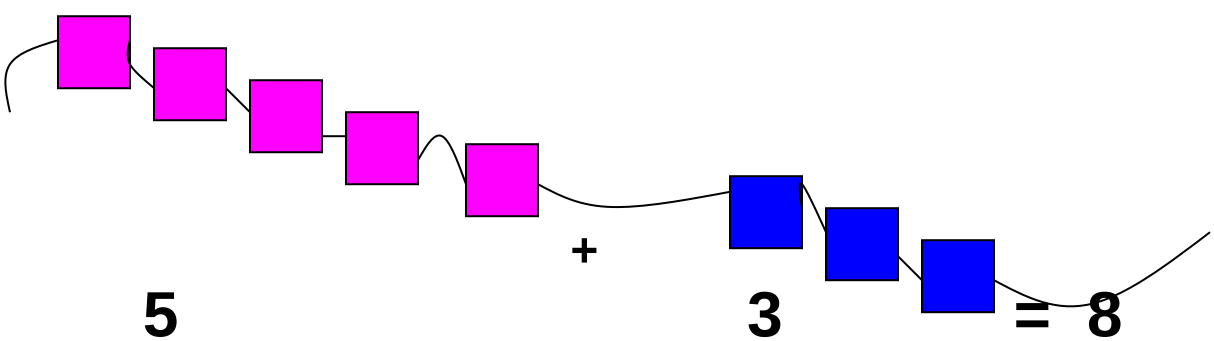


- Recording on a number line using single jumps

5 + 3 = 8

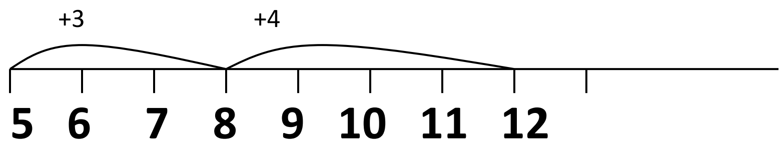


- Using a bead string



- Recording on a number line using larger jumps

5 + 3 + 4 = 12



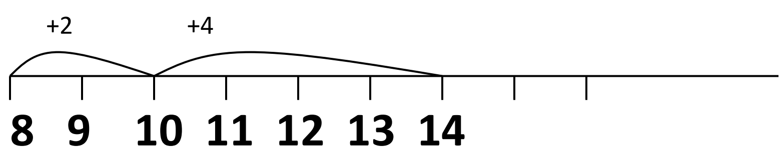
- Use knowledge of bonds to 10 to add 3 or more numbers

3 + 8 + 7 = 10 + 8 = 18

This would apply with bonds to any multiples of 10 at later stages of learning

- Bridge through ten by partitioning

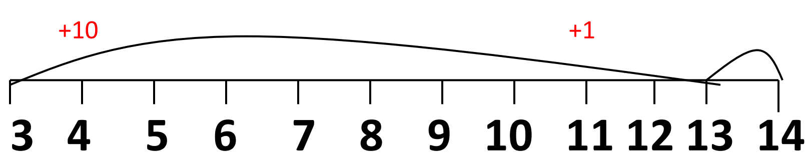
8 + 6 = 8 + 2 + 4 = 14



This would apply to bridging through any multiples of 10 at later stages of learning

- Partition to add 11 or 21

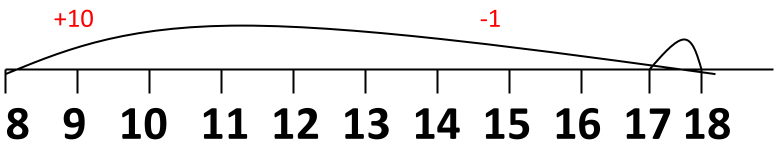
3 + 11 = 3 + 10 + 1 = 14



This would apply to adding any similar near multiple of 10 at later stages of learning

- Adjustment to add 9 or 19

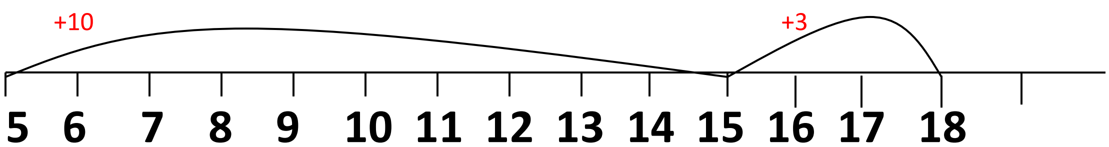
$$8 + 9 = 8 + 10 - 1 = 17$$



This would apply to adding any similar near multiple of 10 at later stages of learning

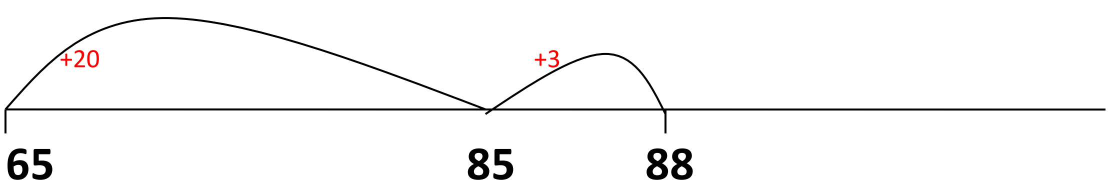
- Partition and recombine to add other numbers

$$5 + 13 = 5 + 10 + 3 = 18$$



And larger numbers …

$$65 + 23 = 65 + 20 + 3$$
$$= 85 + 3$$
$$= 88$$



Ensure that children know addition can be done in any order to do mental calculations more efficiently (Beginning with the largest number etc.).

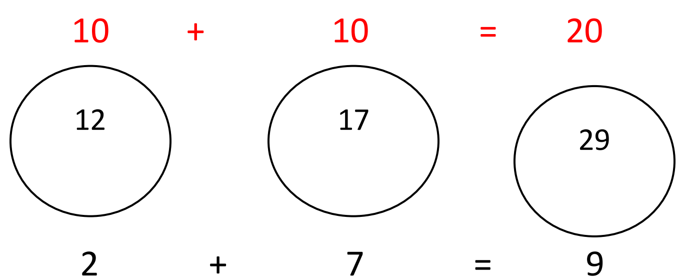
- Identify near doubles using doubles already known

$$8 + 9 = 8 + 8 + 1 = 17$$
$$15 + 16 = 15 + 15 + 1 = 31$$

This can be used with much larger numbers later on too:

$$38 + 36 = (40 + 40) – 6 = 74$$
$$370 + 380 = (400 + 400) – 50 = 750$$

- Partition and recombine using circles to record the partitioning *(only really suitable for 10s and units)*

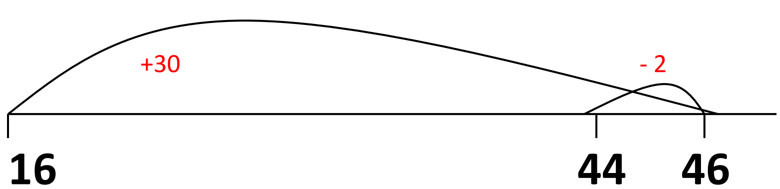


$$12 + 17 = 10 + 10 + 7 + 2 = 29$$

This is developed into the informal mental addition with jottings where hundreds are also recorded more easily

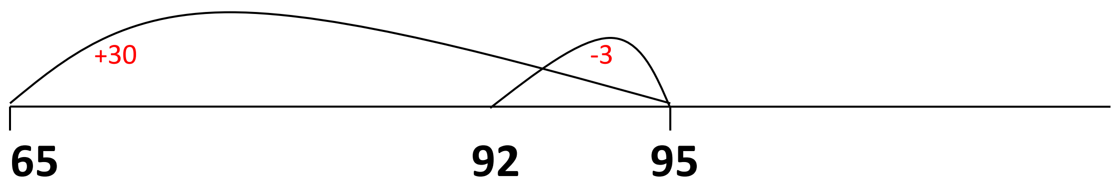
- Adjustment to add other near multiples of 10

$$16 + 28 = 16 + 30 - 2 = 44$$



And with larger numbers either with or without a number line…

65 + 27 = 65 + 30 - 3
= 95 - 3
= 92



Informal mental addition with pencil + paper jottings

- Partitioning

27 + 44 = 20 + 40 + 7 + 4
= 60 + 11
= 71

247 + 438 = 685
247 + 438 = 247 + 400 + 30 + 8
= 647 + 30 + 8
= 677 + 3 + 5 here, the 8 has been split into a 3 and 5 to bridge through the 10.
= 685

or…..

- Adjustment

27 + 44 = 30 + 44 – 3 = 71

or….

27 + 44 = 30 + 41 = 71

Vertical Layouts for Addition

When children can add or subtract *any* pair of two-digit numbers, recall all their bonds to 20 and partition three-digit numbers, they are probably ready for vertical calculation methods. However, the first vertical methods are what we call *expanded* methods and they do not involve 'carrying' figures. The idea behind this is that pupils can complete each stage of an expanded method mentally; they are simply arranging each stage vertically instead of horizontally

- **Partitioning and recombining values of digits**

Adding the least significant digits first in order to work towards compact methods:

Recording Method

68 + 24 = 92

```
     6 8
   + 2 4
   -----
Units first  →   1 2
Then tens    →   8 0
   -----
     9 2
   -----
```

Add mentally from the least significant units:

123 + 425 = 548

```
      1 2 3
    + 4 2 5
    -------
Units first     →       8
Then tens       →     4 0
Then hundreds   →   5 0 0
    -------
      5 4 8
    -------
```

Extend to addition of larger numbers (e.g. Th . H. T. U + T h . H. T. U) and then on into decimals when children are ready.

```
                                  5 6 . 7 5
                                    7 . 8 6
                                  ---------
                                    0 . 1 1
Extension to decimals  →            1 . 5 0
                                  1 3 . 0 0
                                  5 0 . 0 0
                                  ---------
                                  6 4 . 6 1
```

This method helps children understand what is happening when we introduce 'carrying' figures

Compact standard written methods

Adding least significant digits first and carrying under the line

```
  1 6 3
+ 5 7 5
  7 3 8
  1
```

Progression using 'carrying'

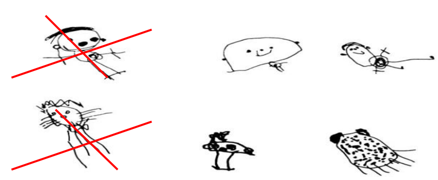
```
  6 7 9        2 5 4 7          5 . 6 8
+ 4 6 5      +   6 7 6  ---->  + 4 . 8 7
1 1 4 4        3 2 2 3         1 0 . 5 5
  1 1          1 1 1             1 1   1
```

Subtraction:

(Methods to follow all the practical suggestions referenced earlier)

- Draw objects and cross them out

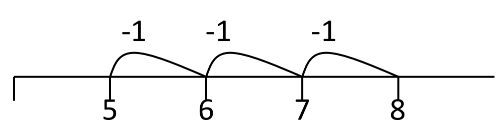
6 - 2 = 4



- Use a number line and count back from the larger number

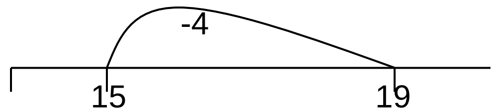
8 - 3 = 5

Using single steps



Using larger jumps

19 - 4 = 15

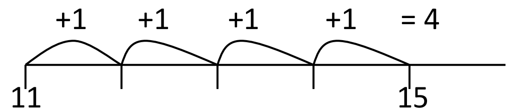


At later stages, with larger numbers, counting back is really best when there's only a small amount to be subtracted.

- Find a <u>small difference</u> by counting up from the smaller to the larger number using a number line *(a good method when the two numbers are near each other in value / size)*

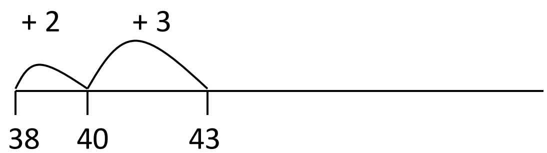
15 - 11 = 4

Using single jumps



Counting on to find a small difference with larger numbers, bigger steps and an empty number line

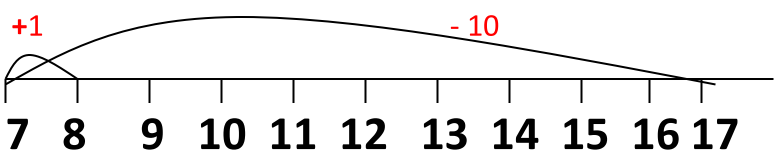
43 - 38 = 5



This develops at later stages, turning subtraction into addition by knowing related calculations e.g. 43 – 38 = ? so 38 + ? = 43 so I just count on from 38 to find the missing value

- Adjustment to subtract 9 or 19

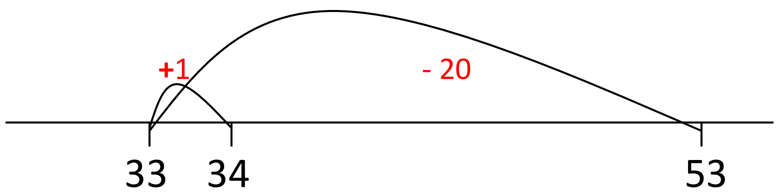
17 - 9 = 17 - 10 + 1 = 8



Add one back on as you took away one too many

Or

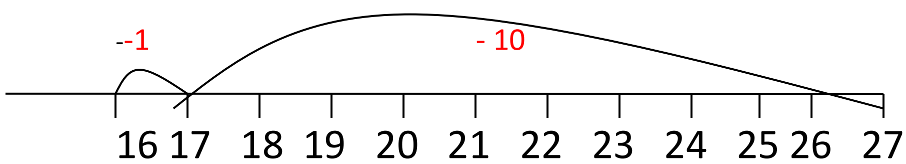
53 - 19 = 53 – 20 + 1 = 34



This method can be used to very effectively to subtract mentally any near multiple of 10 from (or to) any 2 digit number as children become more proficient.

- Adjustment to subtract 11 or 21

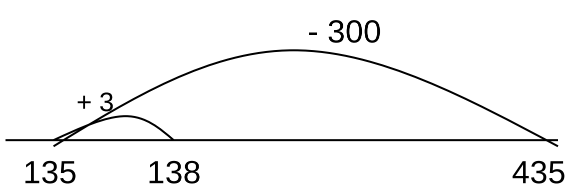
27 - 11 = 27 - 10 - 1 = 16



Or any other similar near multiple of 10 e.g. 31 - 41 – 51 etc. at later stages.

- Compensation/ Adjustment – taking away too much, then adjusting- can be used with much larger numbers later on

435 – 297 = (435 – 300) + 3



- Compensation/ Adjustment – *expanded vertical layout:*

435 – 297

```
  4 3 5
- 2 9 7
  1 3 5 (- 300)
 +    3
  1 3 8
```

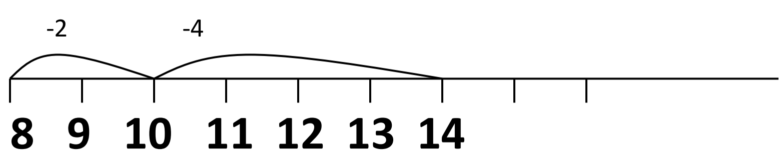
- Partition and recombine

36 - 24 = 12
30 – 20 = 10
6 - 4 = 2

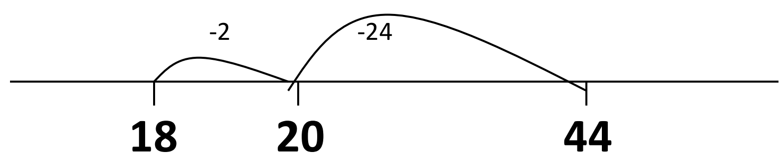
At early stages, avoid calculations like 32 – 17 as we do not want to say " 2 – 7 can't be done". Encourage children to make choices as to when this method is effective. Once children are happy with negative numbers, this method can be used so that 2 – 7 would be -5.

- Bridge through tens by partitioning

14 - 6 = 14 – 4 - 2 = 8



44 - 26 = 44 – 24 - 2 = 8



- Use patterns of similar calculations. Stating the corresponding subtraction to a given addition (and vice versa).

6 + 8 = 14

therefore

14 – 6 = 8

and

14 – 8 = 6

OR …

33 + 15 = 48

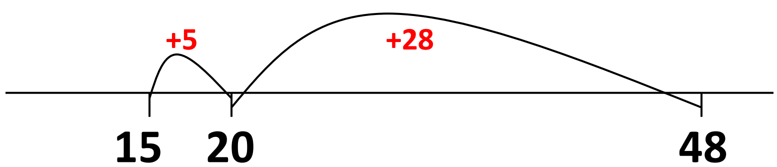
48 – 15 = 33

and

48 – 33 = 15

Later on, be able to use this knowledge to "count up" and so turn a subtraction into an addition. (Children generally find counting on easier and this is the same method as for finding a small difference.)

48 – 15 = 33 (I know 15 + ? = 48 so if I count on from 15, I'll find what ? is)



Mirror these steps used on the number line to transfer skills to expanded vertical layout when using larger numbers and children are ready to move on.

- Count on in an expanded vertical method

480 – 155 = 33

```
 480
 155
----
   5   (to 160)
  40   (to 200)
 280   (to 480)
----
 325
----
1
```

Add up all the steps taken from the smaller to the larger number

Eventually the step sizes should become much smaller so rather than a step of 5 then 40, it would be 1 step of 45 to 200 immediately.

Go on from HTU to THHTU then any number of digits and finally decimals.

Counting up (developmental steps) :

```
   6 6 5
 -  2 7 8
     2 2   (300)
   3 0 0   (600)
     6 5   (665)
   3 8 7
```

leading to →

```
   6 6 5          6 . 6 5
 - 2 7 8      -   2 . 7 8
     2 2(300)     0 . 2 2 (3.00)
   3 6 5(665)     3 . 6 5 (6.65)
   3 8 7          3 . 8 7
```

Extension to decimals

- Use knowledge of negative numbers to aid subtraction when partitioning

324 - 168 = 156
300 - 100 = 200
20 - 60 = - 40
4 - 8 = - 4

200 – 40 – 4 = 156 *do the calculations as shown by answers. If the numbers weren't negative, they'd just all be added together.*

Make sure that addition and subtraction are taught together and plenty of opportunities are given to record using equation styles e.g. 21 + 7 = 31 – 3

Compact traditional algorithm (decomposition)

Whilst decomposition is an efficient subtraction algorithm it is important to emphasise the processes involved when teaching it. An expanded version of this method is recommended to help students *see* the underpinning mathematical principles that make it work:

Decomposition

```
  7 5 3    =    700 + 50 + 3    leading to….
-   7 6       -       70 + 6
                                          1
           =    700 + 40 +13            7 4 3
              -       70 + 6          -   7 6

                                        1 1
           =    600 +140 + 13           6 4 3
              -       70 +  6         -   7 6
                600 + 70 +  7 = 677     6 7 7
```

Multiplication and Division

Understanding Multiplication and Division

Teachers use visual models to help children understand how multiplication and division work. Children are taught that multiplication is essentially repeated addition and division is repeated subtraction. At the same time children will be practising counting forwards and backwards in steps (multiples) and learning multiplication tables. Dot pattern grids help children understand the relationship between multiplication and division. Seeing numbers set out as rectangular patterns of dots (arrays) can help children develop an understanding of repeated addition and division as subtraction of groups.

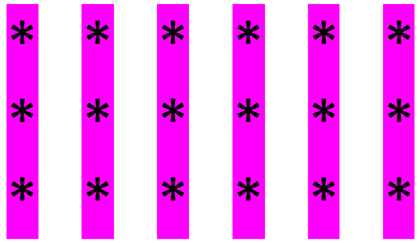
- Arrays

```
        6
    * * * * * *

3   * * * * * *     6 x 3 = 18

    * * * * * *
```

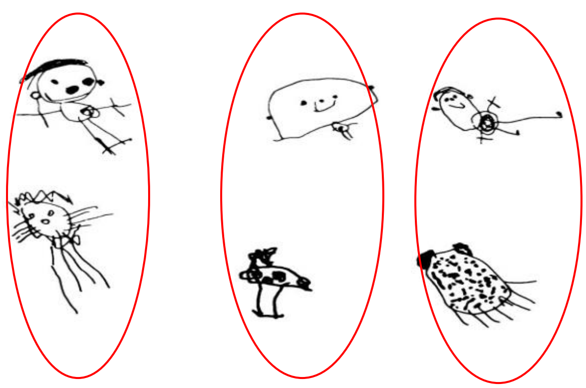
Or 18 ÷ 6 = 3



$3 \times 6 = 18$

or $18 \div 3 = 6$

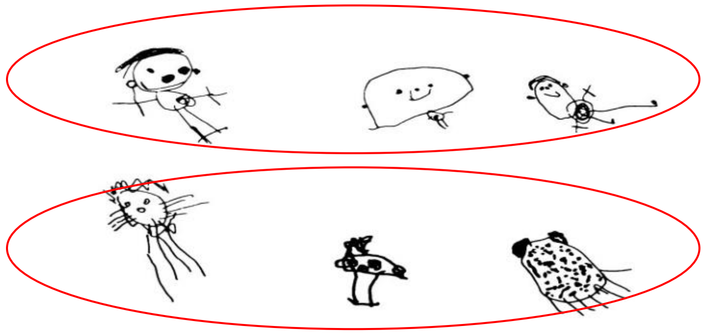
Children can be given pictures of objects and asked to put rings around to show them as 3 lots of 2 or 2 lots of 3



This represents 3 lots of 2 or 2 multiplied 3 times

2 x 3 is therefore to be read as 2 three times over / 3 lots of 2 and shown in the appropriate array

3 x 2 would be drawn like so:

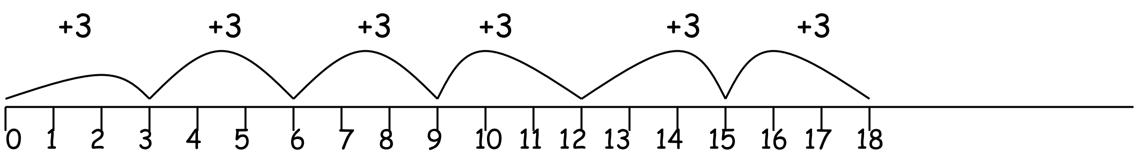


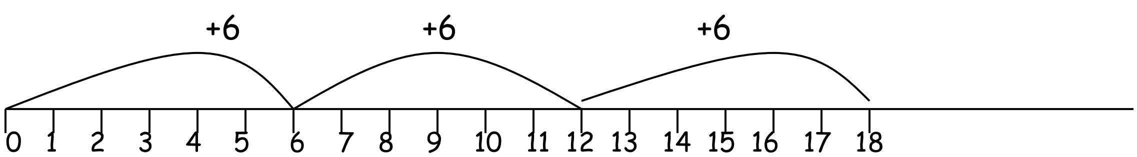
So when we say 7 x 8 we should have in our minds 8 lots of 7 and 8 x 7 would be 7 lots of 8. The answer of 56 is the same but how you arrived at it or represented it visually is different.

Seeing multiplication and division in this visual way will help children develop a better understanding which will enable them to deal with much more complex calculations in Key Stage 2.

Using a number line is also a very effective way of helping children understand multiplication and division. Children are taught that one way of thinking about multiplication is to see it as repeated addition.

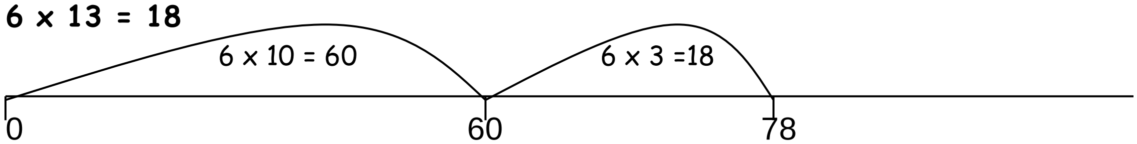
3 x 6 = 18 can be represented on a number line as six jumps of 3 and can also be shown that three jumps of six will give the same answer:



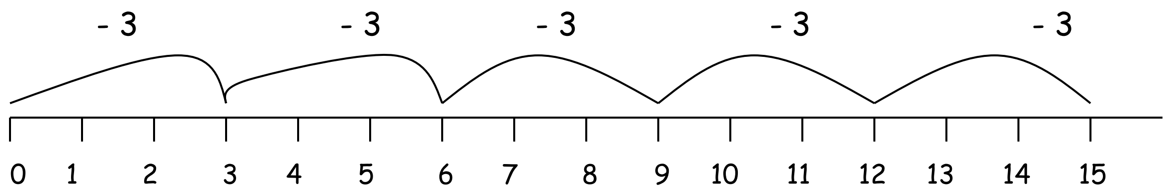


In later year groups these jumps can be put together to do more complicated calculations, partitioning the parts and adding them on

6 x 13 = 18



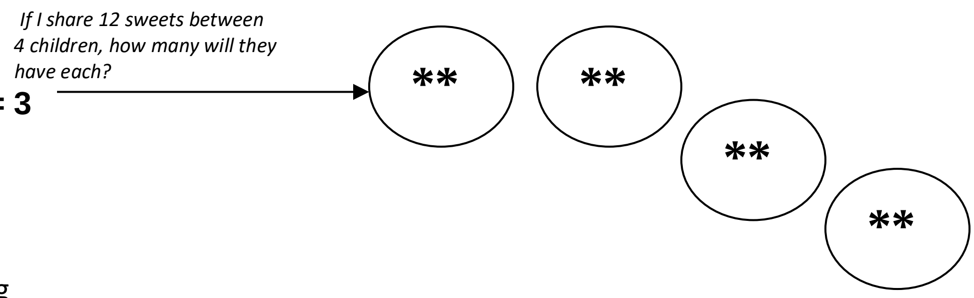
Similarly, **15 ÷ 3** can also be represented as repeated subtraction:



'Sharing' and 'grouping'

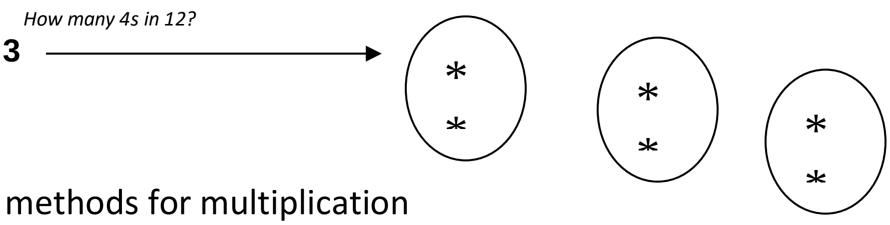
Sharing

12 ÷ 4 = 3



Grouping

12 ÷ 4 = 3



Written methods for multiplication

The grid method (partitioning)

Always remember to approximate first

TU x U

33 x 6 is approximately 30 x 6 = 180

X	30	3	
6	180	18	= 198

26 x 7 is approximately 25 x 7 = 175

x	20	6	
7	140	42	= 182

TU x TU

68 x 37

Approximate first 70 x 40 = 2800

x	60	8	
30	**1800**	**240**	= 2040
7	**420**	**56**	= 476
			2516

If pupils find it hard to x by 7 then they could partition the 7 into 5 and 2 or 3 and 4

HTU x U and HTU x TU

Approximate first
437 x 6 is between 400 x 6 = 2400 and 400 x 7 = 2800

x	400	30	7
6	2400	180	42

2400
180
42
2622

*Pupils who are finding it difficult to recall facts from x6 table can still access this calculation by partitioning the 6 into two easier numbers i.e. 4 and 2, or 5 and 1:

x	400	30	7	
2	800	60	14	= 874
4	1600	120	28	= 1748
				2622

This method can also be used very effectively to multiply using decimals also.

6.93 x 6 **6.93 x 6 is approximately 7 x 6 = 42**

x	6	0.9	0.03
6	36	5.4	0.18

36
5.4
0.18
———
41.58

= 41.58

- Factorising

Simple examples of this are x 4 is double and double again (4 can be broken down to 2x2) and x 8 is double, double then double again (8 is 2 x 2 x 2) while x 20 is x 10 then double (because 20 can be broken down into 10 x 2)

7 x 24 =

7 x 24 = 7 x 12 x 2
= 7 x 6 x 2 x 2
= 7 x 3 x 2 x 2 x 2
= 7 x 3 x 2^3

7 x 3 = 21 21 x 2 = 42 42 x 2 = 84 84 x 2 = 168

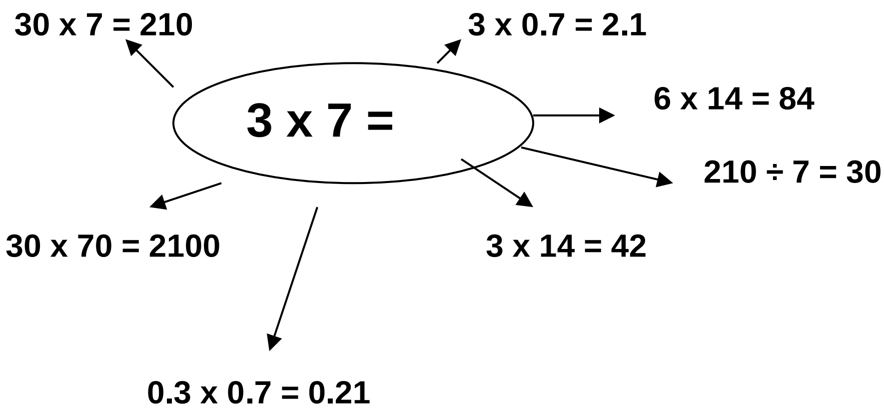
So 7 x 24 = 168

- Special cases *Where we can work out mentally quite easily through our knowledge of number relationships e.g.*

X 50 = x 100 then ÷ 2 *because 50 is ½ of 100*
X 25 = x 100 then ÷ 4 *because 25 is ¼ of 100*
X 20 = x 10 then x 2 *because 20 is 10 x 2*

- Fact webs

These can be very useful for helping children work out new facts from ones they already know:



Wherever possible, make the links between multiplication and division explicit.

Short (x U) and Long multiplication (x T U): informal written methods

26 x 7 is approximately 25 x 8 = 200

```
      2 6
X       7
    1 4 0
      4 2
    1 8 2
```

427 x 8 is approximately 400 x 9 = 3600

```
  427
  X 8
 3200 (400 x 8)
  160 (20 x 8)
   56 (7 x 8)
 3416
```

3578 x 7 =

3578 x 7 is approximately 3500 x 7 = 24500

```
   3 5 7 8
x        7
----------
 2 1 0 0 0    (3000 x 7)
   3 5 0 0    (500 x 7)
     4 9 0    (70 x 7)
       5 6    (8 x 7)
----------
 2 5 0 4 6
```

63 x 46

63 x 46 is approximately 60 x 50 = 3000

```
    6 3
x   4 6
-------
2 4 0 0   (60 x 40 .. 60 x 10 x 2 x 2)
  1 2 0   (3 x 40 )
  3 6 0   (60 x 6 )
    1 8   (3 x 6 )
-------
2 8 9 8
```

Leading to …

63 x 46 *and extending to decimals* 5.7 x 7

63 x 46 is approximately 60 x 50 = 3000

5.7 x 7 is approximately 6 x 7 = 42

```
    6 3
x   4 6
-------
2 5 2 0   (63 x 40 .. 63 x 10 x 2 x 2)
  3 7 8   (63 x 6 ..  63 x 3 x 2 )
-------
2 8 9 8
```

extend to simple decimals →

```
 35.0   (5.0 x 7)
  4.9   (0.7 x 7)
-----
 39.9
```

- Division

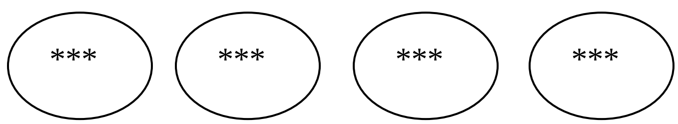
Again practical work on halving and quartering would come first and there would be lots of counting in jumps of ? to say how many lots of an amount is in another given amount e.g. “How many 5s in 25. Let’s count up in 5s and see how many 5s we have.” (Children might count of the number of groups of 5 on their fingers or make tallies etc.)

Sharing –

If I share 12 sweets between 4 children, how many will they have each?

Here the children are sharing in a “one for you one for me” way

$$12 \div 4 = 3$$

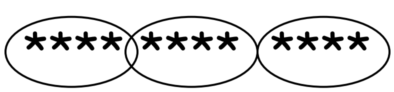


Grouping -

How many 4s in 12 ?

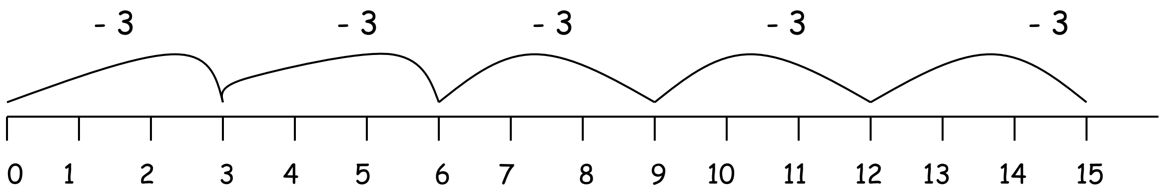
Here the children are splitting a given amount up into groups of the divisor.

$$12 \div 4 = 3$$



- Repeated Subtraction

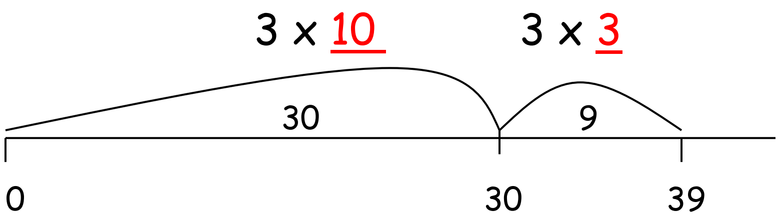
Shown on a number line, initially in single jumps



Moving onto counting forward and jumping in multiples of the divisor *(taking away chunks of the divisor). A method called:*

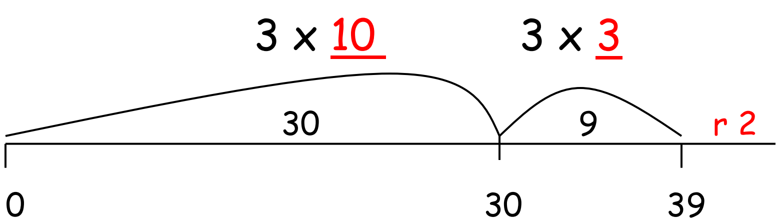
- Chunking

39 ÷ 3 = 13



And remainders

41 ÷ 3 = 13 r 2

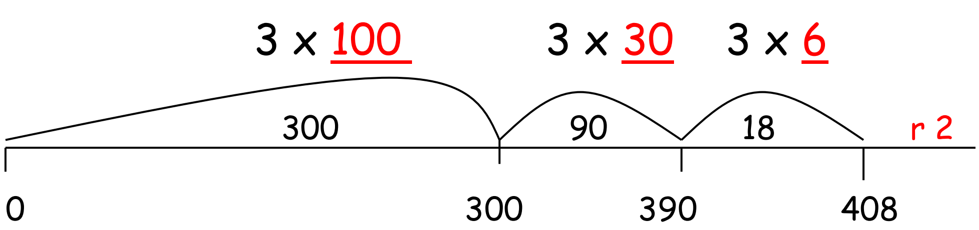


The number of groups of the divisor is underlined so children are reminded of what to add at the end and, if needed, the size of the jump is put into the balloon/jump so they can see what needs to be added on at each step.

And leading on to bigger chunking

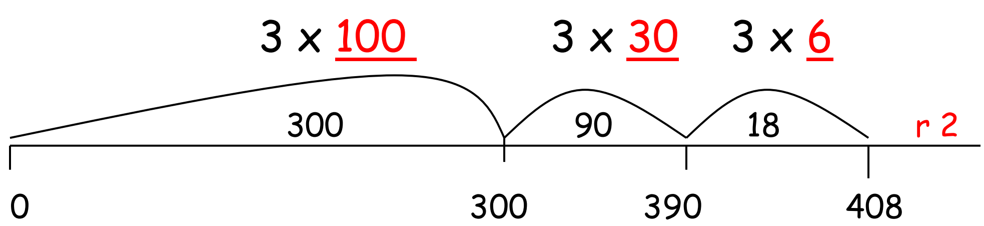
410 ÷ 3 Approx *390 ÷ 3 = 130*

410 ÷ 3 = 136 r 2



And then expressing remainders as fractions

410 ÷ 3 = 136 and 2/3 *(I have 2 out of the 3 needed for another jump of 3 so 2/3)*



The chunking recorded on the number line is then mirrored in a vertical layout

Approx = 900 ÷ 6 = 150

873 ÷ 6

```
6) 8 7 3
 - 6 0 0      (6 x 100)
   2 7 3
 - 2 4 0      (6 x 40)
     3 3
   - 3 0      (6 x 5)
       3
```

Answer: 145 remainder 3 or 145 3/6 or 145 ½

344 ÷ 6 =

344 ÷ 6 is between 300 ÷ 6 = ***50*** *and 360 ÷ 6 =* ***60*** *so the answer must be between 50 and 60*

344 ÷ 6

```
  3 4 4
- 3 0 0    (6 x 50)
    4 4
-   4 2    (6 x 7)
      2
```

Answer: 57 remainder 2

When students are adept at using 'chunking', the number line, or even vertical layouts, become unnecessary, e.g:

5 6 4 ÷ 9

↓

540 + 18 + 6

(9 x <u>60</u>) + (9 x <u>2</u>) + 6 = 62 R6 = 62 6/9 = **62.666**

And extend to decimals

896 ÷ 24 is approximately 900 ÷ 25 = 36

$$24\overline{)\,896}$$

	8 9 6	
-	7 2 0	(24 x 30)
	1 7 6	
-	1 6 8	(24 x 7)
	8. 0	
-	7. 2	(24 x 0.3)
	0. 8 0	
-	0. 7 2	(24 x 0.03)

Answer: **37.33**

Factorising

Long division can often be easily accomplished using knowledge of factors:

896 ÷ 24 = 896 ÷ 6 ÷ 4

$$\begin{array}{r} 1\;4\;9.\;3\;3\;3 \\ 6\overline{)8\;{}^{2}9\;{}^{5}6.\;{}^{2}0\;{}^{2}0\;{}^{2}0} \end{array}$$

then:

$$\begin{array}{r} 3\;7.\;3\;3\;3 \\ 4\overline{)1\;4\;{}^{2}9.\;{}^{1}3\;{}^{1}3\;{}^{1}3} \end{array}$$