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For Integrated Math III Students

Solutions for Summer Math Packet

Math 3 Summer Review Packet

One Variable Statistics:

Mr. Day asks his students in his two history classes how old they were when they rode on a plane for the first time. The responses are in the tables below. Use the tables below to solve problems 1-2.

Class 1	
Student	Age in years
1	10
2	9
3	2
4	11
5	3
6	10
7	7
8	13
9	8
10	8
11	15
12	12
13	2
14	5
15	9
16	7
17	10
18	13
19	11

Class 2	
Student	Age in years
1	3
2	6
3	11
4	10
5	5
6	7
7	1
8	2
9	4
10	3
11	1
12	5
13	3
14	10
15	6
16	2
17	4
18	7
19	1

1. Find the measures of center for each data set. Measure of centers are the mean and median.

Class 1

mean: 8.68
median: 9

Class 2

mean: 4.79
median: 4

2. Find the standard deviation of both data sets and determine which data has a larger spread. **Standard deviation $(\sigma x:)$ can be found in your calculator by putting your data into a List and Spreadsheet, hit menu, 4. Statistics, 1. Stat Calculations, 1. One-Variable Statistics**

Class 1

3.60

Class 2

3.04

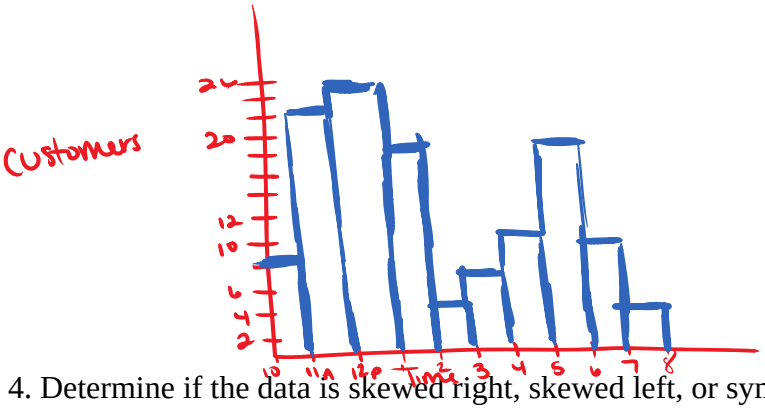
Class 1 has a slightly larger spread than class 2.

Use the following information for problems 3-4

A deli records the number of customers each hour that they are open to determine how many staff members they need working at each time of day. The number of customers is in the table below.

Time Frame	Number of Customers
10:00 A.M. – 11:00 A.M.	8
11:00 A.M. – 12:00 P.M.	22
12:00 P.M. – 1:00 P.M.	25
1:00 P.M. – 2:00 P.M.	18
2:00 P.M. – 3:00 P.M.	4
3:00 P.M. – 4:00 P.M.	6
4:00 P.M. – 5:00 P.M.	11
5:00 P.M. – 6:00 P.M.	18
6:00 P.M. – 7:00 P.M.	10
7:00 P.M. – 8:00 P.M.	3

3. Create a histogram to help the deli staff understand how many customers are in the deli at each time of day.



4. Determine if the data is skewed right, skewed left, or symmetric.

somewhat skewed right because more of the data falls on the left or lower end

5. Billy achieved scores of 87, 88, 93, 90, and 89 on his first five tests in math class. What would he need to score on the sixth test to get a grade of 90% so he can keep his A?

$$\frac{87+88+93+90+89+x}{6} = 90$$

$x = 93$

6. Multiple Choice: For which of the following sets of numbers is the mean greater that the median?

a) $\{4,5,6,7,8\}$ *b*) $\{3,5,6,7,8\}$ *c*) $\{4,6,6,6,8\}$

d) $\{2,6,6,6,6\}$ *e*) $\{4,5,6,7,9\}$

Expressions: Combining like terms

Simplify the following expressions

7. $5x-3x^2+15+11x^2-21x+8$

$8x^2-16x+23$

8. $\frac{2}{3}x+\frac{4}{5}y-\frac{1}{2}x+7y$

$\frac{1}{6}x+\frac{39}{5}y$

9. $2x(3w+4x)+5w(w-2x)+5w^2-2x^2$

$6xw+8x^2+5w^2-10xw+5w^2-2x^2$

$6x^2-4xw+10w^2$

10. $(3x-1)(4x+5)+(x+3)(x-2)$.

$12x^2+15x-4x-5-x^2+2x-3x+6$

$11x^2+10x+1$

Rearranging formulas:

11. The formula to find the rate of change or slope is $m=\frac{y_2-y_1}{x_2-x_1}$. Solve the equation for y_2.

$m(x_2-x_1)=y_2-y_1$

$+y_1 \quad +y_1$

$y_2=m(x_2-x_1)+y_1$

or $y_2=mx_2-mx_1+y_1$

12. The distance formula for finding the distance between two points is $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. Solve the equation for x_1.

$d^2=(x_2-x_1)^2+(y_2-y_1)^2$

$\sqrt{d^2-(y_2-y_1)^2}=x_2-x_1$

$-x_2 \quad -x_2$

$x_1=x_2-\sqrt{d^2-(y_2-y_1)^2}$

Creating analyzing and transforming functions:

13. Write the function that is described by each of the following tables.

a. exponential

x	$f(x)$	
1	6	
2	18	×3
3	54	×3
4	162	×3

$f(x) = 2(3)^x$

b. linear

x	$g(x)$	
-2	14	
0	11	+2, −3
2	8	+2, −3
4	5	+2, −3

$g(x) = -\frac{3}{2}x + 11$

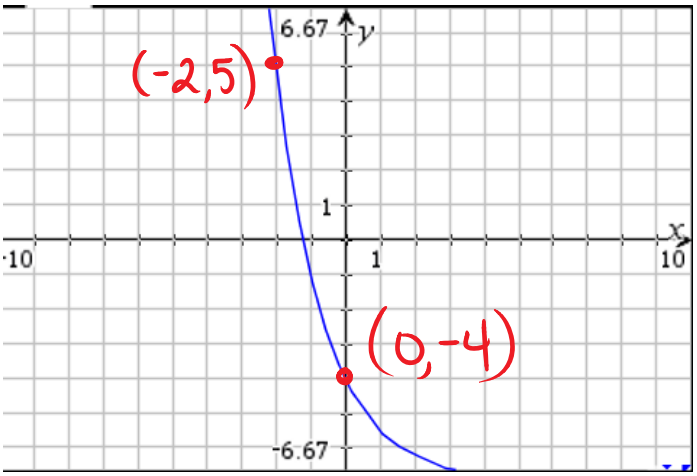
c. quadratic

x	$h(x)$
1	9
2	3
3 (vertex)	1
4	3

$h(x) = 2x^2 - 12x + 19$

or $h(x) = 2(x-3)^2 + 1$

14. Find the rate of change of the graph below over the interval $[-2, 0]$.



Rate of Change $= \frac{-4-5}{0+2} = \boxed{-\frac{9}{2}}$

15. Find the rate of change between the points $(-3, 8)$ and $(5, 4)$.

$\frac{4-8}{5+3} = \frac{-4}{8} = \boxed{-\frac{1}{2}}$

16. A basketball tournament starts with 64 teams. After each round half the teams are eliminated. Write a function to represent the amount of teams remaining after x rounds.

$f(x) = 64\left(\frac{1}{2}\right)^x$

$f(x) \rightarrow$ teams remaining

$x \rightarrow$ # of rounds

17. $f(x)=x-6$ and $g(x)=3x+2$. If $g(x)$ can be written as $a\cdot f(x)+k$, what are the values of a and k?

$a=3$

$3(x-6)+k$

$3x-18+k$

$-18+k=2$

$k=20$

V: (1, 4)

V: (-3, 1)

18. $f(x)=2(x-1)^2+4$ and $g(x)=2(x+3)^2+1$. Describe the transformation of $f(x)$ to $g(x)$?

left 4 and down 3

$(h-4, k-3)$

19. If $f(x)=3x^2-4x+1$ and $g(x)=-5x^2+2x-10$, find $f(2)-g(3)$

$f(2)=5$

$g(3)=-49$

$f(2)-g(3)=-49$

20. What is the domain and range of the function $f(x)=\dfrac{3}{x-5}$?

D: $(-\infty, 5)\cup(5, \infty)$

R: $(-\infty, 0)\cup(0, \infty)$

21. The function $g(x)=160+1.5x$ models the weight gain of a basketball player as he starts a workout program where g is the weight in pounds after x weeks.

a) Explain the meaning of 160 in the context of this problem.

b) Explain the meaning of 1.5 in the context of this problem.

c) Evaluate $g(6)$ and explain its meaning

a) His starting weight

B) pounds gained per week

c) $g(6)=169$

He weighs 169 pounds after 6 weeks

Systems:

22. Given the system of inequalities $\begin{cases} 3x-4y>8 \\ 2x^2+y>5 \\ x\le 7 \\ y\ge -1 \end{cases}$, determine which of the following points is a solution.

a. $(5,1)$ b. $(3,-2)$ c. $(4,1)$

23. Lucille has a total of 23 dimes and pennies. The value of her coins is $1.85.How many dimes does she have? How many pennies does she have?

24. Solve the following system of equations $\begin{matrix} -x-2y=-3 \\ 2x+4y=6 \end{matrix}$.

$2(-x-2y=-3)$

$-2x-4y=-6$

$2x+4y=6$

$0=0$

Infinite amount of solutions

25. Suppose you buy 3 shirts and 2 pairs of slacks on sale at a clothing store for $72. The next day, a friends buys 2 shirts and 4 pairs of slacks for $96. If the shirts you each bought were all the same price and the slacks were also all the same price, then what was the cost of each shirt and each pair of slacks? Set up and solve a system of equations.

x y

$3x+2y=72$

$2x+4y=96$

12 shirts

18 pairs of slacks

Solving exponential equations:

26. Solve $9^x = 27^{x-2}$ for *x*.

$3^{2x} = 3^{3(x-2)}$

$2x = 3x - 6$

$6 = x$

$x = 6$

Quadratics:

27. Explain how to find the zeros of the quadratic function $f(x) = 15x^2 + x - 28$.

Set the function = 0, factor, and solve

28. Factor the following expressions:

a) $25x^2 - 100$

$(5x-10)(5x+10)$

b) $2x^2 + 9x - 5$

$(2x-1)(x+5)$

c) $x^2 + 5x - 36$

$(x+9)(x-4)$

d) $6x^2 + 5x - 4$

$(2x-1)(3x+4)$

29. Use the discriminant to determine the number of solutions to quadratic equation $4x^2 - 5x - 7 = 0$. Then solve it.

$b^2 - 4ac$

$(-5)^2 - 4(4)(-7)$

$25 + 112 = 137$

2 real solutions

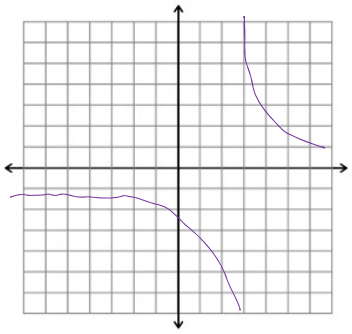
$x = \frac{5 \pm \sqrt{(-5)^2 - 4(4)(-7)}}{2(4)}$

$x = \frac{5 \pm \sqrt{137}}{8}$

30. On your calculator, graph the function $f(x) = -2x^2 + 3x - 6$. What is the highest point on the graph?

(.75, -4.875)

31. Graph the function $g(x) = \frac{7}{x-3}$. As the *x* value gets bigger and bigger, what does the *y* value get closer and closer to?



y gets closer to 0

32. During an experiment, a ball is dropped from a height of 205 feet. The formula $h = -16t^2 + h_0$ can be used to approximate the number of seconds *t* it takes for the ball to reach height *h* from an initial height of h_0 in feet. Find the time it takes the ball to reach the ground.

$h = -16t^2 + 205 = 0$

Graph, zero

T = .503 sec

33. The cheerleaders at Bulls games launch T-Shirts into the crowd every time the Bulls win. The height of the T-shirt can be modeled by the function $h(x) = -16x^2 + 48x + 6$, where $h(x)$ represents the height in feet of the T-shirt after *x* seconds.

a) At what height was the T-shirt launched?

6 feet

b) What is the maximum height of the T-shirt?

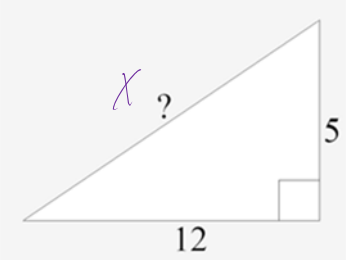
42 feet

c) When was the maximum height reaches?

at 1.5 sec

Geometry/Trig:

34. A right triangle has legs of length 5 and 12. What is the length of the hypotenuse?



$5^2+12^2=x^2$

$169=x^2$

$x=13$

35. The second angle in a triangle is 3° less than twice the first angle. The third angle measures 8° more than twice the first angle. Find each angle.

1st angle x

2nd angle $2x-3$

3rd angle $8+2x$

$x+2x-3+8+2x=180$

$5x+5=180$

$5x=175$

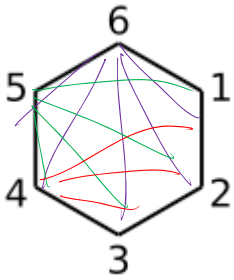
$x=35$

1st = 35°

2nd = 67°

3rd = 78°

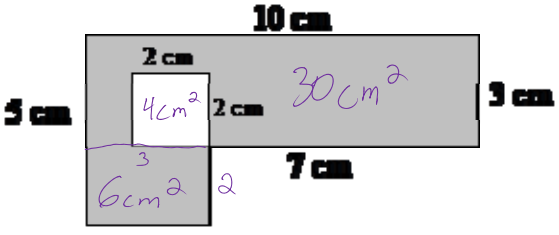
36. Jim cuts a board in the shape of a regular hexagon and pound a nail at each vertex. How many rubber bands will he need to stretch a rubber band across every possible pair of nails?



$5+4+3+2+1$

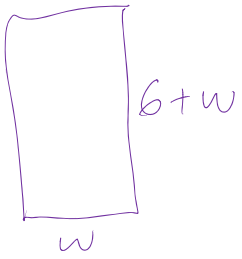
$=15$

37. Find the area of the shaded region:



$30+6-4=32cm^2$

38. Molly is designing a poster. The length of the poster is 6 inches longer than the width. If the poster requires 616 square inches of poster board, find the width *w* of the poster.



$w(6+w) = 616$

$6w + w^2 = 616$

$w^2 + 6w - 616 = 0$

$w = -28, 22$

$w = 22$ in

$L = 28$ in

39. The three angles of a triangle are $3x$, $x+10$, and $2x-40$. Find the measure of the smallest angle in the triangle.

$3x + x + 10 + 2x - 40 = 180$

$6x - 30 = 180$

$6x = 210$

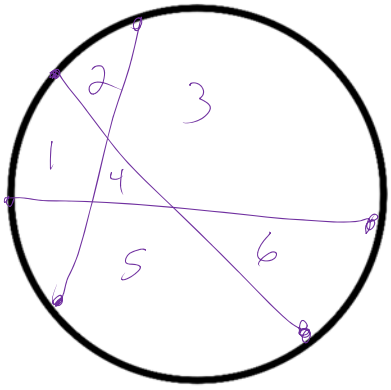
$x = 35$

1st $= 105$

2nd $= 45$

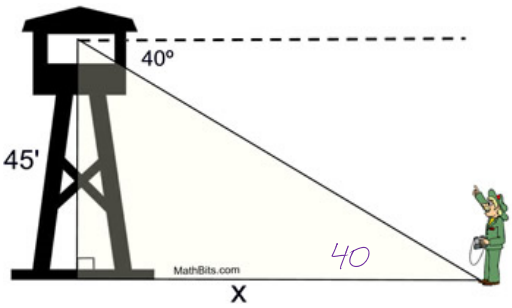
3rd $= 30$ smallest

40. What is the greatest number of regions that can be formed if three distinct lines intersect a circle?



6 regions

41. Roger is looking up a guard tower that is 45 feet tall in the forest. The angle of depression from the top of the guard tower is 40°. How far is Roger from the guard tower?



$\tan 40 = \frac{45}{x}$

$x \tan 40 = 45$

$x = \frac{45}{\tan 40}$

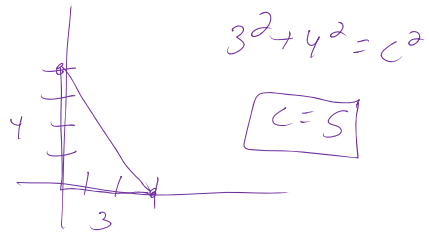
$x = 53.63$ ft

42. On a coordinate grid where each unit represents 1 mile, Isabel's house is located at (3, 0) and a mall is located at (0, 4). What is the distance between Isabel's house and the mall?

$Dist = \sqrt{(4-0)^2 + (0-3)^2}$

$= 5$ miles

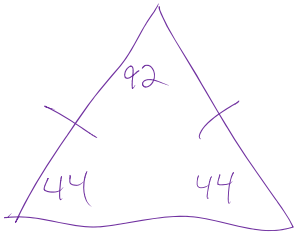
or



$3^2 + 4^2 = c^2$

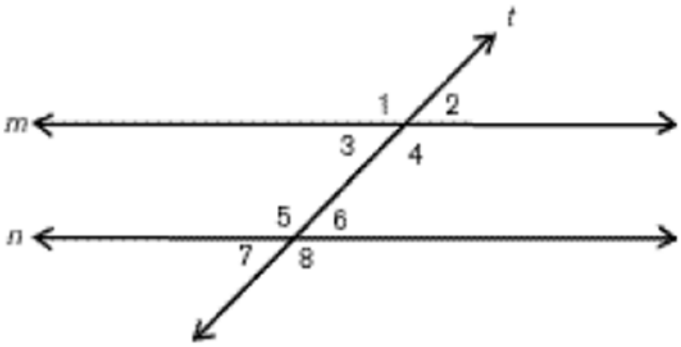
$c = 5$

43. Suppose one base angle of an isosceles triangle has a measurement of 44°. What is the measure of the vertex angle?



$180 - 44 - 44 = 92^\circ$

44. In the following diagram, line *m* and *n* are both parallel to line *t*.



a) If $m\angle 4 = 2x - 17$ and $m\angle 1 = 85$, find *x*.

$2x - 17 = 85$

$2x = 102$

$x = 51$

b) If $m\angle 1 = 4y + 30$ and $m\angle 7 = 7y + 6$, find *y*.

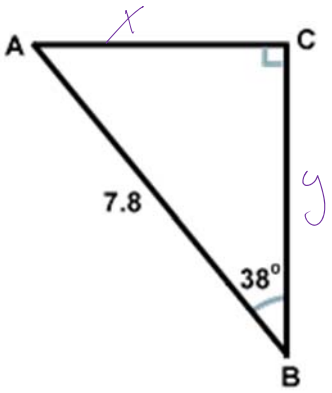
$4y + 30 + 7y + 6 = 180$

$11y + 36 = 180$

$11y = 144$

$y = 13.09$

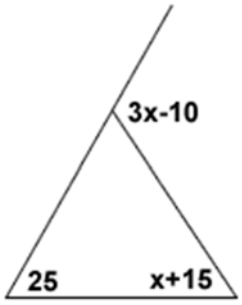
45. Find the length of both AC and BC.



$\sin 38 = \frac{x}{7.8}$ $\quad \cos 38 = \frac{y}{7.8}$

$x = 4.80$ $\quad y = 6.15$

46. Find the measure of all angles shown below.



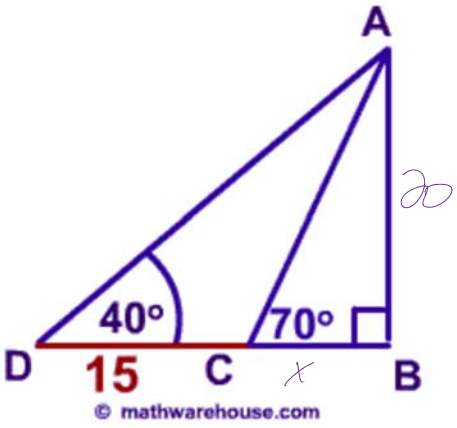
$25 + x + 15 + 3x - 10 = 180$

$4x + 30 = 180$

$4x = 150$

$x = 37.5$

47. Using the diagram below, find the distance between CB if AB is 20.



$\tan 40 = \frac{20}{15+x}$

$(15+x)\tan 40 = 20$

$15 + x = \frac{20}{\tan 40}$

$x = 8.84$

Mixed Review:

48. Explain how to find $\frac{5!}{2!(3!)}$. Check your answer in your calculator.

factorial

$\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)} = 10$

49. What is the degree of the polynomial $4x^3 - 5x^6 + 10x^9$?

Biggest exponent 9

50. What is the next term in the following sequence: 3, 7, 11, 15, 19 ?

+4

51. Divide: $\frac{x^2 + 12x + 36}{x + 6}$ $= \frac{(x+6)(x+6)}{x+6} = x+6$

52. What is the conjugate of $(4 - 3i)$?

$(4 + 3i)$

53. Write the equation for the line parallel to the line $4x-3y=9$ and through the point (3, -1). Write the answer in slope intercept form.

$4x-3y=9$

$-3y=-4x+9$

$y=\frac{4}{3}x-3$

$m=\frac{4}{3}$

$y-y_1=m(x-x_1)$

$y+1=\frac{4}{3}(x-3)$

$y=\frac{4}{3}x-4-1$

$y=\frac{4}{3}x-5$

54. Given a line through (–2, 4) and (8, –1), find the equation of the line perpendicular to the line through the midpoint of those points. Write the answer in slope intercept form.

midpoint $\left(\frac{-2+8}{2}, \frac{4-1}{2}\right)$

$(3, \frac{3}{2})$

$m=\frac{4+1}{-2-8}$

$m=\frac{5}{-10}=-\frac{1}{2}$

$y-\frac{3}{2}=2(x-3)$

$y-\frac{3}{2}=2x-6$

$y=2x-4.5$

For 55 - 56, remember that $x^a \cdot x^b = x^{a+b}$, $(x^a)^b = x^{ab}$, $\frac{x^a}{x^b} = x^{a-b}$, $x^{-1} = \frac{1}{x}$.

55. Simplify: $\frac{-22x^5y^6}{14x^{13}y^{-3}}$

$\frac{-11y^9}{7x^8}$

56. Simplify: $(-a^3b^2)^2(-b^2c^2)^3(-a^3c^4)^4$

$(a^6b^4)(-b^6c^6)(a^{12}c^{16})$

$-a^{18}b^{10}c^{22}$

57. Simplify $3\sqrt{75}+4\sqrt{125}-5\sqrt{50}$.

$3\sqrt{25\cdot3}+4\sqrt{25\cdot5}-5\sqrt{25\cdot2}$

$15\sqrt{3}+20\sqrt{5}-25\sqrt{2}$

58. Find the inverse of the function $f(x)=5x+9$.

$y=5x+9$

$x=5y+9$

$x-9=5y$

$y=\frac{x-9}{5}$

59. Joe can drive 4 miles in m minutes. At this rate, how many minutes will it take him to drive 19 miles? Leave your answer in terms of m.

$D = R \cdot T$

$4 = R \cdot m$

$R = \frac{4}{m}$

$D = R \cdot T$

$19 = \frac{4}{m} \cdot T$

$\frac{19}{\frac{4}{m}} = 1$

$\frac{19m}{4} = T$

60. Multiple choice: Which of the following could NOT be a solution to $5 - 3x < -3$?

a) 2.5 *b*) 3.5

c) 3 *d*) 4

61. Solve $\sqrt{k+1} = k - 1$ for k.

$k + 1 = (k-1)(k-1)$

$k + 1 = k^2 - 2k + 1$

$0 = k^2 - 3k$

$0 = k(k-3)$

$k = 0, 3$

$k = 3$

62. Multiply and simplify: $(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})$

$\sqrt{9} + 2\sqrt{6} - \sqrt{6} - 2\sqrt{4}$

$3 + \sqrt{6} - 4$

$-1 + \sqrt{6}$

63. What is the slope of the line that is perpendicular to the line $4x + 5y = 20$?

$y = -\frac{4}{5}x + 4$

$\perp$ is $\frac{5}{4}$

64. Use the long division process to find the remainder of $\left(4x^2-7x+1\right)\div\left(x-3\right)$.

$4x+5+\frac{16}{x-3}$

65. Simplify completely with no radicals in the denominator of the final answer:

a. $\frac{3+i\sqrt{2}}{1-i\sqrt{2}}$ $\frac{1+i\sqrt{2}}{1+i\sqrt{2}}$ $\frac{3+3i\sqrt{2}+i\sqrt{2}+2i^2}{1+2}=\frac{1+4i\sqrt{2}}{3}$ $\boxed{\frac{1+4i\sqrt{2}}{3}}$

b. $\frac{\sqrt[4]{8}}{\sqrt[4]{6}}=\frac{\sqrt[4]{4}}{\sqrt[4]{3}}=\frac{\sqrt[4]{2^2}}{\sqrt[4]{3}}\cdot\frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}}=\boxed{\frac{\sqrt[4]{108}}{3}}$

c. $\left(5+3\sqrt{2}\right)\left(6-\sqrt{3}\right)$

$30-5\sqrt{3}+18\sqrt{2}-3\sqrt{6}$

d. $\left(\sqrt[3]{4}\right)\left(\sqrt[5]{4}\right)$

$4^{\frac{1}{3}}\cdot4^{\frac{1}{5}}=4^{\frac{5}{15}}\cdot4^{\frac{3}{15}}=4^{\frac{8}{15}}=\boxed{\sqrt[15]{4^8}}$

66. Simplify completely: $\left(5x\sqrt{16x^3}\right)+\left(\sqrt{12x^6}\right)$

$20x^2\sqrt{x}+2x^3\sqrt{3}$

67. Given $f\left(x\right)=x^2+3x-1$ and $g\left(x\right)=4x-5$, then what is the expression to represent $f\left(g\left(x\right)\right)$?

$f(4x-5)=(4x-5)^2+3(4x-5)-1$

$=16x^2-40x+25+12x-15-1$

$=\boxed{16x^2-28x+9}$

68. Given $h\left(x\right)=6x-2$, find the inverse and then use composition of functions to prove they are inverses.

$y=6x-2$

$x=6y-2$

$x+2=6y$

$\frac{x+2}{6}=y$

$h^{-1}(x)=\frac{1}{6}x+\frac{1}{3}$

$h(h^{-1}(x))=6\left(\frac{x+2}{6}\right)-2=x+2-2=x$

$h^{-1}(h(x))=\frac{6x-2+2}{6}=\frac{6x}{6}=x$

69. Factor.

a. over the rationals: $9x^2-25$ $(3x+5)(3x-5)$

b. over the reals: x^2-8 $(x+2\sqrt{2})(x-2\sqrt{2})$

c. over the complex: x^2+25 $(x+5i)(x-5i)$

70. Factor into two binomials $(x-3)^2-6(x-3)-16$

option 1
$a=(x-3)$
$a^2-6a-16$
$(a-8)(a+2)$
$(x-3-8)(x-3+2)$
$(x-11)(x-1)$

option 2 $x^2-6x+9-6x+18-16$
$x^2-12x+11$
$(x-11)(x-11)$

71. Create a binomial that can be factored over the reals but not over the rationals.

many options

example: x^2-10
$(x+\sqrt{10})(x-\sqrt{10})$

72. Rewrite the rational function of $g(x)=\dfrac{x^2-2x+2}{x-3}$ to find any asymptotes. Identify the parts of the function of $g(x)=\dfrac{A}{B}+C$ and what they represent of the asymptotes.

$$\begin{array}{r|l} x-3 & x^2-2x+2 \\ & -x^2+3x \\ \hline & x+2 \\ & -x+3 \\ \hline & 5 \end{array} \quad \text{so } x+1$$

$g(x)=\dfrac{5}{x-3}+x+1$ $y=x+1$ is the oblique asymptote

$x=3$ is vertical asymptote

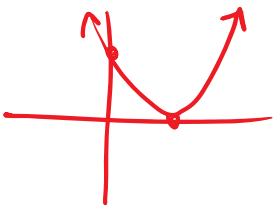
73. Find the quadratic function that goes through the points of $(2,5)$, $(4,15)$ and $(7,-1)$.

$5=4a+2b+c$
$15=16a+4b+c$
$-1=49a+7b+c$
system solver

$y=-2.06x^2+17.4x-21.53$

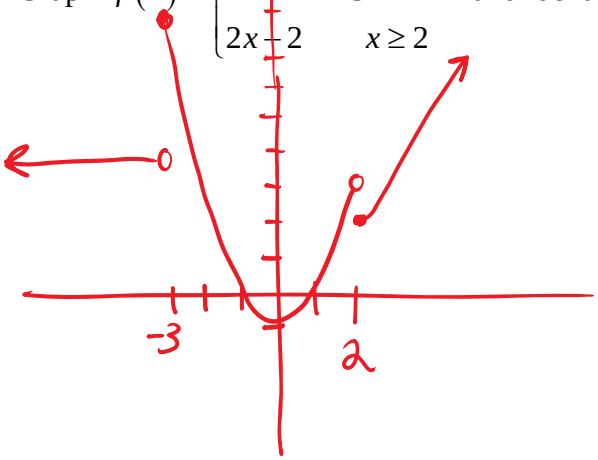
$y=ax^2+bx+c$

74. If a given function has a positive y-intercept and only one x-intercept, what can be said about the values of the general function $f(x)=ax^2+bx+c$.



$a>0$
$c>0$
$b^2-4ac=0$ (Discriminant)

75. Graph $f(x)=\begin{cases} 4 & x<-3 \\ x^2+1 & -3\le x<2 \\ 2x-2 & x\ge 2 \end{cases}$ and identify the domain and range of the function.

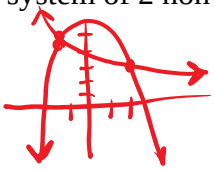


D: all real numbers

R: all real numbers

76. Create a system of 2 non-linear inequalities where both $(2,3)$ and $(-1,5)$ are in the solution set.

example



77. Twelve charms were on a bracelet with a clasp to put the bracelet on and off. The chain broke and when it was replace, the charms were put on a stretchy material without a clasp. Explain the impact of this change in terms of the number of ways the charms can be arranged on the bracelet.

original $12!$ new $\frac{12!}{12}$ ↑ due to no beginning

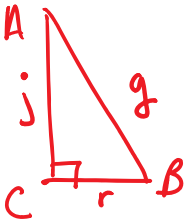
Number of arrangements of new is $\frac{1}{12}$ of old.

78. You are standing a distance away from the large statue of Superman. When looking at the top of the statue your angle of elevation from your eyes is approximately 65°. Because that bothered your neck you stepped back approximately 25 feet and the angle of elevation was only $50^\circ 30'$. How tall is the Superman statue?

$\tan 65 = \frac{y}{x}$ $\tan 50.5 = \frac{y}{x+25}$ $(x,y) = (32.56, 69.8268)$

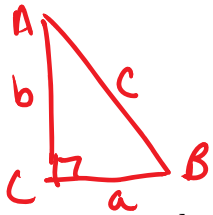
$x \tan 65 = y$ $(x+25)\tan 50.5 = y$ Statue ≈ 70 ft.

79. Given a right triangle with $\measuredangle C = 90^\circ$, when given the $\sin A = \frac{r}{g}$ and $\cos A = \frac{j}{g}$, then find the value of $\cot A$.



$\cot A = \frac{adj}{opp} = \frac{j}{r}$

80. Given a right triangle with $\measuredangle C = 90^\circ$, if the $\sin A > \cos A$, then what is true about $\measuredangle A$?



$\frac{a}{c} > \frac{b}{c}$ $\measuredangle A > \measuredangle B$

$a > b$ $\angle A$ is larger than 45°

81. A frustrum is created with a larger base with a diameter of 16 inches and the smaller base has a diameter of 10 inches. If the slant height of the frustrum is 5 inches, find the volume.

$V = \frac{1}{3}\pi(8^2 + 8\cdot 5 + 5^2)$

$= \boxed{43\pi \text{ in}^3}$

82. Compare the area of the smaller circle with the shaded region that is not part of the smaller circle. Suggestion would be to make the sides of the triangle a number (like 2 cm) to determine the answer. equal