AP Calculus AB Summer Assignment

This assignment is designed to make the transition to AP Calculus AB a smooth one. You will be practicing skills you have acquired in earlier math classes. It is imperative that you are able to do most of these problems with ease, **without the use of a calculator.** **The entire assignment is due on the first day of class.** There will be a test on this material during the first week of school in the Fall.

Directions: In order to receive credit all work must be completed **in pencil**. Remember that we care about process, so show your work carefully on lined paper. **This should include: problem numbers, calculations done neatly, sketches drawn carefully, and labeled answers (circled, underlined, or boxed). Graphs should be done on graph paper.** Organize your work into columns and work down, not across the paper. **No Calculators! No Calculator, unless directed to do so! When appropriate, answers should be in simplified rational or fractional form. Last thing: ABSOLUTELY NO CALCULATORS, unless directed to do so!**

Let $f(x)=x^2$, $g(x)=2x+5$, and $h(x)=x^2-1$. Evaluate:

1. $f\left[g(x-1)\right]$
2. $g\left[h\left(x^3\right)\right]$
3. $g\left(f\left(a+h\right)\right)$

Find $\frac{f(x+h)-f(x)}{h}$ for each given function $f(x)$. Let h represent some constant.

4. $f(x)=9x+3$
5. $f(x)=x^2-10$
6. $f(x)=x^2-3x+2$

Find the x and y intercepts for each equation:

7. $y=x\sqrt{16-x^2}$
8. $y=x^2\left(x-1\right)^2$
9. $y=\frac{x-\frac{1}{2}}{x+2}$

The domain is the set of all the possible values for x (everything you can input), and the range is the set of all possible values for y (every possible output.)

Find the domain and range of each function. Write your answer in interval notation.

10. $f(x)=x^2-5$
11. $f(x)=-\sqrt{x+3}$
12. $f(x)=3\sin x$
13. $f(x)=\frac{x}{x^2-9}$
14. $f(x)=\frac{x-1}{x^2-x-6}$
15. $f(x)=\frac{3x^2}{x^2+1}$
16. $f(x)=\frac{2}{x-1}$

17. Write the equation of a line with a slope of 3 and a y-intercept of 5 in slope-intercept form.
18. Write the equation of a line passing through the point $(0,5)$ with a slope of $\frac{2}{3}$.
19. Write the equation of a line passing through the points $(-3,6)$ and $(1,2)$.
20. Write the equation of a line passing through the points $(-4,2)$ with a slope of zero.
21. Write the equation of a line passing through the points $(-4,2)$ with an undefined slope.
22. Write the equation of a line passing through the points $(2,8)$ and parallel to the line $y=\frac{5}{6}x-1$.
23. Write the equation of a line passing through the point $(2,-3)$ normal to $y=\frac{5}{6}x-1$.

Find the value without a calculator.

24. $\tan\left(\arccos\frac{2}{3}\right)$ 25. $\sec\left(\sin^{-1}\frac{12}{13}\right)$ 26. $\sin\left(\arctan\frac{12}{5}\right)$ 27. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

The vertical asymptote occurs where the function is undefined (where the denominator is equal to zero).

Determine the vertical asymptote(s) for the function.

28. $f(x)=\frac{1}{x^2}$ 29. $f(x)=\frac{x^2}{x^2-4}$ 30. $f(x)=\frac{2+x}{x^2(1-x)}$

Determining the horizontal asymptote.
Case 1: The degree of the numerator is less than the degree of the denominator. The function has a horizontal asymptote at $y=0.$
Case 2: The degree of the numerator is equal to the degree of the denominator. The function has a horizontal asymptote equal to the ration of the leading coefficients.
Case 3: The degree of the numerator is greater than the degree of the denominator. The function has no horizontal asymptote.

Determine the horizontal asymptote.

31. $f(x)=\frac{x^2-4x+10}{x^3+x-5}$ 32. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$ 33. $f(x)=\frac{4x^5}{x^2-7}$

Solve:

34. $x^2-6x+5=0$ 35. $3x^3-4x=1$ 36. $2e^{-t}-2te^{-t}=0$

37. $\sin^2 x-\sin x=0,\ \text{for } 0\le x\le\frac{3\pi}{2}$

Simplify:

38. $\frac{x^2-a^2}{x^4-a^4}$ 39. $\sqrt{x}(x-1)^2$ 40. $\frac{\frac{x+h}{x+h+2}-\frac{x}{x^2}}{h}$ 41. $\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

42. $\left(\frac{1}{x}+\frac{1}{y}\right)(x+y)^{-1}$ 43. $\left(x^{-1}+y^{-1}\right)^{-1}$ 44. $\frac{x^2-2x-8}{x^3+x^2-2x}$

Simplify completely. Leave no variables in the denominator.

45. $\frac{(2a^2)^3}{b}$ 46. $\frac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$ 47. $\frac{ab-a}{b^2-b}$ 48. $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$ 49. $\left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\frac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

Solve for the indicated variable:

50. Solve for ***z*** in terms of ***x*** and ***y***: $3xy^2z+y^3+2x^2yz+2xy^2=0$
51. Solve for ***z*** in terms of ***y***: $z-z\sin y=1$
52. Solve for ***y*** in terms of ***x***: $\frac{1}{2}e^{2y}=x^3+2$

53. If $f(x)=\frac{1}{3}x^3$, then find k so that $f(k)-f(-3)=0$.

54. If $f(x)=3x^2-6x+12$, then find the minimum value of f.

55. If $f(x)=\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$, then find the value of $\frac{1}{2-0}\left[f(2)-f(0)\right]$.

56. If $f(x)=3\ln x+C$, and $f(e)=2$, then find the value of C.

57. If $f(x)=\frac{x^2}{2}\ln x-\frac{3}{4}x^2+C$, and $f(1)=6$, then find the value of C.

58. If $f(x)=\sec^2(2x)$, then find $f\left(\frac{\pi}{6}\right)$.

59. If $f(x)=e^x-\frac{x^2}{2}$, then find the value of $f(4)-f(0)$.

60. If $f(x)=\frac{3}{2}\sqrt{1+\tan x}\bullet\sec^2 x$, then find $f(0)$.

61. If $f(x)=\frac{3}{5}(2x)^{\frac{5}{3}}$, then find $f(4)$.

62. If $f(x)=-x-\frac{x^2}{2}+\frac{x^3}{3}$, then find $f(3)$.

63. If $f(x)=-\frac{x}{2}-x^2+\frac{x^3}{3}$, then find $f(4)-f(2)$.

64. If $f(x)=\begin{cases}2x^2, & when\ \ x<0\\ 3x, & when\ \ 0\le x\le 1\\ |6-x|, & when\ \ x>1\end{cases}$, then find $f(-3)+f(1)$.

Multiply:

65. $2^3\bullet 2^m$ 66. $\left(a^k+3\right)\left(a^k-2\right)$ 67. $x^{\frac{5}{8}}\left(x^{\frac{3}{8}}-10x^{\frac{11}{8}}\right)$

Without using a calculator, evaluate the following: (Imperative that you can do this with ease w/o a calculator quickly!)

68. $\cos 210^\circ$ 69. $\sin\frac{5\pi}{4}$ 70. $\tan^{-1}(-1)$ 71. $\sin^{-1}(-1)$ 72. $\cos\frac{9\pi}{4}$

73. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 74. $\tan\frac{7\pi}{6}$ 75. $\cos^{-1}(-1)$ 76. $\sec\left(\frac{\pi}{6}\right)$

77. Divide $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$.

78. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

79. Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$.

Write as a single equation in ***x*** and ***y***:

80. $\begin{cases} x = t + 1 \\ y = t^2 - t \end{cases}$

81. $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$

82. $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

Graph the following. You should recall how they are graphed from Pre-Cal. Use a calculator only if you absolutely need to:

83. $y = \ln x$

84. $y = \dfrac{|x|}{x}$

85. $y = e^x$

86. $y = e^{-x}$

87. $y = 2^x$

88. $y = \sqrt{x}$

89. $y = \begin{cases} \sqrt{x+1}, & 0 \le x \le 3 \\ 5 - x, & 3 < x \le 5 \end{cases}$

If all of the above was not enough, you <u>MUST</u> be able to crunch fractions in a timely manner! Do the following as efficiently as possible <u>without the use of a calculator</u>!

Given: $f(x) = \dfrac{1}{2}x^2 + 2x - 3$ and $g(x) = \dfrac{2}{3}x^3 - \dfrac{1}{2}x^2 + \dfrac{1}{5}x + 7$, Find:

90. $f\left(\dfrac{1}{5}\right)$

91. $g(3)$

92. $g\left(\dfrac{1}{2}\right) - f\left(\dfrac{1}{2}\right)$

Using the Graphing Calculator: These are the only problems that you should use a calculator for!!!!!!!!!!

93. Solve $6e^{2x} = 18x^2$.

94. How many solutions exist for the equation $\dfrac{\cos^2 x}{x} - \dfrac{1}{5} = 0$ on the interval $0 < x < 10$?

95. If $f(x) = 2xe^{2x}$, then find the absolute minimum of ***f*** and the range of ***f***.

96. If $f(x) = 80 - 10\cos\left(\dfrac{\pi x}{12}\right)$, then (a) graph f over the interval $0 \le x \le 24$, and (b) find x-values such that $f(x) \ge 78$.

97. For $0 \le t \le 10$, how many values of t satisfy the equation $\cos t = -2e^{-2t}$? Do not find specific values.

98. Solve: $1 - 3e^{-0.2\sqrt{x}} = 0$.

99. Find the range of $f(x) = xe^{1-x}$.

100. Find the domain of $y = \ln\left(\dfrac{x}{x-1}\right)$.

I will post an answer key on <u>qayumi.weebly.com</u> so that you can check your answers. Make sure that you show work for any problem that requires work to be shown.