AB CALCULUS AP
SUMMER REVIEW PACKET
THE BOLLES SCHOOL
MRS. REINSCHMIDT

It is expected that students entering AB Calculus AP have a strong mastery of Pre-Calculus. Pre-Calculus will not be reviewed at the start of the year; therefore, it is essential that you come to class prepared to expand upon the topics included in this packet. The problems on pages 1 and 2 of this packet are required. You are to complete these problems and bring them to class on your first day.

If you need further instruction on any given topic, supplemental problems will be attached to the packet. The supplemental problems are to be completed at your discretion if you feel you need the extra practice for a particular skill.

In class, we will take a series of skills tests to determine if you are as prepared for this course as you should be. Please bring Pages 1 and 2 to class with you on the first day of school, as well as any supplemental work which you complete.

Questions are on Pages 1 and 2
Answers and solutions are on Pages 3 through 6.
Supplemental problems begin on Page 7.

Note: This is a new packet. If you find errors, please let me know. You may email me at *reinschmidtj@bolles.org* *and I will correct/clarify. Thank you.*

I. <u>Non-Linear Inequalities</u>: Solve the following by using a number line test.
Solutions on page 5. Supplemental help is on page 7.

(1) $x^3 - 4x^2 \geq 5x$ (2) $\dfrac{x^2 - 4x + 4}{x} < 0$

II. <u>Domain</u>: Find the domain of the following.
Answers on page 3. Supplemental help is on page 8.

(3) $f(x) = x^2 - 4$ (4) $g(x) = \dfrac{x}{x^2 - 4}$ (5) $h(x) = \sqrt{x^2 - 4}$

(6) $f(x) = e^{x^2 - 4}$ (7) $h(x) = \ln\left(x^2 - 4\right)$

III. <u>Graphing Piece-Wise Functions</u>:
Answers on page 3. Supplemental help is on page 9.

(8) $h(x) = \begin{cases} x + 3 , & x < -2 \\ -x^2 + 1 , & -2 \leq x \leq 1 \\ 2x - 2 , & x > 1 \end{cases}$ (9) $h(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x - 1} , & x \neq 1 \\ -1 , & x = 1 \end{cases}$

IV. <u>Rational Expressions</u>: Fill in the chart. Write all intercepts as ordered pairs and all asymptotes as equations. *Answers on page 3. Supplemental help is on pages 9-11.*

(10) $f(x) = \dfrac{x^2 - 9}{x + 6}$ (11) $g(x) = \dfrac{10 - 6x}{3x + 1}$ (12) $h(x) = \dfrac{x - 1}{x^2 - 4}$

	$f(x)$	$g(x)$	$h(x)$
x-intercepts			
y-intercepts			
vertical asym.			
horizontal asym			

V. Equations of lines: *Answers on page 3. Supplemental help is on page 11.*

(13) Using the point/slope form, write an equation of a line which:

(a) Goes through the points $(-3, 4)$ *and* $(5, -2)$.

(b) Goes through the point $(-1, 7)$ and is perpendicular to the line in part (a).

VI. Solving Non-Linear Systems: *Solutions on page 5. Supplemental help is on page 13.*

(14) $\begin{aligned} y &= x^3 - 3x^2 + x \\ y &= -x \end{aligned}$

(15) $\begin{aligned} x^2 + y^2 &= 20 \\ y &= x + 2 \end{aligned}$

VII. Trigonometric Functions: Supplemental problems have not been included. It is suggested that you seek help online if needed.

(16) You must know what the 6 basic trig graphs look like. Study them.

(17) Evaluate: Note, in AP Calculus, only radians are used.

(i) $\sin\left(\frac{2\pi}{3}\right)$ (iii) $\cot\left(\frac{-\pi}{6}\right)$ (v) $\sec\left(\frac{5\pi}{6}\right)$

(ii) $\cos\left(\frac{-5\pi}{3}\right)$ (iv) $\tan\left(\frac{-\pi}{4}\right)$ (vi) $\csc\left(\frac{7\pi}{2}\right)$

VIII. Algebra Practice:

(18) Given: $f(x) = 2x^2 + 3x$ Evaluate: *Answers on page 4.*

(a) $f(x + \Delta x)$ (b) $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Algebraically cleanup and find the zeros: *Solutions on page 6. Supplemental help is on page 14.*

(19) $(2x+4)^5\left[7(x+5)^6\right] = (x+5)^7\left[5(2x+4)^4(2)\right]$

(20) $\dfrac{\frac{1}{x} + \frac{1}{2+x}}{3} = 0$

(21) $\dfrac{\sqrt{6x+4} - (x-1)\left[\frac{1}{2}(6x+4)^{-\frac{1}{2}}(6)\right]}{6x+4} = 0$

Answers: Solutions to the starred problems are on Page 5 and 6.

(1) $[-1, 0] \cup [5, \infty)$ **

(2) $(-\infty, 0)$ **

(3) $(-\infty, \infty)$

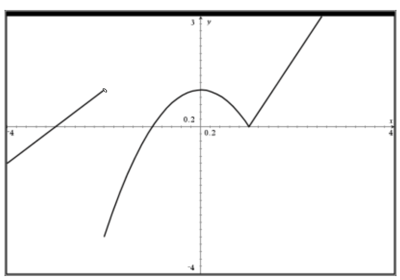
(4) $x \in \mathbb{R},\ x \neq \pm 2$

(5) $(-\infty, -2] \cup [2, \infty)$

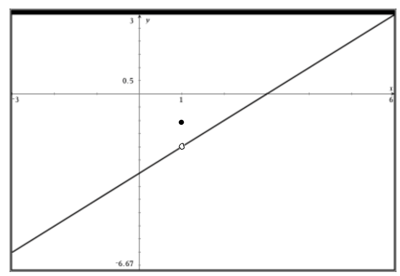
(6) $(-\infty, \infty)$

(7) $(-\infty, -2) \cup (2, \infty)$

(8)



(9)



(10) – (12)

	$f(x)$	$g(x)$	$h(x)$
x-intercepts	(-3, 0) (3, 0)	(5/3, 0)	(1, 0)
y-intercepts	(0, -3/2)	(0, 10)	(0, 1/4)
vertical asym.	X=-6	X=-1/3	X=2, X=-2
horizontal asym	no hor asym	y=-2	y=0

(13) (a) $y-4=\frac{-3}{4}(x+3)$ *or* $y+2=\frac{-3}{4}(x-5)$

(b) $y-7=\frac{4}{3}(x+1)$

(14) $(0,0)$ $(2,-2)$ $(1,-1)$ **

(15) $(-4,-2)$ $(2,4)$ **

(16) Google anyplace to check for correct trig graphs and help on evaluating trig functions.

(17) (i) $\frac{\sqrt{3}}{2}$ (iv) -1

(ii) $\frac{1}{2}$ (v) $\frac{2}{-\sqrt{3}}$

(iii) $-\sqrt{3}$ (vi) -1

(18a) $2x^2+4x(\Delta x)+2(\Delta x)^2+3x+3\Delta x$

(18b) $4x+2\Delta x+3$

(19) $(x+5)^6(2x+4)^4(4x-22)=0$ Zeros at $x=\frac{11}{2},\ -5,\ -2$ **

(20) $\frac{2+2x}{3x(2+x)}=0$ Zeros at $x=-1,\ 0,\ -2$ **

(21) $\frac{3x+7}{(6x+4)^{3/2}}=0$ Zeros at $x=\frac{-7}{3},\ \frac{-2}{3}$ **

Solutions for Selected Problems:

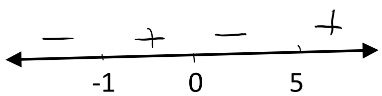
(1)

$$x^3 - 4x^2 - 5x \ge 0$$

$$x\left(x^2 - 4x - 5\right) \ge 0$$

$$x(x-5)(x+1) \ge 0$$

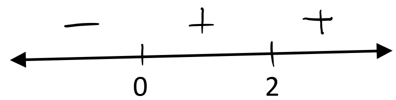
$$x = 0,\ 5,\ -1$$



$$[-1,\ 0] \cup [5,\ \infty)$$

(2)

$$\frac{(x-2)^2}{x} < 0$$



$$(-\infty,\ 0)$$

(14) $y = x^3 - 3x^2 + x$

$$x^3 - 3x^2 + x = -x$$

$$x^3 - 3x^2 + 2x = 0$$

$$x\left(x^2 - 3x + 2\right) = 0$$

$$x(x-2)(x-1) = 0$$

$$x = 0,\ 2,\ 1$$

$y = -x$

$(0,\ 0)$

$(2,\ -2)$

$(1,\ -1)$

(15) $y = x + 2$

$$x^2 + y^2 = 20$$

$$x^2 + (x+2)^2 = 20$$

$$x^2 + x^2 + 4x + 4 = 20$$

$$2x^2 + 4x - 16 = 0$$

$$2\left(x^2 + 2x - 8\right) = 0$$

$$2(x+4)(x-2) = 0$$

$$x = -4,\ 2$$

$(-4,\ -2)$

$(2,\ 4)$

(19) $(2x+4)^5\left[7(x+5)^6\right] - (x+5)^7\left[5(2x+4)^4(2)\right] = 0$

Note... 2 terms which each have at least six of the $(x+5)$'s and four of the $(2x+4)$'s. Factor out those common terms.

$(x+5)^6(2x+4)^4\left[7(2x+4) - 10(x+5)\right] = 0$

$(x+5)^6(2x+4)^4\left[14x+28-10x-50\right] = 0$

$(x+5)^6(2x+4)^4(4x-22) = 0$

Zeros @ $x = -5, -2, 11/2$

(20)

$\frac{x(2+x)}{x(2+x)}\left[\frac{\frac{1}{x} + \frac{1}{2+x}}{3} = 0\right]$

Find a common denominator for all terms. In this case it is $x(2+x)$. Multiply complex fraction by $\frac{x(2+x)}{x(2+x)}$

$\frac{x+2+x}{3x(2+x)} = 0$

Zeros @ $x = 0, -1, -2$

$\frac{2x+2}{3x(x+2)} = 0$

(21) $\frac{\sqrt{6x+4} - (x-1)\left[\frac{1}{2}(6x+4)^{-1/2}(6)\right]}{6x+4} = 0$

Clean up this part

$\frac{\sqrt{6x+4}}{\sqrt{6x+4}}\left[\frac{\sqrt{6x+4} - \frac{3(x-1)}{\sqrt{6x+4}}}{6x+4} = 0\right]$

mult. by $\frac{\sqrt{6x+4}}{\sqrt{6x+4}}$

$\frac{6x+4-3(x-1)}{(6x+4)^{3/2}} = \frac{3x+7}{(6x+4)^{3/2}} = 0$

Zeros at $x = -\frac{7}{3}, -\frac{2}{3}$

SUPPLEMENTAL PROBLEMS

I. <u>Non-linear Inequalities</u>:

To solve a non-linear inequality, place all terms on the left of the inequality symbol with 0 on the right. Factor the numerator and denominator (if any) of each equation so you can determine all zeros. Once you have the zeros, place these values on a number line. Next, pick a number which lies within each interval on your number line. Plug that number into your equation. If the result is a positive number, then put a + sign on that portion of the number line. Continue until all intervals have a plus or minus sign. Where you have + signs is where the equation is considered positive (greater than zero); and likewise, where you have − signs is where the equation is considered negative (less than zero).

<u>QUESTIONS</u>:

1. $(2x+1)(3x-5)<0$
2. $\dfrac{2x-1}{x+3}>0$
3. $x^2+80\leq 18x$
4. $(3x+2)(x-3)>7x-1$
5. $\dfrac{(x-1)^3}{(x+5)^2}>0$
6. $x^2e^x \geq 2xe^x$

<u>ANSWERS</u>:

1. *interval*: $\left(\dfrac{-1}{2}, \dfrac{5}{3}\right)$
2. *interval*: $(-\infty, -3)\cup\left(\dfrac{1}{2}, \infty\right)$
3. *interval*: $[8, 10]$
4. *interval*: $\left(-\infty, -\dfrac{1}{3}\right)\cup(5, \infty)$
5. *interval*: $(1, \infty)$
6. *interval*: $(-\infty, 0]\cup[2, \infty)$

Hint for problem # 4: You will need to multiply the binomials on the left side of the inequality. Bring both terms from the right side of the inequality over to the left side and combine like terms. Refactor, and solve as directed above.

II. <u>Domain Supplemental Practice</u>:

The five most common functions of which we will find the domain are the following.

<u>Polynomials</u>: The domain will always be $(-\infty,\ \infty)$ as polynomials go infinitely far in each direction.

<u>Exponential Functions</u>: The domain will always be $(-\infty,\ \infty)$ as exponential functions go infinitely far in each direction.

<u>Rational Functions</u>: Fractions are not allowed to have denominators equal to zero. Therefore, the zeros of the denominator make the rational function undefined at that x-value. The domain will always be the set of Real Numbers with the exception of the zeros of the denominator.

<u>Radicals</u>: In the set of Real Numbers, we are not allowed to have a negative under the radical. Therefore, when asked to find the domain of a radical, set the expression under the radical ≥ 0. This inequality will be solved by completing a number line sign test as we did in Part I on page 7. Once you have completed your number line test, the domain is where you have the + signs inclusive of the end points of the interval, since we are allowed to have zero values under the radical.

<u>Logarithmic Functions</u>: In the set of Real Numbers, we are not allowed to have the log of a negative number or the log of 0. If given $\log(u)$, then $u > 0$. When asked to find the domain of a log or natural log, first set up your inequality. Next, solve the inequality as we did in Part I by doing a number line sign test. Once you have completed your number line test, the domain is where you have the + signs, exclusive of the end points since zero values are not allowed.

<u>QUESTIONS</u>:

(7) $f(x)=\ln\left(\dfrac{x-2}{x+1}\right)$

(8) $h(x)=\dfrac{x-3}{x^2-9}$

(9) $f(x)=\sqrt{\sin x}$ Over the interval of $[0,\ 2\pi]$. *It may help to think of the graph of this function.*

(10) $g(x)=\sqrt{\dfrac{x-4}{x^2+9}}$

(11) $g(x)=e^{x^3-5x^2}$

<u>ANSWERS</u>:

(7) $(-\infty,\ -1)\ (2,\ \infty)$

(8) $x\in\mathbb{R},\ x\neq\pm3$

(9) $[0,\ \pi]$

(10) $[4,\ \infty)$

(11) $(-\infty,\ \infty)$

III. <u>Piece-Wise Functions Supplemental Problems</u>:

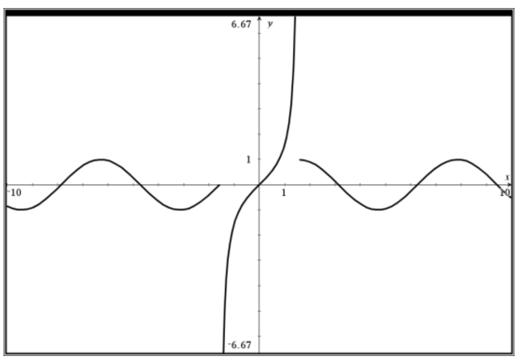
In AB Calculus AP, it is expected that you are able to graph trig functions and rational functions at any time throughout the year.

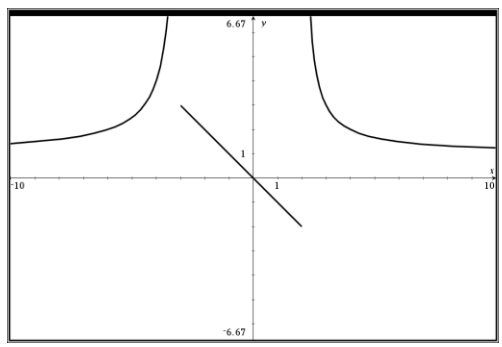
(12)

$$f(x)=\begin{cases} \cos x , & x \le \dfrac{-\pi}{2} \\ \tan x , & \dfrac{-\pi}{2} < x < \dfrac{\pi}{2} \\ \sin x , & x \ge \dfrac{\pi}{2} \end{cases}$$

(13)

$$h(x)=\begin{cases} \dfrac{x}{x+3} , & x < -3 \\ -x , & -3 \le x \le 2 \\ \dfrac{x}{x-2} , & x > 2 \end{cases}$$





For help in graphing rational functions on (13), see Part IV below.

IV. <u>Rational Functions Supplemental Problems</u>:

Graphing rational functions (polynomial divided by a polynomial):

(i) First fully factor the function. A factor which can be reduced will be called a removable factor. A removable factor will appear as a hole on a graph. To find the y-value of the hole, plug the x-value into the reduced fraction.

(ii) Find all x-intercepts by setting $y=0$ then solving for x.

(iii) Find all y-intercepts by setting $x=0$ and solving for y.

(iv) Find vertical asymptotes by finding the zeros of the denominator. The zeros of the denominator are your vertical asymptotes. (Undefined x-values in the domain.)

Continued……….

(v) Finding horizontal asymptotes is much more involved. *Please note that in this packet I will explain a "short-cut" version. The first few weeks of Calculus will be spent developing the mathematical reasoning through the study of limits.*

Short cut: First, compare the leading term of the numerator and the denominator.

If your leading terms are the same, then the leading coefficients of each would be your horizontal asymptote. Example:

$f(x)=\dfrac{2x+3}{5x-4}$ The horizontal asymptote will be $y=\dfrac{2}{5}$.

If your leading terms are not the same, one of two things will happen. Either the numerator will have a leading term with the highest power or the denominator will. Depending upon which is higher decides what your asymptote will be. See below.

If the leading term is higher in the denominator: $g(x)=\dfrac{5x-2}{x^2+3}$

Some students simply memorize that if the highest power is in the denominator, then your horizontal asymptote will always be $y=0$. Others fill in a missing term as shown below:

$g(x)=\dfrac{0x^2+5x-2}{x^2+3}$. The short cut gives $y=\dfrac{0}{1}=0$ as the horizontal asymptote.

If the leading term in the numerator is higher such as: $h(x)=\dfrac{x^3-3x}{x^2-2}$

As above, some students simply memorize that if the highest power is in the numerator, then a horizontal asymptote will not exist. Others fill in a missing term as shown below:

$h(x)=\dfrac{x^3-3x}{0x^3+x^2-2}$. The short cut rule gives $y=\dfrac{1}{0}$ $or\ \varnothing$; hence, no horizontal asymptote.

NOTE: If a horizontal asymptote does not exist, there will be a slant asymptote, a parabolic asymptote, or some other asymptotic behavior which follows a polynomial curve. We will discuss this in class.

Problems on the next page:

(14) $f(x)=\frac{x^2-16}{x+3}$ (15) $g(x)=\frac{2x^2+6x-8}{x^2-5x+4}$ (16) $h(x)=\frac{x-8}{x^2-4}$

	$f(x)$	$g(x)$	$h(x)$
x-intercepts			
y-intercepts			
vertical asym			
horizontal asym			

Graph (15) and (16) above. *Answers are on the next page.*

V. <u>Equations of Lines in Point/Slope</u>:

Point/Slope formula: $y-y_1=m(x-x_1)$

Given a point $(-3, 5)$ and a slope of $\frac{1}{2}$, simply substitute the point and the slope as follows:

$y-5=\frac{1}{2}(x-(-3))$ or $y-5=\frac{1}{2}(x+3)$

<u>QUESTIONS</u>:

(17) Write an equation of a line which goes through the point $(7, -2)$ and is parallel to the line $3x+y=8$.

(18) Write an equation of a line which goes through the point $(=6, 3)$ and is perpendicular to the line given in (17).

Answers to Page 11.

(14) $f(x)=\dfrac{x^2-16}{x+3}$ (15) $g(x)=\dfrac{2x^2+6x-8}{x^2-5x+4}$ (16) $h(x)=\dfrac{x-8}{x^2-4}$

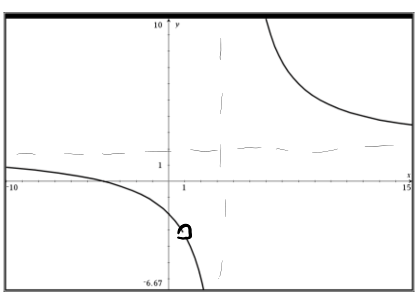
	$f(x)$	$g(x)$	$h(x)$
x-intercepts	(4, 0) (−4, 0)	(−4, 0)	(8, 0)
y-intercepts	(0, −16/3)	(0, −2)	(0, 2)
vertical asym	$x=-3$	$x=4$	x=2 x=−2
horizontal asym	none	y=2	y=0

Note: (15) reduces to be $g(x)=\dfrac{2(x+4)(x-1)}{(x-4)(x-1)}=\dfrac{2(x+4)}{(x-4)}$.

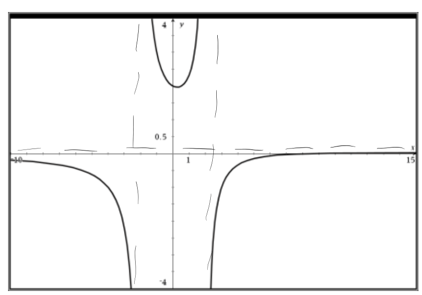
We removed the factor of $(x-1)$ so we will have a hole at $\left(1,\dfrac{-10}{3}\right)$

Graphs:

(15)



(16)



(17) $y+2=-3(x-7)$

(18) $y-3=\dfrac{1}{3}(x+6)$

VI. Systems of Non-Linear Equations:

(19) $$\begin{aligned} x+2y&=11 \\ xy&=5 \end{aligned}$$

(20) $$\begin{aligned} y^3&=2x+7-x^2 \\ y^3&=3x+5 \end{aligned}$$

Answers:

(19)

$$y=\frac{5}{x} \qquad x+2y=11$$

$$\left[x+2\left(\frac{5}{x}\right)=11\right]$$

$$x^2+10=11x$$

$$(x-10)(x-1)=0$$

$$x=10 \quad x=1$$

$$\left(10,\ \frac{1}{2}\right)$$

$$(1,\ 5)$$

(20)

$$y^3=2x+7-x^2 \qquad y^3=3x+5$$

$$2x+7-x^2=3x+5$$

$$0=x^2-x-2$$

$$0=(x-2)(x+1)$$

$$x=2, \quad x=-1$$

$$\left(2,\ \sqrt[3]{11}\right)$$

$$\left(-1,\ \sqrt[3]{2}\right)$$

VIII. <u>Algebra Practice</u>: Solve for x

(21) $(x-4)^8\left[5(3x-1)^4\right]+(3x-1)^5\left[3(x-4)^7(2)\right]=0$

(22) $\dfrac{5\sqrt{x-3}-(3x+1)\left[(x-3)^{-\frac{1}{2}}(2)\right]}{x-3}=0$

(23) $\dfrac{\dfrac{4}{x^2-1}+\dfrac{8}{x+1}}{\dfrac{4}{x^2-1}+\dfrac{8}{x-1}}=0$

ANSWERS:

(21)

$$(x-4)^7(3x-1)^4\left[5(x-4)+6(3x-1)\right]=0$$

$$(x-4)^7(3x-1)^4(23x-26)=0$$

$$x=4,\ \frac{1}{3},\ \frac{26}{23}$$

(22)

$$\frac{5\sqrt{x-3}-(3x+1)\left[(x-3)^{-\frac{1}{2}}(2)\right]}{x-3}=0$$

$$\frac{5\sqrt{x-3}-\dfrac{2(3x+1)}{\sqrt{x-3}}}{x-3}=0$$

$$\frac{\dfrac{\sqrt{x-3}}{1}}{\dfrac{\sqrt{x-3}}{1}}\left[\frac{5\sqrt{x-3}-\dfrac{2(3x+1)}{\sqrt{x-3}}}{x-3}=0\right]$$

$$\frac{5(x-3)-2(3x+1)}{(x-3)^{3/2}}=\frac{-x-17}{(x-3)^{3/2}}=0$$

$$x=-17,\ 3$$

(23)

$$\frac{\dfrac{(x-1)(x+1)}{1}}{\dfrac{(x-1)(x+1)}{1}}\left[\frac{\dfrac{4}{x^2-1}+\dfrac{8}{x+1}}{\dfrac{4}{x^2-1}+\dfrac{8}{x-1}}\right]=0$$

$$\frac{4+8(x-1)}{4+8(x+1)}=\frac{4+8x-8}{4+8x+8}=0$$

$$\frac{8x-4}{8x+12}=\frac{4(2x-1)}{4(2x+3)}=0$$

$$x=\frac{1}{2},\ \frac{-3}{2}$$