SUMMER WORK FOR STUDENTS ENTERING CALCULUS

Dear Calculus students--your summer work assignment is here!

To make your transition into Calculus as smooth as possible, you will need to master these preliminary topics covered in the following handout and practice sheets.

All of the material reviewed here should be familiar to you. However, if some of it seems new, or at best a distant memory, use the links at the bottom as a resource.

You will be tested on mastery in September. The goal is not to "Just get it Right," it is to master the processes. Pay particular attention to orders of operation(PEMDAS), and all operations when using rational expressions. Differentiate between expressions, equations, functions. The package will be collected and assessed your first week back to school You are not expected to spend more than about 6 hours spread out over the summer to complete this. Do take the time to annotate your work and write down questions when you have them. There will be time to go over the packet and strengthen any weak areas.

We hope you have a wonderful summer! We will look forward to seeing you back to Pingree in late August.

Here are a few additional resources that might be helpful for you.

Pingree Quant Center: www.qc.pingree.org
Khan Academy: www.khanacademy.org
Purplemath: www.puplemath.com
Math is fun: www.mathsisfun.com

Summer Work for Students
Entering Calculus

For the problems sets which start on **page 6** write out all solutions clearly on a separate sheet of paper. Show all steps and circle your answer.

The following are examples of different types of problems. Use these for reference where appropriate when working on the problem sets.

1) Lines

slope-intercept form: $y = mx + b$

general form: $ax + by = c$

$\text{point} - \text{slope form}: y - y_1 = m(x - x_1)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

2) Exponents

- $x^m \cdot x^n = x^{m+n}$
- $x^{\frac{1}{n}} = \sqrt[n]{x}$
- $\dfrac{x^m}{x^n} = x^{m-n}$
- $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$
- $(x^m)^n = x^{mn}$
- $x^{-m} = \dfrac{1}{x^m}$

3) Factoring

- Always factor out a common factor first
- Look for a difference of two squares : $a^2 - b^2 = (a-b)(a+b)$
- Look for a sum or difference of two cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

- Trinomials
- Grouping

4) Rational Expressions and Equations

- Reducing: Factor numerator and denominator
  Cancel when possible

$$\text{Example}: \frac{x^3 - 4x}{x^2 + x - 2} = \frac{x(x^2 - 4)}{(x+2)(x-1)} = \frac{x(x-2)}{x-1} \quad , x \neq -2, x \neq 1$$

(be sure to include what values of x are not in the expression's domain!)

Summer Work for Students
Entering Calculus

Rational Expressions and Equations (con't)

- Multiplying: Factor numerator and denominator of each fraction
  Cancel where possible
  Multipy
- Dividing: Keep first fraction
  Change sign to multiplication and multiply as above

$$\text{Example}: \frac{2x^2+x-6}{x^2+4x-5}\cdot\frac{x^3-3x^2+2x}{4x^2-6x}=\frac{(2x-3)(x+2)}{(x+5)(x-1)}\cdot\frac{x(x-2)(x-1)}{2x(2x-3)}=\frac{(x+2)(x-2)}{2(x+5)},\ x\neq 0,-5,1,\frac{3}{2}$$

$$\text{Example}: \frac{x^3-8}{x^2-4}\div\frac{x^2+2x+4}{x^3+8}=\frac{x^3-8}{x^2-4}\cdot\frac{x^3+8}{x^2+2x+4}=\frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)}\cdot\frac{(x+2)(x^2-2x+4)}{x^2+2x+4}=x^2-2x+4$$

- Adding/Subtracting: Find LCD
  Get both fractions into form with the LCD
  Add or subtract numerator, keep denominator

$$\text{Example}: \frac{x}{x-3}-\frac{2}{3x+4}\quad LCD=(x-3)(3x+4)$$

$$=\frac{x(3x+4)}{(x-3)(3x+4)}-\frac{2(x-3)}{(x-3)(3x+4)}=\frac{3x^2+4x-2x+6}{(x-3)(3x+4)}=\frac{3x^2+2x-6}{(x-3)(3x+4)}\quad x\neq 3,\frac{-4}{3}$$

- Adding/Subtracting: Find LCD
  Get both fractions into form with the LCD
  Add or subtract numerator, keep denominator

$$\text{Example}: \frac{x}{x-3}-\frac{2}{3x+4}\quad LCD=(x-3)(3x+4)$$

$$=\frac{x(3x+4)}{(x-3)(3x+4)}-\frac{2(x-3)}{(x-3)(3x+4)}=\frac{3x^2+4x-2x+6}{(x-3)(3x+4)}=\frac{3x^2+2x-6}{(x-3)(3x+4)}\quad x\neq 3,\frac{-4}{3}$$

Summer Work for Students
Entering Calculus

- Solving Rational Equations: Find the LCD
  Multiply every term by the LCD
  Solve your new equation for x

$$\text{Example}: \frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4} \quad LCD = (x+2)(x-2) \quad \text{Domain}: \{x \mid x \neq 2, -2\}$$

$$\Rightarrow (x+2)(x-2)\frac{1}{x-2} = \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

$$\Rightarrow (x+2) = 3(x-2) - 6x$$

$$\Rightarrow x+2 = 3x-6-6x$$

$$\Rightarrow x = -2 \quad \text{But this answer is not in your domain; therefore, we have NO SOLUTION}$$

5) Radicals and Rationalizing Denominators

Simplifying Radicals: 1) **Simplifying** a radical means that: a) no perfect square factors (other than 1) are left inside the radical, and b) No radicals are left in the denominator.
2) **"Combine"** means to simplify and then add or subtract like radicals
Examples:

$$\sqrt{52x^3} = \sqrt{4(13)x^2 x} = 2x\sqrt{13x}$$

$$\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\frac{4}{\sqrt{6}-2} = \frac{4}{\sqrt{6}-2}\frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{4(\sqrt{6}+2)}{6-4} = \frac{4(\sqrt{6}+2)}{2} = 2(\sqrt{6}+2)$$

$$4\sqrt{27} + \sqrt{9} - 2\sqrt{48} = 4\sqrt{9}\sqrt{3} + 3 - 2\sqrt{16}\sqrt{3} = 12\sqrt{3} + 3 - 8\sqrt{3} = 4\sqrt{3} + 3$$

6) Functions

- Domain: denominator can not be zero
  what is under a square root can not be negative

$$\text{Example}: f(x) = \frac{4}{x+2} \quad \text{Domain}: \{x \mid x \neq -2\}$$

$$f(x) = \sqrt{x+1} \quad \text{Domain}: \{x \mid x \geq -1\}$$

- Range
- Function notation: plug what is in () for x in your function

$$\text{Example}: f(x) = x^2 - 4 \Rightarrow f(3) = 3^2 - 4 = 5$$

7) Piecewise Functions

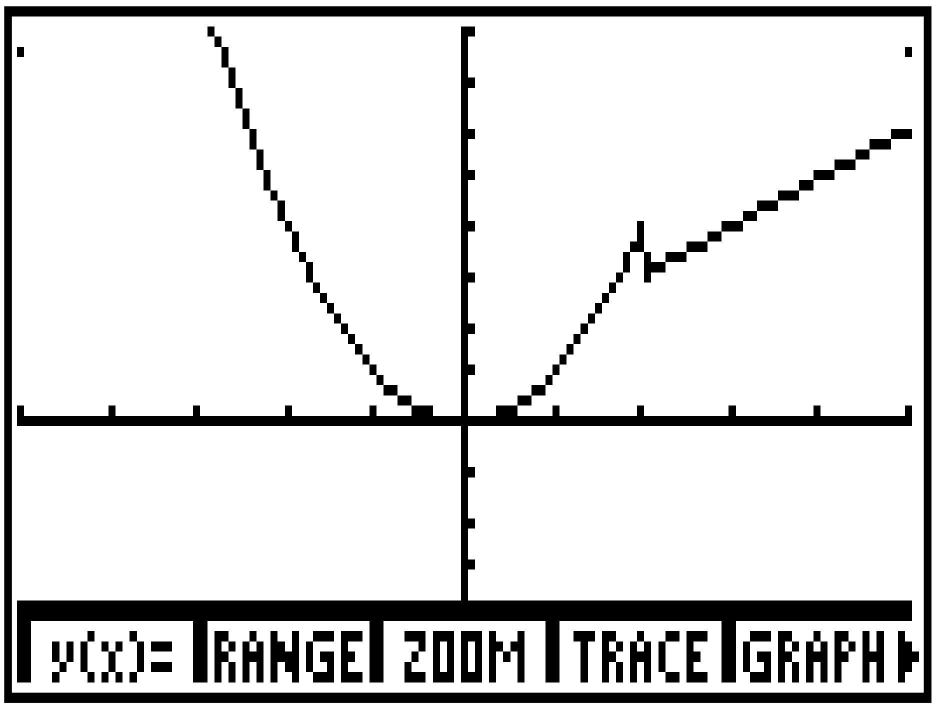
- Evaluating: look at the conditions of each piece of the function evaluate as usual

Example : $f(x)=\begin{cases} x+1 & x>2 \\ x^2 & x\le 2 \end{cases}$

Then $f(3)=3+1=4$ and $f(1)=1^2=1$

- Sketching: sketch each piece as usual in the given domain

Example: Given the function above, the graph will be the line $y = x + 1$ for all values of $x > 2$ while it will be the parabola $y = x^2$ for all values of $x \le 2$



8) Sketching Rational Functions

- find discontinuities (where the denominator=0)
  - -removable is a hole in your graph
  - -nonremovable is a vertical asymptote (be sure to look at the function's behavior from the left an right of vertical asymptotes
- horizontal asymptotes
  - if the degree of the numerator is larger than the denominator, no horizontal asymptotes
  - if the degree of the numerator is smaller than the denominator, there will be a horizontal asymptote at the x-axis
  - if the degrees of the numerator and denominator are the same, a horizontal asymptote will exist at $\dfrac{\text{leading coefficient of numerator}}{\text{leading coefiicient of denominator}}$
- x and y intercepts

Summer Work for Students
Entering Calculus

9) Logarithmic and Exponential Graphs and Equations

- Know what the graphs look like
- Definition of a logarithmic function: for $x > 0, 0 \le a \ne 1$

  $y = \log_a x$ if and only if $x = a^y$

  Example : $\log_2 32 = 5$ since $2^5 = 32$ Example : $\log_2 32 = 5$ since $2^5 = 32$
- The natural logarithmic function : $f(x) = \log_e x = \ln x \ , x > 0$
- Change of base formula : Let a, b, x be real numbers such that $a, b \ne 1$

  Then $\log_a x = \dfrac{\log_b x}{\log_b a}$

  Example : $\log_4 30 = \dfrac{\log 30}{\log 4} = \dfrac{\log_2 30}{\log_2 4}$
- Properties of logs : $\log_a (uv) = \log_a u + \log_a v$

  $$\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$$

  $$\log_a u^n = n \log_a u$$
- Solving Exponential Equations: first isolate the exponential expression, then take the logarithms of both sides and solve for the variable
- Solving logarithmic equations: rewrite the equation in an exponential form and solve for the variable

Example : $e^x + 5 = 60$

$$e^x = 55 \ \Rightarrow \ \ln e^x = \ln 55 \ \Rightarrow \ x = \ln 55 \ \Rightarrow \ x \approx 4.007$$

$$\ln x = 2 \ \Rightarrow \ e^{\ln x} = e^2 \ \Rightarrow \ x = e^2 \ \Rightarrow \ x \approx 7.389$$

11) Trigonometry

- Know the unit circle in degrees and radians
- Convert from degrees to radians : Multiply by $\dfrac{\pi}{180}$
- Convert from radians to degrees: Multiply by $\dfrac{180}{\pi}$
- Know the sine, cosine and tangent of quadrantal angles and 30º,45º,60º
- Know the graphs of the trig functions
- Know your trig identities

Calculus
Problem Set

Write out all solutions clearly on a separate sheet of paper. Show all steps and circle your answer.

1) Factor the following completely:

$a)\ 16x^2-9$ $b)\ (x-1)^2-4$ $c)\ x^3-27$ $d)\ 3x^2-5x-2$

$e)\ 8t^3+1$ $f)\ 3x^3+x^2+15x+5$ $g)\ 4x(2x-1)+(2x-1)^2$

$h)\ 2(x+1)(x-3)^2-3(x+1)^2(x-3)$ $i)\ \dfrac{x^2}{2}(x^2+1)^4-(x^2+1)^5$

2) Write the rational expression in reduced form.

$a)\ \dfrac{x^2-25}{5-x}$ $b)\ \dfrac{x^2-7x+12}{x^2+3x-18}$ $c)\ \dfrac{x^3-2x^2-3x}{x^3+1}$

3) Multiply or divide the following and simplify

$a)\ \dfrac{4x-16}{5x+15}\cdot\dfrac{2x+6}{4-x}$ $b)\ \dfrac{x^3-1}{x+1}\div\dfrac{x^2+1}{x^2-1}$ $c)\ \dfrac{\left[\dfrac{x^2-1}{x}\right]}{\left[\dfrac{(x-1)^2}{x}\right]}$

4) Add or subtract the following and simplify

$$\frac{2}{x^2-4}-\frac{1}{x^2-3x+2}$$

5) Solve the following for x

$$\frac{1}{x-2}+\frac{3}{x+3}=\frac{4}{x^2+x-6}$$

6) Simplify the following:

$a)\ 3\sqrt{4a^2+4b^2}$ $b)\ \dfrac{1}{\sqrt{2}+1}$ $c)\ \dfrac{6}{3-\sqrt{5}}$ $d)\ (3\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})$

7) Find the domain and range of the following functions.

$a)\ f(x)=\dfrac{1}{x-3}$ $b)\ f(x)=\sqrt{x+4}$ $c)\ f(x)=\dfrac{1}{(x+2)^2}$ $d)\ f(x)=5x^2+8x^6-2$

$e)\ f(x)=\sqrt{x^2-x-6}$ $f)\ f(x)=e^x+3$ $g)\ f(x)=\ln(x-2)+1$

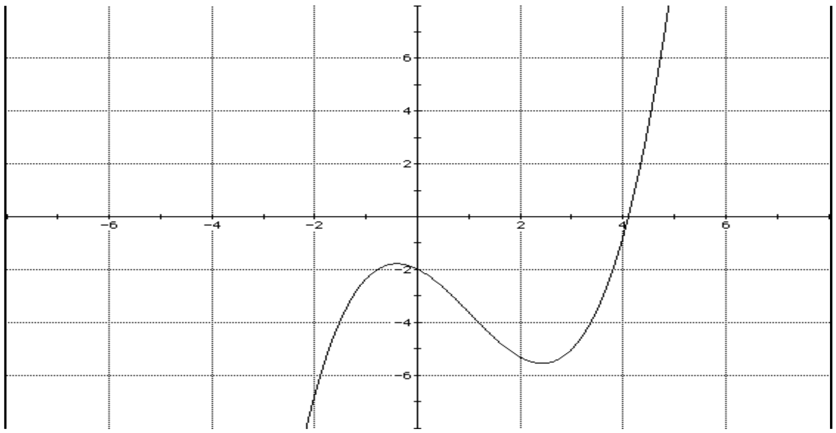
Calculus
Problem Set

8) Find $f(2)$, $f(0)$, $f(x+h)$, $f(x+h)-f(x)$ for the following functions

$a)\ f(x)=x^2+4x-2 \qquad b)\ f(x)=\sqrt{x-2}+4$

9) Given the graph below, determine the following

a) maximums and mininmums
b) domain and range
c) where $f(x)>0$ and where $f(x)<0$
d) $f(5)$ and $f(2)$



10) Evaluate

Given $f(x)=\begin{cases} x^2+3x & x>0 \\ 2x-1 & x\le 0 \end{cases}$, find the following:

$a)\ f(0) \qquad b)\ f(1) \qquad c)\ f(-3)$

11) Sketch the following functions

$a)\ f(x)=\begin{cases} x^2+1 & x\ge 0 \\ x-2 & x<0 \end{cases} \qquad b)\ f(x)=\begin{cases} 2 & x<3 \\ (x-3)^2 & x\ge 3 \end{cases}$

12) Sketch the graphs of the following functions

$a)\ f(x)=\dfrac{1}{x-2} \qquad b)\ f(x)=\dfrac{2x^2+6x}{(x+3)(x-2)} \qquad c)\ f(x)=\dfrac{x^2-x-6}{x+2}$

13) Sketch the graphs of the following

$a)\ y=e^x \qquad b)\ y=\ln x \qquad c)\ y=2^{x-3}+1 \qquad d)\ y=\log_2 x-3$

14) Evaluate using the change of base formula.

$a)\ \log_2 14 \qquad b)\ \log_7 23$

15) Evaluate without a calculator

$a)\ \ln e^5 \qquad b)\ \log_2 16 \qquad c)\ \log_{16} 4 \qquad d)\ \log_7 1 \qquad e)\ \log_8 32$

Calculus
Problem Set

16) Rewrite as a sum or difference of logs

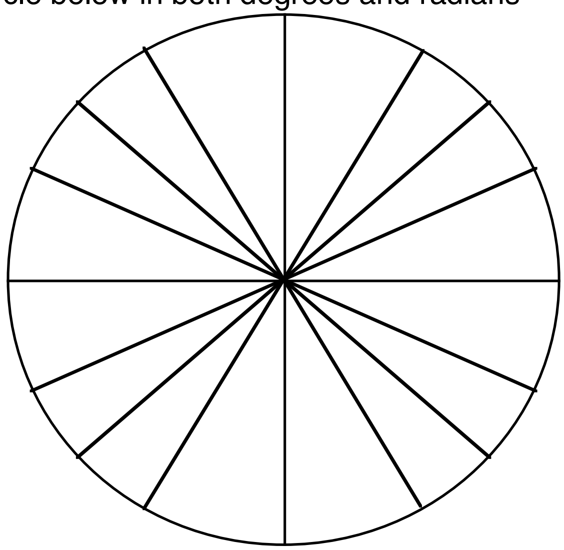
$a)\ \ln xyz \qquad b)\ \ln \sqrt[3]{\dfrac{x}{y}} \qquad c)\ \ln \dfrac{x^4\sqrt{y}}{z^5}$

17) Solve the following equations for x.

$a)\ 4^x = 16 \qquad b)\ 7^x = \dfrac{1}{49} \qquad c)\ \log_4 x = 3 \qquad d)\ \ln(2x-1) = 0 \qquad e)\ 7 - 2e^x = 5$

$f)\ 3^{x-1} = 27 \qquad g)\ \ln x + \ln(x-2) = 1 \qquad h)\ \log_4 x - \log_4(x-1) = \dfrac{1}{2}$

18) Fill in the unit circle below in both degrees and radians



19) Find the sine, cosine and tangent of the following angles

$a)\ 45^\circ \quad b)\ \dfrac{2\pi}{3} \quad c)\ \dfrac{\pi}{6} \quad d)\ 300^\circ \quad e)\ \dfrac{5\pi}{6} \quad f)\ \dfrac{3\pi}{2} \quad g)\ \pi \quad h)\ \dfrac{5\pi}{4}$

20) Sketch the graphs of the following **without** the assistance of a calculator:

$a)\ y = \sin x \qquad b)\ y = \cos x \qquad c)\ y = \tan x$

21) Prove the following equations using trig identities.

$a)\ \tan x \cos x = \sin x \qquad b)\ \dfrac{\tan\theta}{\theta} = \dfrac{\sin\theta}{\theta\cos\theta}$

$c)\ \sin^2 x - \cos^2 x = 2\sin^2 x - 1 \qquad d)\ \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \csc\theta\sec\theta$