



## **Lemont High School**

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For Integrated Math III Students

Solutions for Summer Math Packet

### Math 3 Summer Review Packet

#### One Variable Statistics:

Mr. Day asks his students in his two history classes how old they were when they rode on a plane for the first time. The responses are in the tables below. Use the tables below to solve problems 1-2.

Class 1	
Student	Age in years
1	10
2	9
3	2
4	11
5	3
6	10
7	7
8	13
9	8
10	8
11	15
12	12
13	2
14	5
15	9
16	7
17	10
18	13
19	11

Class 2	
Student	Age in years
1	3
2	6
3	11
4	10
5	5
6	7
7	1
8	2
9	4
10	3
11	1
12	5
13	3
14	10
15	6
16	2
17	4
18	7
19	1

1. Find the measures of center for each data set. Measure of centers are the mean and median.

class 1  
mean: 8.68

median: 9

class 2  
mean: 4.79

median: 4

2. Find the standard deviation of both data sets and determine which data has a larger spread. **Standard deviation ( $\sigma_x$ ) can be found in your calculator by putting your data into a List and Spreadsheet, hit menu, 4. Statistics, 1. Stat Calculations, 1. One-Variable Statistics**

class 1  
3.60

class 2  
3.04

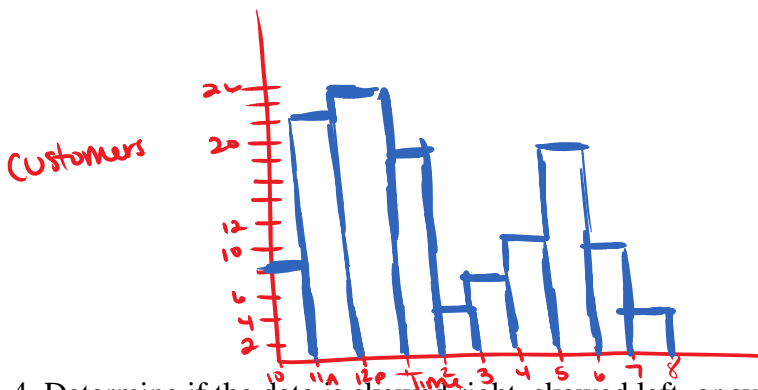
Class 1 has a slightly larger spread than class 2.

Use the following information for problems 3-4

A deli records the number of customers each hour that they are open to determine how many staff members they need working at each time of day. The number of customers is in the table below.

Time Frame	Number of Customers
10:00 A.M. – 11:00 A.M.	8
11:00 A.M. – 12:00 P.M.	22
12:00 P.M. – 1:00 P.M.	25
1:00 P.M. – 2:00 P.M.	18
2:00 P.M. – 3:00 P.M.	4
3:00 P.M. – 4:00 P.M.	6
4:00 P.M. – 5:00 P.M.	11
5:00 P.M. – 6:00 P.M.	18
6:00 P.M. – 7:00 P.M.	10
7:00 P.M. – 8:00 P.M.	3

3. Create a histogram to help the deli staff understand how many customers are in the deli at each time of day.



4. Determine if the data is skewed right, skewed left, or symmetric.

some what skewed right because more of the data falls on the left or lower end

5. Billy achieved scores of 87, 88, 93, 90, and 89 on his first five tests in math class. What would he need to score on the sixth test to get a grade of 90% so he can keep his A?



$$\frac{87 + 88 + 93 + 90 + 89 + x}{6} = 90$$

$$x = 93$$

6. Multiple Choice: For which of the following sets of numbers is the mean greater than the median?

a) {4, 5, 6, 7, 8}

b) {3, 5, 6, 7, 8}

c) {4, 6, 6, 6, 8}

d) {2, 6, 6, 6, 6}

e) {4, 5, 6, 7, 9}

### Expressions: Combining like terms

Simplify the following expressions

7.  $5x - 3x^2 + 15 + 11x^2 - 21x + 8$

$$8x^2 - 16x + 23$$

8.  $\frac{2}{3}x + \frac{4}{5}y - \frac{1}{2}x + 7y$

$$\frac{1}{6}x + \frac{39}{5}y$$

9.  $2x(3w + 4x) + 5w(w - 2x) + 5w^2 - 2x^2$

$$6xw + 8x^2 + 5w^2 - 10xw + 5w^2 - 2x^2$$

$$6x^2 - 4xw + 10w^2$$

10.  $(3x - 1)(4x + 5) + (x + 3)(x - 2)$

$$12x^2 + 15x - 4x - 5 - x^2 + 2x - 3x + 6$$

$$11x^2 + 10x + 1$$

### Rearranging formulas:

11. The formula to find the rate of change or slope is  $m = \frac{y_2 - y_1}{x_2 - x_1}$ . Solve the equation for  $y_2$ .

$$m(x_2 - x_1) = y_2 - y_1$$

$$y_2 = m(x_2 - x_1) + y_1$$

$$y_2 = mx_2 - mx_1 + y_1$$

12. The distance formula for finding the distance between two points is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ . Solve the equation for  $x_1$ .

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\sqrt{d^2 - (y_2 - y_1)^2} = x_2 - x_1$$

$$x_1 = x_2 - \sqrt{d^2 - (y_2 - y_1)^2}$$

### Creating analyzing and transforming functions:

13. Write the function that is described by each of the following tables.

a. *exponential*

$x$	$f(x)$
1	6
2	18
3	54
4	162

$$f(x) = 2(3)^x$$

b. *linear*

$x$	$g(x)$
-2	14
0	11
2	8
4	5

$$g(x) = -\frac{3}{2}x + 11$$

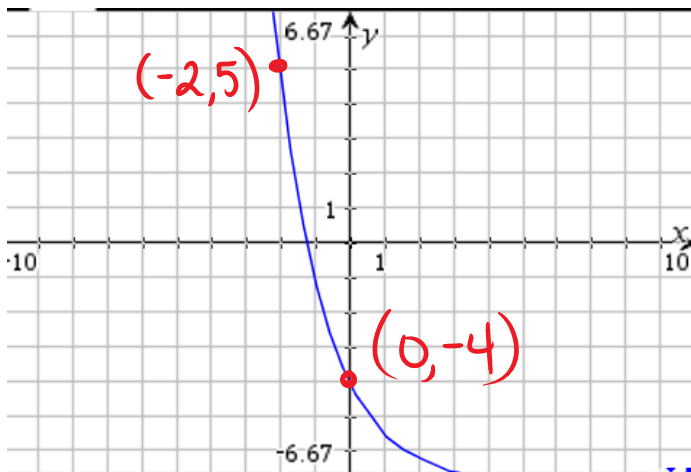
c. *quadratic*

$x$	$h(x)$
1	9
2	3
3	1
4	3

$$h(x) = 2x^2 - 12x + 19$$

or  $h(x) = 2(x-3)^2 + 1$

14. Find the rate of change of the graph below over the interval  $[-2, 0]$ .



$$\text{Rate of Change} = \frac{-4-5}{0-(-2)} = \boxed{-\frac{9}{2}}$$

15. Find the rate of change between the points  $(-3, 8)$  and  $(5, 4)$ .

$$\frac{4-8}{5-(-3)} = \frac{-4}{8} = \boxed{-\frac{1}{2}}$$

16. A basketball tournament starts with 64 teams. After each round half the teams are eliminated. Write a function to represent the amount of teams remaining after  $x$  rounds.

$$f(x) = 64\left(\frac{1}{2}\right)^x$$

$f(x) \rightarrow$  teams remaining  
 $x \rightarrow$  # of rounds

17.  $f(x) = x - 6$  and  $g(x) = 3x + 2$ . If  $g(x)$  can be written as  $a \cdot f(x) + k$ , what are the values of  $a$  and  $k$ ?

$$a = 3$$

$$3(x - 6) + k$$

$$3x - 18 + k$$

$$-18 + k = 2$$

$$k = 20$$

18.  $f(x) = 2(x - 1)^2 + 4$  and  $g(x) = 2(x + 3)^2 + 1$ . Describe the transformation of  $f(x)$  to  $g(x)$ ?

$$v: (1, 4)$$

$$v: (-3, 1)$$

left 4 and down 3

$$(h - 4, k - 3)$$

19. If  $f(x) = 3x^2 - 4x + 1$  and  $g(x) = -5x^2 + 2x - 10$ , find  $f(2) - g(3)$

$$f(2) = 5$$

$$g(3) = -49$$

$$f(2) - g(3) = -44$$

20. What is the domain and range of the function  $f(x) = \frac{3}{x - 5}$ ?

$$D: (-\infty, 5) \cup (5, \infty)$$

$$R: (-\infty, 0) \cup (0, \infty)$$

21. The function  $g(x) = 160 + 1.5x$  models the weight gain of a basketball player as he starts a workout program where  $g$  is the weight in pounds after  $x$  weeks.

a) Explain the meaning of 160 in the context of this problem.

b) Explain the meaning of 1.5 in the context of this problem.

c) Evaluate  $g(6)$  and explain its meaning



a) His starting weight

b) pounds gained per week

$$c) g(6) = 169$$

He weighs 169 pounds after 6 weeks

## Systems:

22. Given the system of inequalities  $\begin{cases} 3x - 4y > 8 \\ 2x^2 + y > 5 \\ x \leq 7 \\ y \geq -1 \end{cases}$ , determine which of the following points is a solution.

a. (5,1)

b. (3,-2)

c. (4,1)

23. Lucille has a total of 23 dimes and pennies. The value of her coins is \$1.85. How many dimes does she have? How many pennies does she have?

24. Solve the following system of equations  $\begin{cases} -x - 2y = -3 \\ 2x + 4y = 6 \end{cases}$ .

$$\begin{array}{r} -2x - 4y = -6 \\ 2x + 4y = 6 \\ \hline 0 = 0 \end{array}$$

Infinite amount of solutions

25. Suppose you buy 3 shirts and 2 pairs of slacks on sale at a clothing store for \$72. The next day, a friend buys 2 shirts and 4 pairs of slacks for \$96. If the shirts you each bought were all the same price and the slacks were also all the same price, then what was the cost of each shirt and each pair of slacks? Set up and solve a system of equations.

$$3x + 2y = 72$$

$$2x + 4y = 96$$

12 shirts

18 pairs of slacks



### Solving exponential equations:

26. Solve  $9^x = 27^{x-2}$  for  $x$ .

$$3^{2x} = 3^{3(x-2)}$$

$$2x = 3x - 6$$

$$6 = x$$

**Quadratics:**

$$x = 6$$

27. Explain how to find the zeros of the quadratic function  $f(x) = 15x^2 + x - 28$ .

Set the function = 0, factor, and solve

28. Factor the following expressions:

a)  $25x^2 - 100$

$$(5x - 10)(5x + 10)$$

b)  $2x^2 + 9x - 5$

$$(2x - 1)(x + 5)$$

c)  $x^2 + 5x - 36$

$$(x + 9)(x - 4)$$

d)  $6x^2 + 5x - 4$

$$(2x - 1)(3x + 4)$$

29. Use the discriminant to determine the number of solutions to quadratic equation  $4x^2 - 5x - 7 = 0$ . Then solve it.

$$b^2 - 4ac$$

$$(-5)^2 - 4(4)(-7)$$

$$25 + 112 = 137$$

2 real solutions

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(4)(-7)}}{2(4)}$$

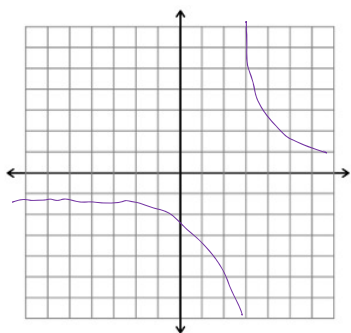
$$x = \frac{5 \pm \sqrt{137}}{8}$$



30. On your calculator, graph the function  $f(x) = -2x^2 + 3x - 6$ . What is the highest point on the graph?

$$(.75, -4.875)$$

31. Graph the function  $g(x) = \frac{7}{x-3}$ . As the  $x$  value gets bigger and bigger, what does the  $y$  value get closer and closer to?



$y$  gets closer to 0

32. During an experiment, a ball is dropped from a height of 205 feet. The formula  $h = -16t^2 + h_0$  can be used to approximate the number of seconds  $t$  it takes for the ball to reach height  $h$  from an initial height of  $h_0$  in feet. Find the time it takes the ball to reach the ground.

$$h = -16t^2 + 205 = 0$$

Graph, zero

$$T = .503 \text{ sec}$$

33. The cheerleaders at Bulls games launch T-Shirts into the crowd every time the Bulls win. The height of the T-shirt can be modeled by the function  $h(x) = -16x^2 + 48x + 6$ , where  $h(x)$  represents the height in feet of the T-shirt after  $x$  seconds.

a) At what height was the T-shirt launched?

6 feet

b) What is the maximum height of the T-shirt?

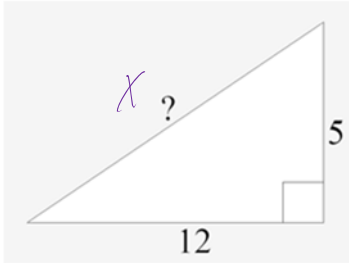
42 feet

c) When was the maximum height reaches?

at 1.5 sec

## Geometry/Trig:

34. A right triangle has legs of length 5 and 12. What is the length of the hypotenuse?



$$5^2 + 12^2 = X^2$$

$$169 = X^2$$

$$X = 13$$

35. The second angle in a triangle is  $3^\circ$  less than twice the first angle. The third angle measures  $8^\circ$  more than twice the first angle. Find each angle.

1<sup>st</sup> angle  $x$

2<sup>nd</sup> angle  $2x - 3$

3<sup>rd</sup> angle  $8 + 2x$

$$x + 2x - 3 + 8 + 2x = 180$$

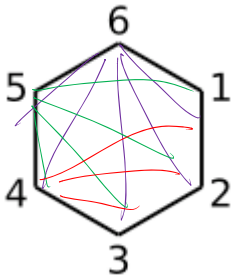
$$5x + 5 = 180$$

$$5x = 175$$

$$x = 35$$

$$\begin{aligned} 1^{st} &= 35^\circ \\ 2^{nd} &= 67^\circ \\ 3^{rd} &= 78^\circ \end{aligned}$$

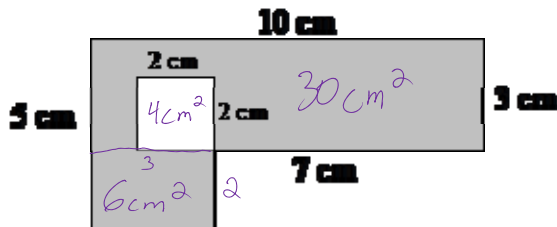
36. Jim cuts a board in the shape of a regular hexagon and pound a nail at each vertex. How many rubber bands will he need to stretch a rubber band across every possible pair of nails?



$$5 + 4 + 3 + 2 + 1$$

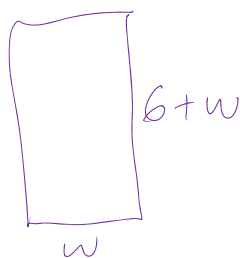
$$= 15$$

37. Find the area of the shaded region:



$$30 + 6 - 4 = 32 \text{ cm}^2$$

38. Molly is designing a poster. The length of the poster is 6 inches longer than the width. If the poster requires 616 square inches of poster board, find the width  $w$  of the poster.



$$\begin{aligned} w(6+w) &= 616 \\ 6w + w^2 &= 616 \\ w^2 + 6w - 616 &= 0 \\ w &= -28, 22 \end{aligned}$$

$$\begin{aligned} w &= 22 \text{ in} \\ L &= 28 \text{ in} \end{aligned}$$

39. The three angles of a triangle are  $3x$ ,  $x+10$ , and  $2x-40$ . Find the measure of the smallest angle in the triangle.

$$3x + x + 10 + 2x - 40 = 180$$

$$6x - 30 = 180$$

$$6x = 210$$

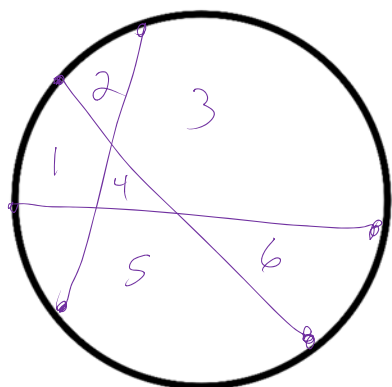
$$x = 35$$

$$1^{st} = 105$$

$$2^{nd} = 45$$

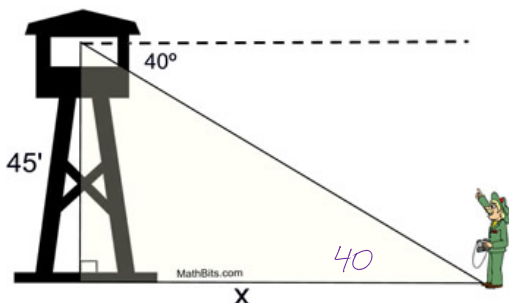
$$3^{rd} = 30 \text{ smallest}$$

40. What is the greatest number of regions that can be formed if three distinct lines intersect a circle?



6 regions

41. Roger is looking up a guard tower that is 45 feet tall in the forest. The angle of depression from the top of the guard tower is  $40^\circ$ . How far is Roger from the guard tower?



$$\tan 40 = \frac{45}{x}$$

$$x \tan 40 = 45$$

$$x = \frac{45}{\tan 40}$$

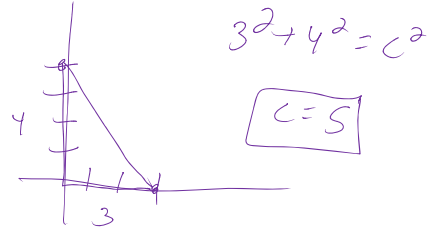
$$x = 53.63 \text{ ft}$$

42. On a coordinate grid where each unit represents 1 mile, Isabel's house is located at (3, 0) and a mall is located at (0, 4). What is the distance between Isabel's house and the mall?

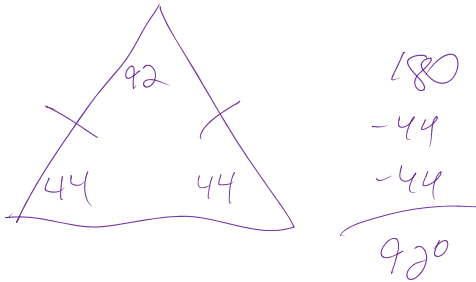
$$\text{Dist} = \sqrt{(4-0)^2 + (0-3)^2}$$

$$= \boxed{5 \text{ miles}}$$

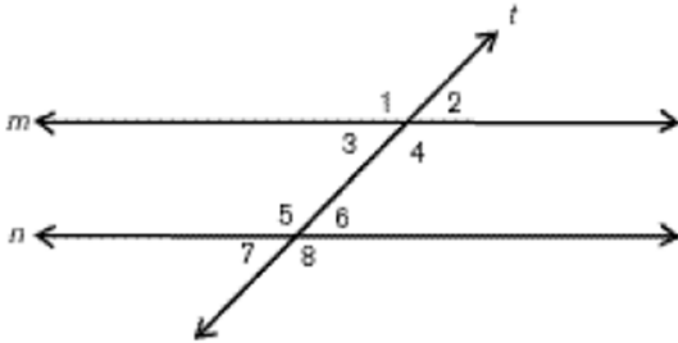
or



43. Suppose one base angle of an isosceles triangle has a measurement of  $44^\circ$ . What is the measure of the vertex angle?



44. In the following diagram, line  $m$  and  $n$  are both parallel to line  $t$ .



a) If  $m\angle 4 = 2x - 17$  and  $m\angle 1 = 85$ , find  $x$ .

$$2x - 17 = 85$$

$$2x = 102$$

$$\boxed{x = 51}$$

b) If  $m\angle 1 = 4y + 30$  and  $m\angle 7 = 7y + 6$ , find  $y$ .

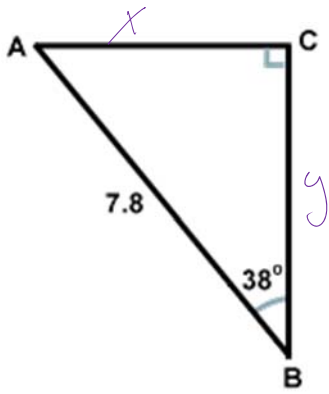
$$4y + 30 + 7y + 6 = 180$$

$$11y + 36 = 180$$

$$11y = 144$$

$$\boxed{y = 13.09}$$

45. Find the length of both AC and BC.



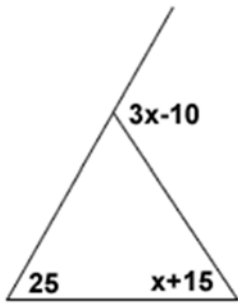
$$\sin 38 = \frac{x}{7.8}$$

$$\cos 38 = \frac{y}{7.8}$$

$$x = 4.80$$

$$y = 6.15$$

46. Find the measure of all angles shown below.



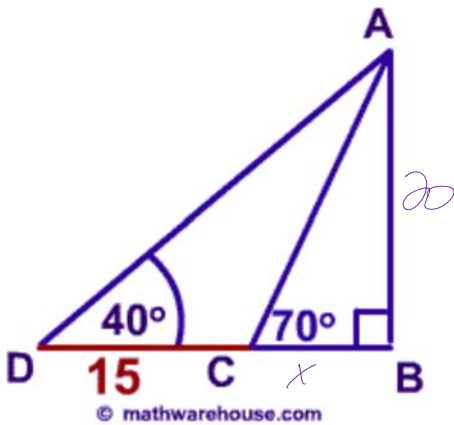
$$25 + x + 15 + 3x - 10 = 180$$

$$4x + 30 = 180$$

$$4x = 150$$

$$x = 37.5$$

47. Using the diagram below, find the distance between CB if AB is 20.



$$\tan 40 = \frac{20}{15+x}$$

$$(15+x) \tan 40 = 20$$

$$15+x = \frac{20}{\tan 40}$$

$$x = 8.84$$

**Mixed Review:**

48. Explain how to find  $\frac{5!}{2!(3!)}$ . Check your answer in your calculator.



**factorial**

$$\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)} = \boxed{10}$$

49. What is the degree of the polynomial  $4x^3 - 5x^6 + 10x^9$ ?

Biggest exponent  $\boxed{9}$

50. What is the next term in the following sequence: 3, 7, 11, 15, 19?

+4

51. Divide:  $\frac{x^2 + 12x + 36}{x + 6} = \frac{(x+6)(x+6)}{x+6} = \boxed{x+6}$

52. What is the conjugate of  $(4 - 3i)$ ?

$$\boxed{(4 + 3i)}$$

53. Write the equation for the line parallel to the line  $4x - 3y = 9$  and through the point  $(3, -1)$ . Write the answer in slope intercept form.

$$4x - 3y = 9$$

$$-3y = -4x + 9$$

$$y = \frac{4}{3}x - 3$$

$$m = \frac{4}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{4}{3}(x - 3)$$

$$y = \frac{4}{3}x - 4 - 1$$

$$y = \frac{4}{3}x - 5$$

54. Given a line through  $(-2, 4)$  and  $(8, -1)$ , find the equation of the line perpendicular to the line through the midpoint of those points. Write the answer in slope intercept form.

$$\text{midpoint } \left( \frac{-2+8}{2}, \frac{4-1}{2} \right)$$

$$(3, \frac{3}{2})$$

$$m = \frac{4-1}{-2-8}$$

$$m = \frac{3}{-10} = -\frac{1}{2}$$

$$y - \frac{3}{2} = 2(x - 3)$$

$$y - \frac{3}{2} = 2x - 6$$

$$y = 2x - 4.5$$

For 55 - 56, remember that  $x^a \cdot x^b = x^{a+b}$ ,  $(x^a)^b = x^{ab}$ ,  $\frac{x^a}{x^b} = x^{a-b}$ ,  $x^{-1} = \frac{1}{x}$ .

55. Simplify:  $\frac{-22x^5y^6}{14x^{13}y^{-3}}$

$$\frac{-11y^9}{7x^8}$$

56. Simplify:  $(-a^3b^2)^2(-b^2c^2)^3(-a^3c^4)^4$

$$(a^6b^4)(-b^6c^6)(a^{12}c^{16})$$

$$-a^{18}b^{10}c^{22}$$

57. Simplify  $3\sqrt{75} + 4\sqrt{125} - 5\sqrt{50}$ .

$$3\sqrt{25 \cdot 3} + 4\sqrt{25 \cdot 5} - 5\sqrt{25 \cdot 2}$$

$$15\sqrt{3} + 20\sqrt{5} - 25\sqrt{2}$$

58. Find the inverse of the function  $f(x) = 5x + 9$ .

$$y = 5x + 9$$

$$x = 5y + 9$$

$$x - 9 = 5y$$

$$y = \frac{x-9}{5}$$

59. Joe can drive 4 miles in  $m$  minutes. At this rate, how many minutes will it take him to drive 19 miles? Leave your answer in terms of  $m$ .



$$D = R \cdot T$$

$$D = R \cdot T$$

$$4 = R \cdot m$$

$$19 = \frac{4}{m} \cdot T$$

$$R = \frac{4}{m}$$

$$\frac{19}{\frac{4}{m}} = 1$$

$$\boxed{\frac{19m}{4} = T}$$

60. Multiple choice: Which of the following could NOT be a solution to  $5 - 3x < -3$ ?

a) 2.5  
c) 3

b) 3.5  
d) 4

61. Solve  $\sqrt{k+1} = (k-1)^2$  for  $k$ .

$$k+1 = (k-1)(k-1)$$

$$k+1 = k^2 - 2k + 1$$

$$0 = k^2 - 3k$$

$$0 = k(k-3)$$

$$k = 0, 3$$

$$\boxed{k=3}$$

62. Multiply and simplify:  $(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})$

$$\sqrt{9} + 2\sqrt{6} - \sqrt{6} - 2\sqrt{4}$$

$$3 + \sqrt{6} - 4$$

$$\boxed{-1 + \sqrt{6}}$$

63. What is the slope of the line that is perpendicular to the line  $4x + 5y = 20$ ?

$$5y = -4x + 20$$

$$y = -\frac{4}{5}x + 4$$

$$m = -\frac{4}{5}$$

$$\boxed{m_{\perp} = \frac{5}{4}}$$