To: Rising Calculus students
From: Your Math Teachers
Re: Optional Summer Work

Attached are some materials that you might find helpful in gearing up for your class next year. We want you to know two important things about these materials:

- Please know that we do not expect students to *have* to do this work in order to be prepared for class – if you had a good understanding (you didn't just memorize and mimic procedures but instead understood the concepts) of course content from last year, you should be in a good position for next year's class. But if you could use some additional understanding and/or practice, or if you want to refresh your memory and make sure your skills are sharp, this is meant to be helpful.
- Also, please understand that this packet is not, by any means, "the ticket" to strong achievement in math class next year. What's really important is to develop and use good study skills throughout each unit – adequately preparing nightly homework, sustaining your effort when initially stuck, participating and pushing yourself to understand during class, and thoughtfully studying for unit tests.

We've prepared a few handouts covering important content from last year. We will post solutions to these problems on the math department website by mid-summer. In addition, the Khan Academy website has helpful explanatory videos and problems sets:

To practice trig: https://www.khanacademy.org/math/trigonometry (in particular the Unit Circle and Trig Identities sections)

To practice parametric equations:
https://www.khanacademy.org/math/precalculus/parametric_equations

To practice exponential and logarithmic functions:
https://www.khanacademy.org/math/algebra2/exponential_and_logarithmic_func

We hope you make the time to do many non-math activities over the summer too – to relax, and to refresh and rejuvenate yourself. Enjoy!

TRIG KEY

& Exp/Log Key & parametrics Key

Trigonometry Worksheet

Do the circled ones

1. (a) Convert 75° and 225° to radians.

(b) Convert $\frac{\pi}{15}$ and $\frac{7\pi}{9}$ to degrees.

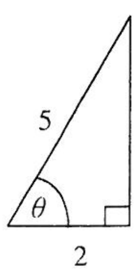
2. Fill in the table from memory.

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
$0 = 0°$	1	0	0
$\frac{\pi}{6} = 30°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/3$
$\frac{\pi}{4} = 45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\frac{\pi}{3} = 60°$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}$
$\frac{\pi}{2} = 90°$	0	1	und.
$\pi = 180°$	-1	0	0
$\frac{3\pi}{2} = 270°$	0	-1	und.
$\frac{5\pi}{6} = 150°$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}/3$
$\frac{4\pi}{3} = 240°$	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}$

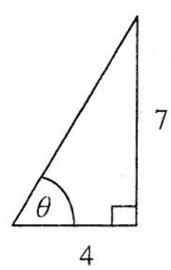
3. Given that $\tan\theta = 3$, find the exact values of the remaining five trigonometric functions of θ. [Hint: draw the appropriate triangle.]

4. Find the cosine, sine, and tangent of θ.

(a)

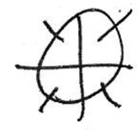


(b)



5. (a) Find all values of θ between 0 and 2π (in radians) such that $4\sin^2\theta - 2 = 0$.

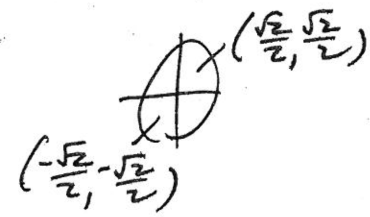
$$4\sin^2\theta = 2$$
$$\sin^2\theta = \frac{1}{2}$$
$$\sin\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$



$$\boxed{\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}}$$

(b) Find all values of θ between 0 and 2π (in radians) such that $\sin\theta = \cos\theta$.

$$\tan\theta = 1$$



$$\boxed{\theta = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}}$$

6. Find the difference quotient of $f(x) = \sin x$.

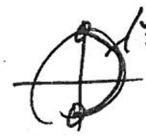
2\. (a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \underline{\frac{\pi}{6}}$ (30°)

i.e. the ∡ from $[0, \pi]$ whose cosine is $\sqrt{3}/2$

(b) $\sin^{-1}(1) = \underline{\frac{\pi}{2}}$ (90°)

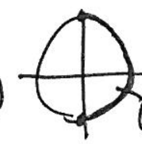
i.e. the ∡ from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is 1

(c) $\tan^{-1}(1) = \underline{\frac{\pi}{4}}$ (45°)



i.e. the ∡ from $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is 1

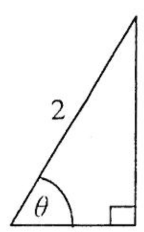
(d) $\sin^{-1}\left(-\frac{1}{2}\right) = \underline{\frac{-\pi}{6}}$ (-30°)



i.e. the ∡ from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is $-\frac{1}{2}$

3\. Find θ.

(a)



i.e. 30-60-90 △

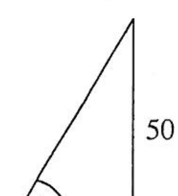
$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = 60^\circ}$

could use $\cos^{-1}\left(\frac{1}{2}\right)$ to find θ, but shouldn't need to!

(b)



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{50}{20}$

$\tan^{-1}\left(\frac{5}{2}\right) = \theta$

$\boxed{\theta \approx 68.2^\circ}$

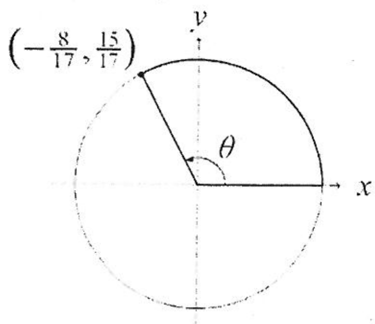
6. Find the length of the arc on a circle with a radius of 9 feet and a central angle of 60°.

7. A car is moving at a rate of 40 miles per hour, and the diameter of its wheels is 2.5 feet.
   (a) Find the linear speed of the tires in feet per minute.
   (b) Find the number of revolutions per minute the wheels are rotating.
   (c) Find the angular speed of the wheels in radians per minute.

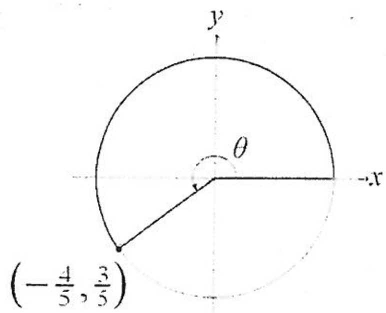
Section 4.2 - Unit Circle

1. Determine the exact values of the six trigonometric functions of the angle θ.

(a)



(b)



2. Find the point on the unit circle that corresponds to the angle θ.

(a) $\theta = \dfrac{\pi}{3}$ (b) $\theta = \dfrac{5\pi}{4}$ (c) $\theta = \pi$

3. Find the sine, cosine and tangent for the angle θ.

(a) $\theta = \dfrac{-3\pi}{4}$ (b) $\theta = \dfrac{-4\pi}{3}$ (c) $\theta = \dfrac{11\pi}{6}$

4. Find the six trigonometric functions for the angle θ.

(a) $\theta = \dfrac{5\pi}{6}$ (b) $\theta = \dfrac{3\pi}{2}$

see answer key

5. Find the value of the given trigonometric function.

(a) $\cos 7\pi$ (b) $\sin \dfrac{9\pi}{4}$ (c) $\sin\left(-\dfrac{19\pi}{6}\right)$

6. For the given trigonometric function, find the value of the indicated function.

(a) Given $\cos\theta = -\dfrac{3}{4}$, find $\cos(-\theta)$ (b) Given $\sin(-\theta) = \dfrac{3}{8}$, find $\csc(\theta)$

7. A bocce ball suspended from a Slinky bobs up and down, but because of friction the ball moves up and down less with each cycle. This is called damped harmonic motion, and in this case the vertical position of the ball y (in feet) is given by the function $y(t) = \frac{1}{4}e^{-t}\cos 6t$, where t is the elapsed time (in seconds). Find the position of the ball at the following points in time.

(a) $t = 0$ (b) $t = \frac{1}{4}$ (c) $t = \frac{1}{2}$

For problems 4-9, sketch the graph of the function. Show two full periods.

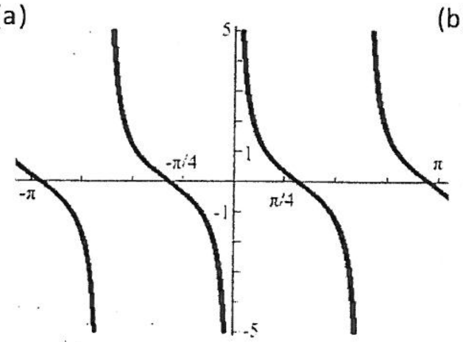
4. $y = \frac{1}{4}\sec x$ 5. $y = 2\csc \pi x$ 6. $y = -2\sec 4x + 2$ 7. $y = -\csc\frac{x}{3}$

8. $y = \sec(x + \pi)$ 9. $y = \csc(2x - \pi)$

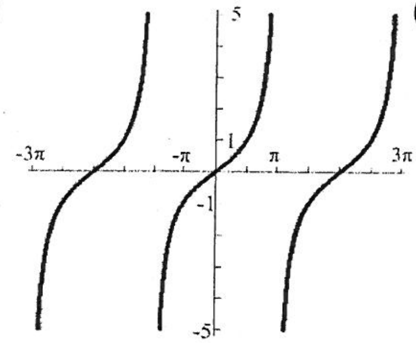
Section 4.6 - Tangent and Cotangent Functions

For problems 1-3, match the function with its graph. Find the period of the function.

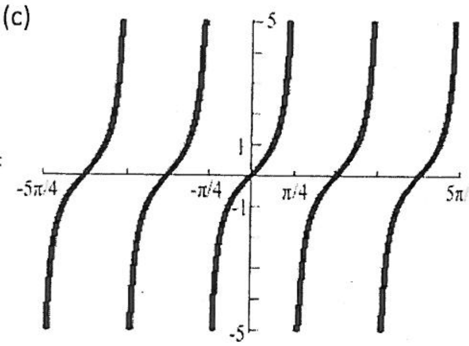
(a)



(b)



(c)



1. $y = \tan\frac{x}{2}$ 2. $y = \tan 2x$ 3. $y = \cot\frac{\pi x}{2}$

For problems 4-9, sketch the graph of the function. Show two full periods.

4. $y = \cot x$ 5. $y = -3\tan 4x$ 6. $y = 3\cot \pi x$ 7. $y = -\frac{1}{2}\tan \pi x$

8. $y = \frac{1}{4}\cot(x + \pi)$ 9. $y = \frac{1}{2}\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$

Chapter 4 Review

1. For a right triangle, $\tan\theta = \sqrt{3}$. Find the value of the other five trig functions.

2. For a right triangle, $\frac{1}{\csc\theta} = \frac{1}{2}$. Find the value of the six trig functions.

3. Name the quadrant that the angle *x* lies in given that sec *x* > 0, and sin *x* < 0.

4. Name the quadrant that the angle *x* lies in given that csc *x* > 0, and tan *x* < 0.

5. Suppose for an angle θ, cot θ = 7/24 and sec θ < 0. Find the exact value of sin θ.

see answer key

6. Convert 75° to radian measure.

7. Convert -7π/4 to its exact degree measure and determine the quadrant of the terminal side of the angle.

8. Find the complement and supplement of π/5.

9. Find the reference angle for θ = -2π/3. Then find the exact values of the six trigonometric functions of θ=2π/3.

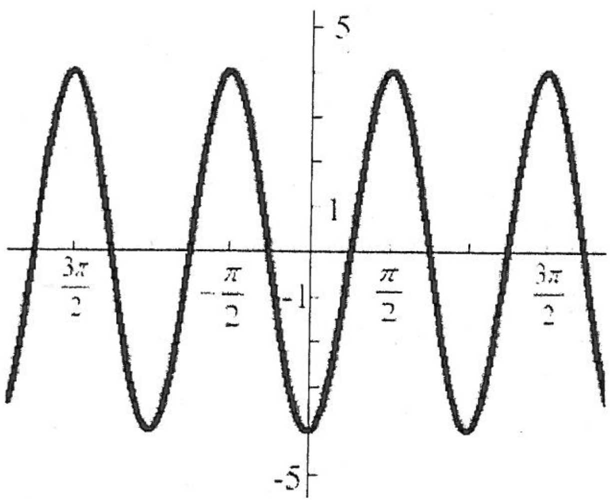
10. If tan θ = 1/2 and sin θ < 0, find the quadrant of θ and the exact values of the remaining five trigonometric functions of θ.

11. If sec θ = -5/4 and tan θ < 0, find the quadrant of θ and the exact values of the remaining five trigonometric functions of θ.

see answer key

12. Identify the phase shift (horizontal shift), amplitude, and period of y = 4sin(3x – π).

13. Find the amplitude and period of the sinusoidal graph given, then write an equation of the graph.



14. Given that sec x = 3, find cot x.

15. An escalator 152 feet in length rises to a platform and makes a 37° angle with the ground. Find the height of the platform.

16. A man at the top of a ramp 30 feet in length looks down to the end of the ramp which rises to a loading platform 3 feet off the ground. For a safe ramp, the $\tan\theta \le 0.15$. Is this ramp safe?

17. cos 240° = 18. cot 13π/6 = 19. sin 120° = 20. tan π/4 =

21. sec 330° = 22. csc 5π/3 =

23. Use the fundamental identities to determine the simplified form of the expression.

a) cos θ csc θ = b) tan θ cot θ =

c) sin θ cot θ =

24. Sketch one period of the graph of $f(x) = -\sin(x - \pi) + 1$. Find the period, amplitude, and phase shift.

25. Sketch one period of the graph of $f(x) = 2\tan(x + \frac{\pi}{2})$. Find the period and phase shift.

26. Sketch one period of the graph of $f(x) = \sec(x - \frac{\pi}{4}) - 2$. Find the period and phase shift.

Chapter 5: Analytic Trigonometry

Essential Questions

How can we us different rules and relationships to represent the same function in different ways?

How can we represent the same angle in different ways mathematically?

Learning Targets

Simplify Trigonometric Functions

Develop Additional Trigonometric Identities

Solve Trigonometric Equations

Evaluate Trigonometric Functions

Homework

Section 5.1 – Using Fundamental Identities

1. Use the given values to evaluate all six trigonometric functions.

a. csc (-x) = - 5 b. $\sec\theta = -5$ c. $\tan\theta$ *is undefined*, $\sin\theta > 0$

2. Use the fundamental identities to simplify the expression.

a. $\cos\beta\tan\beta$ b. $\sec^2 x(1-\sin^2 x)$ c. $\dfrac{\sec\theta}{\csc\theta}$

See answer key.

d. $\dfrac{\tan^2\theta}{\sec^2\theta}$ e. $\cot\left(\dfrac{\pi}{2}-x\right)\cos x$ f. $\dfrac{1}{\tan^2 x+1}$

3. Verify the identity algebraically.

a. $(\sec\theta-\tan\theta)(\csc\theta+1)=\cot\theta$

b. $\dfrac{1+\csc\theta}{\cot\theta+\cos\theta}=\sec\theta$

c. $\cos\theta\sec\theta-\cos^2\theta=\sin^2\theta$

d. $\dfrac{\tan\theta}{1+\sec\theta}+\dfrac{1+\sec\theta}{\tan\theta}=2\csc\theta$

4. Use the fundamental identities to simplify.

a. $\sec^2 x\tan^2 x+\sec^2 x$

b. $\dfrac{\csc^2 x-1}{\csc x-1}$

c. $1-2\sin^2 x+\sin^4 x$

d. $\sec^4 x-\tan^4 x$

e. $(\sin x+\cos x)^2$

see answer key

5.2 - Verifying Trigonometric Identities

Verify the Identity:

1. $\sec^6 x(\sec x\tan x)-\sec^4 x(\sec x\tan x)=\sec^5 x\tan^3 x$

2. $\dfrac{\cos\left[\dfrac{\pi}{2}-x\right]}{\sin\left[\dfrac{\pi}{2}-x\right]}=\tan x$

3. $(1+\sin y)[1+\sin(-y)]=\cos^2 y$

4. $\dfrac{\tan x+\tan y}{1-\tan x\tan y}=\dfrac{\cot x+\cot y}{\cot x\cot y-1}$

5. $\dfrac{\tan x+\cot y}{\tan x\cot y}=\tan y+\cot x$

6. $\csc x(\csc x-\sin x)+\dfrac{\sin x-\cos x}{\sin x}+\cot x=\csc^2 x$

7. $\dfrac{1+\csc\theta}{\sec\theta}-\cot\theta=\cos\theta$

8. $\sin x\left(1-2\cos^2 x+\cos^4 x\right)=\sin^5 x$

9. $\csc^4\theta-\cot^4\theta=2\csc^2\theta-1$

10. $\dfrac{\sin^3\beta+\cos^3\beta}{\sin\beta+\cos\beta}=1-\sin\beta\cos\beta$

5.3 - Solving Trigonometric Equations

Solve the equation

1. $\sqrt{2}\sin x + 1 = 0$

2. $\cot x + 1 = 0$

3. $3\cot^2 x - 1 = 0$

4. $\cos x(\cos x - 1) = 0$

5. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

see answer key

Find all solutions of the equation in the interval [0, 2π) algebraically.

6. $\tan^2 x - 1 = 0$

7. $\sec x \csc x = 2\csc x$

8. $\sec x + \tan x = 1$

9. $\sin^2 x + \cos x + 1 = 0$

10. $2\sec^2 x + \tan^2 x - 3 = 0$

11. $\csc^2 x = 3\csc x + 4$

Solve the multiple angle equation.

12. $\sec 4x = 2$

13. $\tan^2 3x = 3$

14. $\tan\frac{x}{3} = 1$

Section 5.4 - Sum and Difference Formulas

1. Find the exact values of the sine, cosine, and tangent of the angle.

a. $165°$

b. $285°$

c. $\frac{17\pi}{12}$

d. $-\frac{19\pi}{12}$

e. $15°$

f. $-105°$

g. $\frac{5\pi}{12}$

h. $-\frac{13\pi}{12}$

2. Write the expression as the sine, cosine, or tangent of an angle.

a. $\sin 110° \cos 80° - \cos 110° \sin 80°$

b. $\dfrac{\tan 176° - \tan 86°}{1 + \tan 176° \tan 86°}$

c. $\cos 0.84\pi \cos 0.34\pi + \sin 0.84\pi \sin 0.34\pi$

d. $\sin\dfrac{4\pi}{9}\cos\dfrac{\pi}{8} + \cos\dfrac{4\pi}{9}\sin\dfrac{\pi}{8}$

3. Find the exact value of the trigonometric function given that $\sin u = \dfrac{5}{13}$ and $\cos v = -\dfrac{3}{5}$.

a. cos (v – u)

b. sin (u – v)

4. Find the exact value of the trigonometric function given that $\sin u = -\dfrac{7}{25}$ and $\cos v = -\dfrac{4}{5}$

a. tan (u + v)

b. cos (u – v)

5. Verify the identity.

a. cos(x+y) + cos (x – y) = 2cosxcosy

b. $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

5.5 – Multiple-Angle and Product –to – Sum Formulas

Find the exact values of sin2u, cos2u, and ~~tan2u~~ using the double-angle formulas.

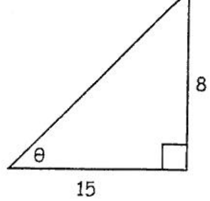
1. $\cos u = -\dfrac{2}{7},\ \dfrac{\pi}{2} < u < \pi$

2. $\cot u = -6,\ \dfrac{3\pi}{2} < u < 2\pi$

3. $\csc u = 3,\ \dfrac{\pi}{2} < u < \pi$

see answer key

Use the figure below to find the exact value of the trigonometric function.



4. $\sin\dfrac{\theta}{2}$

5. $\sec\dfrac{\theta}{2}$

6. $\cot\dfrac{\theta}{2}$

7. $2\cos\dfrac{\theta}{2}\tan\dfrac{\theta}{2}$

Section 4.2 - Unit Circle

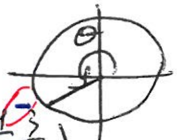
① a) $\left(-\frac{8}{17}, \frac{15}{17}\right)$

$\sin\theta = \frac{15}{17}$ $\qquad$ $\csc\theta = \frac{17}{15}$

$\cos\theta = \frac{-8}{17}$ $\qquad$ $\sec\theta = -\frac{17}{8}$

$\tan\theta = -\frac{15}{8}$ $\qquad$ $\cot\theta = -\frac{8}{15}$

① b)



$\left(-\frac{4}{5}, \frac{3}{5}\right)$

Typo

Should say $\left(-\frac{4}{5}, -\frac{3}{5}\right)$ since in QIII

$\sin\theta = \frac{-3}{5}$ $\qquad$ $\csc\theta = -\frac{5}{3}$

$\cos\theta = \frac{-4}{5}$ $\qquad$ $\sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

③ a) $\theta = \frac{-3\pi}{4}$

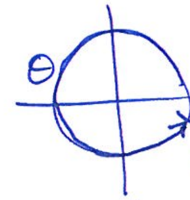
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin\theta = -\frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\tan\theta = 1$

③ c) $\theta = \frac{11\pi}{6}$



$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sin\theta = -\frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = -\frac{\sqrt{3}}{3}$

③ b) $\theta = -\frac{4\pi}{3}$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

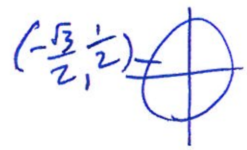
$\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{-1}{2}$

$\tan\theta = -\sqrt{3}$

Section 4.2 – Unit Circle

④ a) $\theta = \frac{5\pi}{6}$



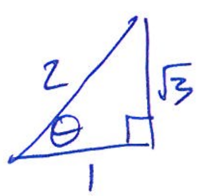
$$\boxed{\begin{array}{ll} \sin\theta = \frac{1}{2} & \csc\theta = 2 \\ \cos\theta = -\frac{\sqrt{3}}{2} & \sec\theta = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \\ \tan\theta = -\frac{\sqrt{3}}{3} & \cot\theta = -\sqrt{3} \end{array}}$$

④ b) $\theta = \frac{3\pi}{2}$

$$\boxed{\begin{array}{ll} \sin\theta = -1 & \csc\theta = -1 \\ \cos\theta = 0 & \sec\theta = \text{und.} \\ \tan\theta = \text{und.} & \cot\theta = 0 \end{array}}$$

Chapter 4 Review

① $\tan\theta = \sqrt{3} = \frac{opp}{adj}$



so...

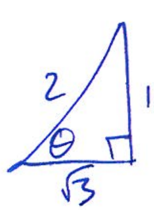
$$\boxed{\begin{array}{ll} \sin\theta = \frac{\sqrt{3}}{2} & \csc\theta = \frac{2\sqrt{3}}{3} \\ \cos\theta = \frac{1}{2} & \sec\theta = 2 \\ \tan\theta = \sqrt{3} & \cot\theta = \frac{\sqrt{3}}{3} \end{array}}$$

Note: $\theta = 60°$
30-60-90 Δ

② $\frac{1}{\csc\theta} = \frac{1}{2}$

$\csc\theta = 2$

$\sin\theta = \frac{1}{2} = \frac{opp}{hyp}$



so...

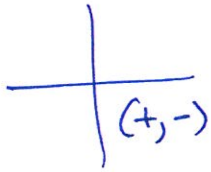
$$\boxed{\begin{array}{ll} \sin\theta = \frac{1}{2} & \csc\theta = 2 \\ \cos\theta = \frac{\sqrt{3}}{2} & \sec\theta = \frac{2\sqrt{3}}{3} \\ \tan\theta = \frac{\sqrt{3}}{3} & \cot\theta = \sqrt{3} \end{array}}$$

Note: $\theta = 30°$
30-60-90 Δ

Chapter 4 Review

(3) $\sec x > 0 \Rightarrow \cos x > 0$

$\sin x < 0$

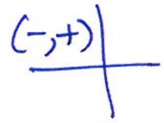


$\boxed{QIV}$

(4) $\csc x > 0 \Rightarrow \sin x > 0$

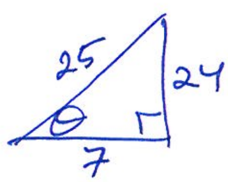
$\Downarrow$

$\tan x < 0 \Rightarrow \cos x < 0$



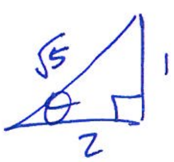
$\boxed{QII}$

(5) $\cot\theta = \frac{7}{24}$ and $\sec\theta < 0 \Rightarrow \cos\theta < 0$ and $\cot\theta > 0$ $\therefore$ QIII



$\sin\theta = \frac{24}{25}$ but QIII so $\boxed{\sin\theta = -\frac{24}{25}}$

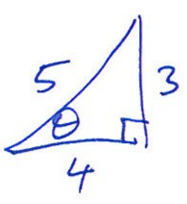
(10) $\tan\theta = \frac{1}{2}$ and $\sin\theta < 0 \Rightarrow \boxed{QIII}$



$$\boxed{\begin{array}{ll} \sin\theta = -\frac{\sqrt{5}}{5} & \csc\theta = -\sqrt{5} \\ \cos\theta = -\frac{2\sqrt{5}}{5} & \sec\theta = -\frac{\sqrt{5}}{2} \\ \tan\theta = \frac{1}{2} & \cot\theta = 2 \end{array}}$$

(11) $\sec\theta = \frac{-5}{4}$ and $\tan\theta < 0 \Rightarrow \boxed{QII}$

$\cos\theta = -\frac{4}{5}$



$$\boxed{\begin{array}{ll} \sin\theta = \frac{3}{5} & \csc\theta = \frac{5}{3} \\ \cos\theta = -\frac{4}{5} & \sec\theta = -\frac{5}{4} \\ \tan\theta = -\frac{3}{4} & \cot\theta = -\frac{4}{3} \end{array}}$$

Section 5.1 – Using Fundamental Identities

2a) $\cos\beta \tan\beta$

$\cancel{\cos\beta} \cdot \frac{\sin\beta}{\cancel{\cos\beta}}$

$\boxed{\sin\beta}$

2b) $\sec^2 x(1-\sin^2 x)$

$\frac{1}{\cancel{\cos^2 x}} \cancel{(\cos^2 x)}$

$\boxed{1}$

2c) $\frac{\sec\theta}{\csc\theta}$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1}$

$\boxed{\tan\theta}$

2d) $\frac{\tan^2\theta}{\sec^2\theta}$

$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{1}$

$\boxed{\sin^2\theta}$ *

or

$\boxed{\frac{1-\cos 2\theta}{2}}$

2e) $\cot\left(\frac{\pi}{2}-x\right)\cos x$

$\tan x \cdot \cos x$

$\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}$

$\boxed{\sin x}$

2f) $\frac{1}{\tan^2 x + 1}$

$\frac{1}{\sec^2 x}$

$\boxed{\cos^2 x}$ *

or

$\boxed{\frac{1+\cos 2x}{2}}$

* Lowering powers useful in some Calculus contexts.

Section 5.1 - Using Fundamental Identities

(4)
a) $\sec^2 x \tan^2 x + \sec^2 x$

$\sec^2 x(\tan^2 x + 1)$

$\sec^2 x \cdot \sec^2 x$

$\boxed{\sec^4 x}$

(4)
b) $\dfrac{\csc^2 x - 1}{\csc x - 1}$

$\dfrac{(\csc x+1)\cancel{(\csc x-1)}}{\cancel{(\csc x - 1)}}$

$\boxed{\csc x + 1}$

(4)
c) $1 - 2\sin^2 x + \sin^4 x$

$(1-\sin^2 x)^2$

$(\cos^2 x)^2$

$\boxed{\cos^4 x}$

(4)
d) $\sec^4 x - \tan^4 x$

$(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

$\boxed{\sec^2 x + \tan^2 x}$

$\tan^2 x + 1 = \sec^2 x$
$\therefore \sec^2 x - \tan^2 x = 1$

(4)
e) $(\sin x + \cos x)^2$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$\sin^2 x + \cos^2 x + 2\sin x \cos x$

$\boxed{1 + \sin 2x}$

5.3 – Solving Trig Equations

① $\sqrt{2}\sin x + 1 = 0$

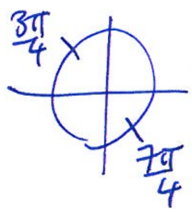
$\sin x = -\frac{1}{\sqrt{2}}$

$\sin x = -\frac{\sqrt{2}}{2}$

$$\boxed{x = \left\{ \frac{5\pi}{4} + 2k\pi,\ \frac{7\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \right\} \text{ or } x = \left\{ \frac{5\pi}{4}, \frac{7\pi}{4} \right\} \text{ if } x \in [0, 2\pi)}$$

② $\cot x + 1 = 0$

$\cot x = -1$

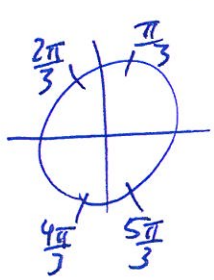


$$\boxed{x = \left\{ \frac{3\pi}{4} + k\pi,\ k \in \mathbb{Z} \right\} \text{ or } x = \left\{ \frac{3\pi}{4}, \frac{7\pi}{4} \right\} \text{ if } x \in [0, 2\pi)}$$

③ $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}$



$$\boxed{x = \left\{ \frac{\pi}{3} + k\pi,\ \frac{2\pi}{3} + k\pi,\ k \in \mathbb{Z} \right\} \text{ or } x = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} \text{ if } x \in [0, 2\pi)}$$

5.3 - Solving Trig Equns

④ $\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x - 1 = 0$

$\cos x = 1$

$x = \{\frac{\pi}{2} + k\pi,\ 2k\pi,\ k \in \mathbb{Z}\}$

or

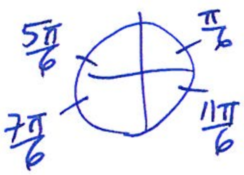
$x = \{0, \frac{\pi}{2}, \frac{3\pi}{2}\}$ if $x \in [0, 2\pi)$

⑤ $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$\tan^2 x = \frac{1}{3}$ or $\tan^2 x = 3$

$\tan x = \pm\frac{1}{\sqrt{3}} = \frac{\frac{1}{2}}{\sqrt{3}/2}$

$\tan x = \pm\sqrt{3} = \frac{\sqrt{3}/2}{1/2}$



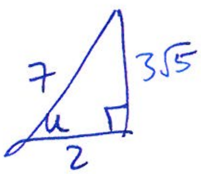
$x = \{\frac{\pi}{6} + k\pi,\ \frac{5\pi}{6} + k\pi,\ \frac{\pi}{3} + k\pi,\ \frac{2\pi}{3} + k\pi,\ k \in \mathbb{Z}\}$

or

$x = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\}$ if $x \in [0, 2\pi)$

5.5 – Multiple Angle and Product-to-Sum Formulas

① $\cos u = -\frac{2}{7}$, $\frac{\pi}{2} < u < \pi$
i.e. QII



$2^2 + n^2 = 7^2$
$4 + n^2 = 49$
$n^2 = 45$
$n = \sqrt{45}$
$n = 3\sqrt{5}$

$\sin 2u = 2\sin u \cos u$
$\sin 2u = 2 \cdot \frac{3\sqrt{5}}{7} \cdot \frac{-2}{7}$

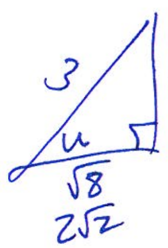
$\boxed{\sin 2u = \frac{-12\sqrt{5}}{49}}$

$\cos 2u = 2\cos^2 u - 1$
$= 2\left(-\frac{2}{7}\right)^2 - 1$
$= 2 \cdot \frac{4}{49} - 1$
$= \frac{8}{49} - 1$

$\boxed{\cos 2u = \frac{-41}{49}}$

② $\csc u = 3$ $\frac{\pi}{2} < u < \pi$
i.e. QII

$\sin u = \frac{1}{3}$



$\sin 2u = 2\sin u \cos u$
$= 2 \cdot \frac{1}{3} \cdot \frac{-2\sqrt{2}}{3}$

$\boxed{\sin 2u = \frac{-4\sqrt{2}}{9}}$

$\cos 2u = 1 - 2\sin^2 u$
$= 1 - 2\left(\frac{1}{3}\right)^2$
$= 1 - 2 \cdot \frac{1}{9}$
$= 1 - \frac{2}{9}$

$\boxed{\cos 2u = \frac{7}{9}}$

Exponential Functions & Graphs

5) D

6) A

7) C

8) B

Logarithmic Functions & Graphs

11) $x = 1$

12) $x = -1$

13) $x = 3\log_3 4$

19) B

20) C

21) D

22) A

23) 2

24) -1

25) 1.8

26) 0

Logarithmic Properties

4) $\log_2 4^2 + \log_2 3^4 = \log_2 16 + 4\log_2 3 = 4 + 4\log_2 3$

5) $\ln 6 - \ln e^2 = \ln(6) - 2$

6) $\ln 5 + \ln e^6 = \ln(5) + 6$

7) $\log 10 + \log X = 1 + \log X$

8) $\log Y - \log 2$

9) $-3\log_6 n$

10) $\frac{1}{3}\ln t$

Logarithmic Properties cont.

11) $\ln xy - \ln t = \ln x + \ln y - \ln t$

12) $\ln g + \ln (g-1)^2 = \ln g + 2\ln (g-1)$

13) $\ln \frac{\sqrt{x^2}}{\sqrt{y^3}} = \ln \sqrt{x^2} - \ln \sqrt{y^3} = \ln |x| - \ln y^{\frac{3}{2}} = \ln |x| - \frac{3}{2} \ln y$

14) $\ln x - \ln \sqrt{x^2+1} = \ln x - \ln (x^2+1)^{\frac{1}{2}} = \ln x - \frac{1}{2} \ln (x^2+1)$

15) $\ln \left(\sqrt{x^2} \cdot \sqrt{x+2}\right) = \ln \sqrt{x^2} + \ln \sqrt{x+2} = \ln |x| + \ln (x+2)^{\frac{1}{2}} = \ln |x| + \frac{1}{2} \ln (x+2)$

16) $\log_b \sqrt{x} + \log_b y^4 - \log_b z^4 = \frac{1}{2} \log_b x + 4 \log_b y - 4 \log_b z$

17) $\ln (xy)$

18) $\log_5 \frac{8}{t}$

19) $\log_7 (h-4)^{\frac{5}{2}} = \log_7 \sqrt{(h-4)^5}$

20) $\log_6 (2x)^{-6} = \log_6 \frac{1}{64x^6}$

21) $\ln 8^2 + \ln x^5 = \ln 64 + \ln x^5 = \ln 64x^5$

22) $\ln x^3 + \ln y^2 - \ln z^4 = \ln x^3y^2 - \ln z^4 = \ln \frac{x^3y^2}{z^4}$

23) $4 \ln x(x+5) - \ln (x-5)^2 = \ln [x(x+5)]^4 - \ln (x-5)^2 =$

24) $2\left[\ln \frac{x}{x+1} - \ln (x-1)\right] = 2 \ln \frac{x}{(x+1)(x-1)} = \ln \left(\frac{x}{(x+1)(x-1)}\right)^2 = \ln \frac{x^2}{(x+1)^2(x-1)^2}$

25) $\frac{1}{2}\left[\ln (x+1) + \ln (x-1)^2\right] + \ln x^3 = \frac{1}{2} \ln (x+1)(x-1)^2 + \ln x^3 = \ln (x+1)^{\frac{1}{2}}|x-1| + \ln x^3$

$= \ln (x+1)^{\frac{1}{2}}|x-1|x^3 = \ln \sqrt{x+1}\,|x-1|x^3$

$= \ln \sqrt{x+1}\,(x-1)x^3$

Note: We know $x-1>0$ b/c of they way the problem was stated, so no need for the absolute value

Logarithmic Properties cont.

26) $\log_6 6^{\frac{1}{3}} = \frac{1}{3}$

27) $\log_5 5^{-3} = -3$

28) DNE

29) $\log_4 (2 \cdot 32) = \log_4 64 = 3$

30) $6 - 2 \cdot 5 = 6 - 10 = -4$

31) 4.5

32) $\ln e^{\frac{3}{5}} = \frac{3}{5}$

Solving Exponential & Logarithmic Equations

1) $3^x = 3^5$

$x = 5$

2) $7^x = 7^{-2}$

$x = -2$

3) $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5}$

$x = -5$

4) $\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^3$

$x = 3$

5) $\ln x = -1$

$x = e^{-1} = \frac{1}{e}$

6) $\ln x = \ln 2$

$x = 2$

7) $\ln(3x+5) = 8$

$3x + 5 = e^8$

$3x = e^8 - 5$

$x = \frac{e^8 - 5}{3}$

8) $\log_x 25 = 2$

$x^2 = 25$

$x = \pm 5$

but the base of a log can't be negative

so $x = 5$

9) $\log x = -\frac{1}{2}$

$x = 10^{-\frac{1}{2}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

10) $e^x = 0$

$\emptyset$

Solving Exponential & Logarithmic Equations cont.

11) $6^{5x} = 3000$

$5x = \log_6 3000$

$x = \frac{1}{5}\log_6 3000$

12) $e^{2x} = 10$

$2x = \ln 10$

$x = \frac{1}{2}\ln 10$

13) $e^{-4x} = \frac{75}{1000}$

$e^{-4x} = \frac{3}{40}$

$-4x = \ln \frac{3}{40}$

$x = -\frac{1}{4}\ln \frac{3}{40}$

14) $3e^{x} = 25$

$e^{x} = \frac{25}{3}$

$x = \ln \frac{25}{3}$

15) $4^{-3t} = .1$

$-3t = \log_4 .1$

$t = -\frac{1}{3}\log_4 .1$

16) $-2 - x = \log_8 431$

$-x = \log_8(431) + 2$

$x = -\log_8(431) - 2$

17) $\frac{525}{275} = 1 + e^{-x}$

$\frac{21}{11} = 1 + e^{-x}$

$e^{-x} = \frac{10}{11}$

$-x = \ln \frac{10}{11}$

$x = -\ln \frac{10}{11}$

18) $16.034^{3t} = 30$

$3t = \log_{16.034} 30$

$t = \frac{1}{3}\log_{16.034} 30 \approx .409$

19) $x = e^{-2}$

20) $2x = e^{1.5}$

$x = \frac{1}{2}e^{1.5}$

Solving Exponential & Logarithmic Equations cont.

21) $2\ln x = 7$

$\ln x = \frac{7}{2}$

$x = e^{\frac{7}{2}}$

22) $x^2 = 10^6$

$x = \pm 10^3 = \pm 1000$

23) $\log(x-6) = \frac{11}{4}$

$x - 6 = 10^{\frac{11}{4}}$

$x = 10^{\frac{11}{4}} + 6$

24) $\sqrt{x-8} = e^5$

$x - 8 = e^{10}$

$x = e^{10} + 8$

25) $x^2 + 1 = e^8$

$x^2 = e^8 - 1$

$x = \pm\sqrt{e^8 - 1}$

26) $\log_3 x(x-8) = 2$

$x(x-8) = 3^2$

$x^2 - 8x = 9$

$x^2 - 8x - 9 = 0$

$(x-9)(x+1) = 0$

$x = 9, -1$

but $x = -1$ is extraneous!

so $x = 9$

27) $\ln \frac{x+1}{x-2} = \ln x$

$\frac{x+1}{x-2} = x$

$x + 1 = x^2 - 2x$

$0 = x^2 - 3x - 1$

$x = \frac{3 \pm \sqrt{9 - 4(1)(-1)}}{2(1)} = \frac{3 \pm \sqrt{13}}{2}$

But $\frac{3 - \sqrt{13}}{2} < 0$

so $x = \frac{3 + \sqrt{13}}{2}$

28) $\log \frac{4x^2}{x+4} = 2$

$\frac{4x^2}{x+4} = 10^2$

$\frac{4x^2}{x+4} = 100$

$4x^2 = 100x + 400$

$4x^2 - 100x - 400 = 0$

$x^2 - 25x - 100 = 0$

$x = \frac{25 \pm \sqrt{625 - 4(1)(-100)}}{2(1)}$

$x = \frac{25 \pm 5\sqrt{41}}{2}$

But $\frac{25 - 5\sqrt{41}}{2} + 4 < 0$

so $x = \frac{25 + 5\sqrt{41}}{2}$

Solving Exponential & Logarithmic Equations cont.

29) a) $600 = 5000\left(1 - \frac{4}{4+e^{-.0002x}}\right)$

$\frac{3}{25} = 1 - \frac{4}{4+e^{-.0002x}}$

$-\frac{22}{25} = -\frac{4}{4+e^{-.0002x}}$

$\frac{22}{25} = \frac{4}{4+e^{-.0002x}}$

$22(4+e^{-.0002x}) = 100$

$11(4+e^{-.0002x}) = 50$

$44 + 11e^{-.0002x} = 50$

$11e^{-.0002x} = 6$

$e^{-.0002x} = \frac{6}{11}$

$-.0002x = \ln\frac{6}{11}$

$x = \frac{\ln\frac{6}{11}}{-.0002x} \approx 3030.679$

b) $800 = 5000\left(1 - \frac{4}{4+e^{-.0002x}}\right)$

$\frac{4}{25} = \left(1 - \frac{4}{4+e^{-.0002x}}\right)$

$-\frac{21}{25} = -\frac{4}{4+e^{-.0002x}}$

$\frac{21}{25} = \frac{4}{4+e^{-.0002x}}$

$21(4+e^{-.0002x}) = 100$

$4 + e^{-.0002x} = \frac{100}{21}$

$e^{-.0002x} = \frac{16}{21}$

$-.0002x = \ln\frac{16}{21}$

$x = \frac{\ln\frac{16}{21}}{-.0002} \approx 1359.669$

Exponential & Logarithmic models

1) C

2) E

3) B

4) A

5) D

6) F

Exponential & Logarithmic Models

7) a) $40000 = 20000e^{.105t}$

$2 = e^{.105t}$

$.105t = \ln 2$

$t = \frac{\ln 2}{.105} \approx 6.601$ yrs

b) $A = 20000e^{.105 \cdot 10} \approx \$57153.02$

8) a) $1505 = 600e^{10k}$

$e^{10k} = \frac{301}{120}$

$10k = \ln \frac{301}{120}$

$k = \frac{1}{10} \ln \frac{301}{120} \approx .092$

about 9.2%

b) $1800 = 600e^{.092t}$

$3 = e^{.092t}$

$.092t = \ln 3$

$t = \frac{\ln 3}{.092} \approx 11.946$ yrs

9) $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{5730}}$

$A = 3\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

$A = 3\left(\frac{1}{2}\right)^{\frac{1000}{5730}} \approx 2.658g$

10) $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{1600}}$

$1.5 = A_0 \left(\frac{1}{2}\right)^{\frac{1000}{1600}}$

$1.5 = A_0 \left(\frac{1}{2}\right)^{\frac{5}{8}}$

$A_0 = \frac{1.5}{\left(\frac{1}{2}\right)^{\frac{5}{8}}} \approx 2.313g$

Exponential & Logarithmic Models cont.

11) a)

$y = ae^{bx}$

$y = e^{bx}$

$10 = e^{3b}$

$3b = \ln 10$

$b = \frac{\ln 10}{3}$

$y = e^{\frac{\ln 10}{3}x}$

b)

$y = ae^{bx}$

$y = \frac{1}{2}e^{bx}$

$5 = \frac{1}{2}e^{4b}$

$10 = e^{4b}$

$4b = \ln 10$

$b = \frac{\ln 10}{4}$

$y = \frac{1}{2}e^{\frac{\ln 10}{4}x}$

c)

$y = ae^{bx}$

$y = e^{bx}$

$\frac{1}{4} = e^{3b}$

$3b = \ln \frac{1}{4}$

$b = \frac{\ln \frac{1}{4}}{3}$

$y = e^{\frac{\ln \frac{1}{4}}{3}x}$

12)

$N = 250e^{kt}$

$280 = 250e^{10k}$

$\frac{28}{25} = e^{10k}$

$10k = \ln \frac{28}{25}$

$k = \frac{1}{10}\ln \frac{28}{25}$

$500 = 250e^{\frac{1}{10}\ln \frac{28}{25}t}$

$2 = e^{\frac{1}{10}\ln \frac{28}{25}t}$

$\frac{1}{10}\ln \frac{28}{25}t = \ln 2$

$t = \frac{\ln 2}{\frac{1}{10}\ln\left(\frac{28}{25}\right)} \approx 61.163$ hrs

$$8^{2x+1} = 4^{1-x}$$

$$(2^3)^{2x+1} = (2^2)^{1-x}$$

$$6x+3 = 2-2x$$

$$8x+3 = 2$$

$$8x = -1$$

$$x = -\frac{1}{8}$$

Review

6) 5

7) 4

8) $\frac{1}{3}$

9) $\ln \frac{1}{e^{\frac{7}{2}}} = \ln e^{-\frac{7}{2}} = -\frac{7}{2}$

12) $\ln x + \ln y - \ln z$

13) $\log_3 \sqrt{x} + \log_3 y^4 - \log_3 z^4 = \log_3 x^{\frac{1}{2}} + 4\log_3 y - 4\log_3 z$

$= \frac{1}{2}\log_3 x + 4\log_3 y - 4\log_3 z$

14) $\ln 8^2 + \ln z^5 = \ln 64 + \ln z^5 = \ln 64z^5$

15) $3[\ln(x-1) + \ln(x+1)^2] - \ln(x^2-1)^2 = 3\ln(x-1)(x+1)^2 - \ln(x+1)^2(x-1)^2$

$= \ln(x-1)^3(x+1)^6 - \ln(x+1)(x-1)^2 = \ln \frac{(x-1)^3(x+1)^6}{(x+1)(x-1)^2} = \ln(x-1)(x+1)^5$

32) This is a repeat!

33) $2.5 = 10(1-e^{5k})$

$\frac{1}{4} = 1-e^{5k}$

$-\frac{3}{4} = -e^{5k}$

$\frac{3}{4} = e^{5k}$

$5k = \ln \frac{3}{4}$

$k = \frac{1}{5}\ln\frac{3}{4}$

$S = 10(1-e^{\frac{1}{5}\ln(\frac{3}{4})x})$

$S = 10(1-e^{\frac{1}{5}\ln(\frac{3}{4})\cdot 7}) \approx 3.315$ so 3,315 units

36) E

37) C

38) A

39) B

More Equation Practice

1) $x+2=8$

$x=6$

2) $2x+3=3$

$2x=0$

$x=0$

3) $\log_3 x^{\frac{1}{2}} = \log_3 2^2$

$x^{\frac{1}{2}} = 2^2$

$\sqrt{x} = 4$

$x = 16$

4) $\log_4 x^{-2} = \log_4 9$

$x^{-2} = 9$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$

but $x>0$ so $x=\frac{1}{3}$

5) $\log_5 x^2 = \log_5 4^3$

$x^2 = 4^3$

$x^2 = 64$

$x = \pm 8$

but $x>0$ so $x=8$

6) $\log_2 x^3 = \log_2 27^{-1}$

$x^3 = 27^{-1}$

$x^3 = \frac{1}{27}$

$x = \frac{1}{3}$

7) $3\log_2(x-1)+2=5$

$3\log_2(x-1)=3$

$\log_2(x-1)=1$

$x-1=2$

$x=3$

8) $2\log_3(x+4)-2=2$

$2\log_3(x+4)=4$

$\log_3(x+4)=2$

$x+4=3^2$

$x+4=9$

$x=5$

9) $\log x(x+15)=2$

$x(x+15)=10^2$

$x^2+15x=100$

$x^2+15x-100=0$

$(x+20)(x-5)=0$

$x=5,-20$

but $x>0$ so $x=5$

10) $\log_4 x(x-3)=1$

$x(x-3)=4^1$

$x^2-3x=4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4,-1$

but $x>0$ so $x=4$

More Equation Practice

11) $\ln \frac{x+1}{x} = 2$

$$\frac{x+1}{x} = e^2$$

$$x+1 = e^2 x$$

$$1 = e^2 x - x$$

$$1 = x(e^2 - 1)$$

$$x = \frac{1}{e^2 - 1}$$

12) $x = \log_2 10$

13) $3^x = 14$

$$x = \log_3 14$$

14) $-x = \log_8 1.2$

$$x = -\log_8 1.2$$

15) $-x = \log_2 1.5$

$$x = -\log_2 1.5$$

16) $1 - 2\cancel{x} = \log_3 4^x$

$$1 - 2x = x\log_3 4$$

$$1 = 2x + x\log_3 4$$

$$1 = x(2 + \log_3 4)$$

$$x = \frac{1}{2 + \log_3 4}$$

17) $\log_7 \left(\frac{3}{5}\right)^x = 1 - x$

$$x \log_7 \left(\frac{3}{5}\right) = 1 - x$$

$$x + x\log_7\left(\frac{3}{5}\right) = 1$$

$$x\left(1 + \log_7\left(\frac{3}{5}\right)\right) = 1$$

$$x = \frac{1}{1 + \log_7\left(\frac{3}{5}\right)}$$

18) $1 - x = \log_{\frac{4}{3}} 5^x$

$$1 - x = x\log_{\frac{4}{3}} 5$$

$$1 = x + x\log_{\frac{4}{3}} 5$$

$$1 = x(1 + \log_{\frac{4}{3}} 5)$$

$$x = \frac{1}{1 + \log_{\frac{4}{3}} 5}$$

More Equation Practice cont

19) $1.2^x = 2^x$

$\log_2 1.2^x = x$

$x \log_2 1.2 = x$

$x - x\log_2 1.2 = 0$

$x(1 - \log_2 1.2) = 0$

$x = 0$

20) $.3^{1+x} = 1.7^{2x-1}$

$1+x = \log_{.3} 1.7^{2x-1}$

$1+x = (2x-1)\log_{.3} 1.7$

$1+x = 2x\log_{.3} 1.7 - \log_{.3} 1.7$

$x - 2x\log_{.3} 1.7 = -1 - \log_{.3} 1.7$

$x(1 - 2\log_{.3} 1.7) = -1 - \log_{.3} 1.7$

$x = \dfrac{-1 - \log_{.3} 1.7}{1 - 2\log_{.3} 1.7}$

21) $\ln \pi^{1-x} = x$

$(1-x)\ln\pi = x$

$\ln\pi - x\ln\pi = x$

$\ln\pi = x + x\ln\pi$

$\ln\pi = x(1+\ln\pi)$

$x = \dfrac{\ln\pi}{1+\ln\pi}$

22) $\pi^x = e^{x+3}$

$\ln\pi^x = x+3$

$x\ln\pi = x+3$

$x\ln\pi - x = 3$

$x(\ln\pi - 1) = 3$

$x = \dfrac{3}{\ln\pi - 1}$

23) $2^{3x} = \frac{8}{5}$

$3x = \log_2\left(\frac{8}{5}\right)$

$x = \frac{1}{3}\log_2\left(\frac{8}{5}\right)$

24) $4^{.2x} = \frac{2}{3}$

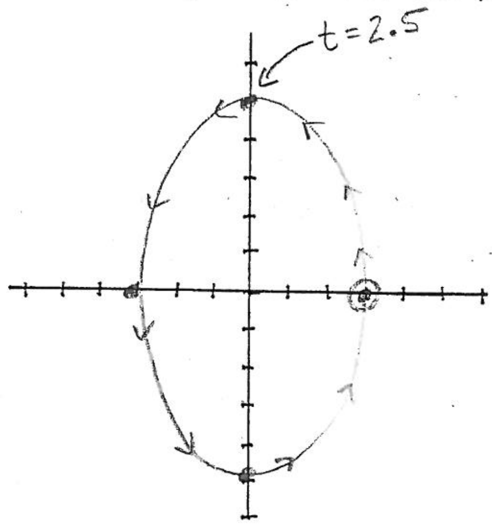
$.2x = \log_4\left(\frac{2}{3}\right)$

$x = 5\log_4\left(\frac{3}{2}\right)$

Key

Parametrics

1. During the time period from t = 0 to t = 6 seconds, a particle moves along the path given by $x(t) = 3\cos(\pi t)$ and $y(t) = 5\sin(\pi t)$.
   a. Find the position of the particle when t = 2.5.
   b. On the axes provided, sketch the graph of the path of the particle from t = 0 to t = 6. Indicate the direction of the particle along its path.
   c. How many times does the particle pass through the point found in part (a)?



a. $t = 2.5 \Rightarrow x = 0$, $y = 5$

b. ↻

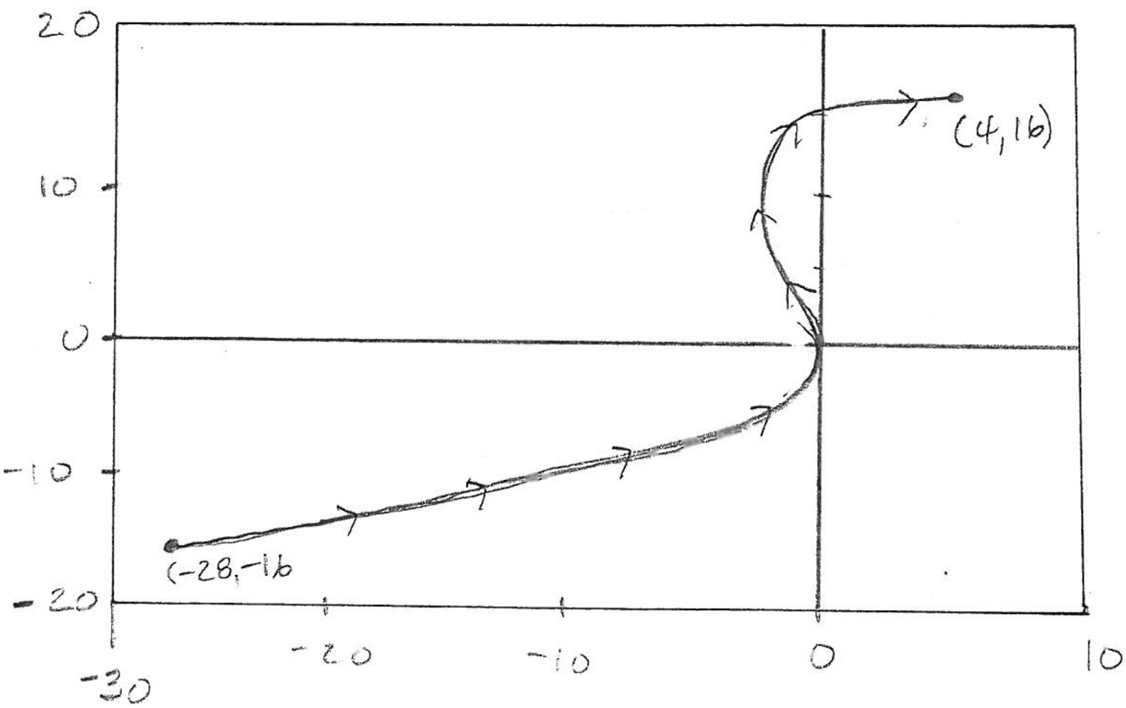
c. 3

One complete trip for $0 \le t \le 2$.

2. Consider the curve given by the parametric equations $x(t) = 2t^3 - 3t^2$ and $y(t) = -t^3 + 12t$ for $-2 \le t \le 2$.

   Find window dimensions suitable for viewing a complete graph. Label the window dimensions you used, sketch the graph, and indicate the direction of motion of a particle moving on this curve for $t \in [-2, 2]$.

Use table to help det window



Key FOWK handout

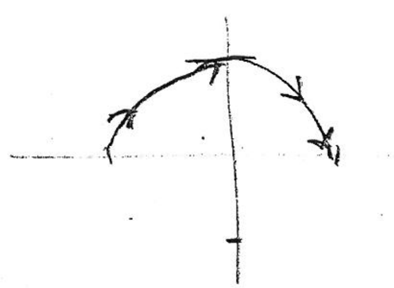
1. c $x \in [-4, 4]$ $t \in [0, 2\pi]$
   $y \in [-3, 3]$

2. a $x \in [-2, 2]$ $t \in [0, 2\pi]$
   $y \in [-2, 2]$

3. d $x \in [-10, 10]$ $t \in [0, 2\pi]$
   $y \in [-10, 10]$

4. b $x \in [-15, 15]$ $t \in [0, 2\pi]$
   $y \in [-15, 15]$

9. $x = \cos(\pi - t) \longrightarrow x = -\cos t$ $0 \le t \le \pi$
   $y = \sin(\pi - t)$ $y = \sin t$



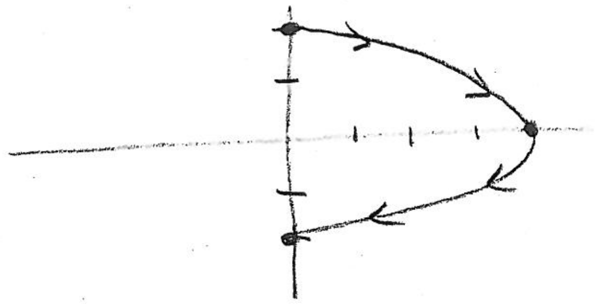
$x^2 + y^2 = 1$

FDWK handout

11. $x = 4\sin t \qquad 0 \le t \le \pi$

$y = 2\cos t$

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$



16. $x = \sec^2 t - 1 \qquad -\pi/2 < t < \pi/2$

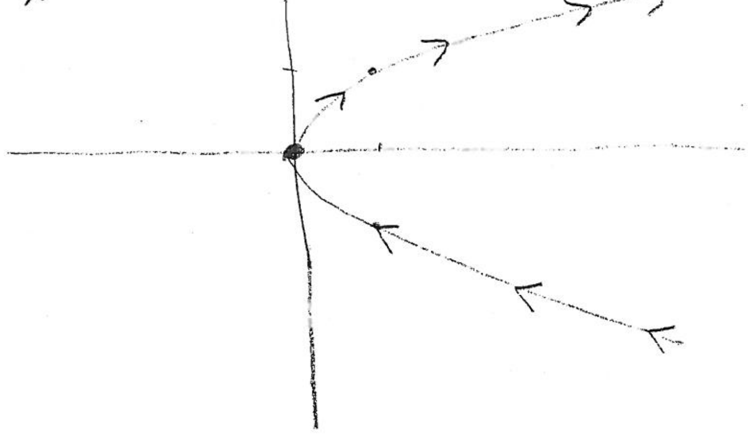
$y = \tan t$

$$\sec^2 t - 1 = \tan^2 t$$

$$x = y^2$$

R_y: $-\infty < y < \infty$

R_x: $x \ge 0$



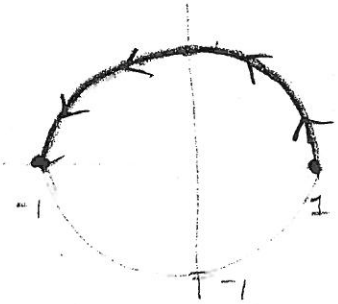
t	0	$\pi/4$
x	0	1
y	0	1

Handout
FDWK

7. $x = \cos t$

$y = \sin t$

$0 \le t \le \pi$

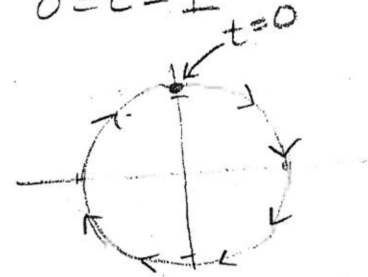


$x^2 + y^2 = 1$

8. $x = \sin(2\pi t)$ per $= 1$

$y = \cos(2\pi t)$

$0 \le t \le 1$

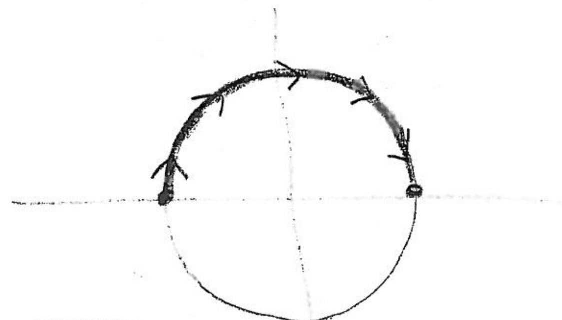


exactly 1 rev.
$t \in [0,1]$

9. $x = \cos(\pi - t) = -\cos t$

$y = \sin(\pi - t) = \sin t$

$0 \le t \le \pi$



$x^2 + y^2 = 1$

10. $x = 4\cos t$

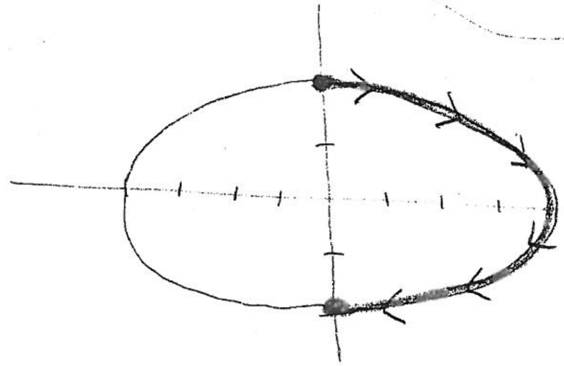
$y = 2\sin t$ 1 complete rev

$0 \le t \le 2\pi$

11. $x = 4\sin t$

$y = 2\cos t$

$0 \le t \le \pi$

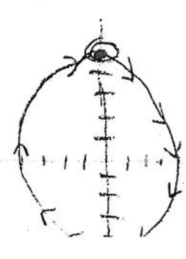


12. $x = 4\sin t$

$y = 5\cos t$

$0 \le t \le 2\pi$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

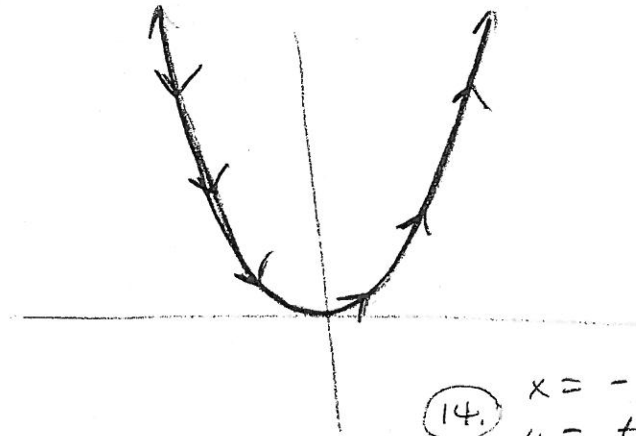


Hamilton
FDWK

(13.) $x = 3t$

$y = 9t^2$

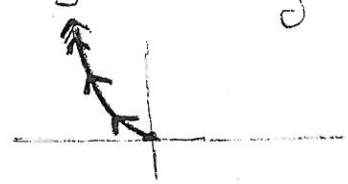
$-\infty < t < \infty$



$y = x^2$

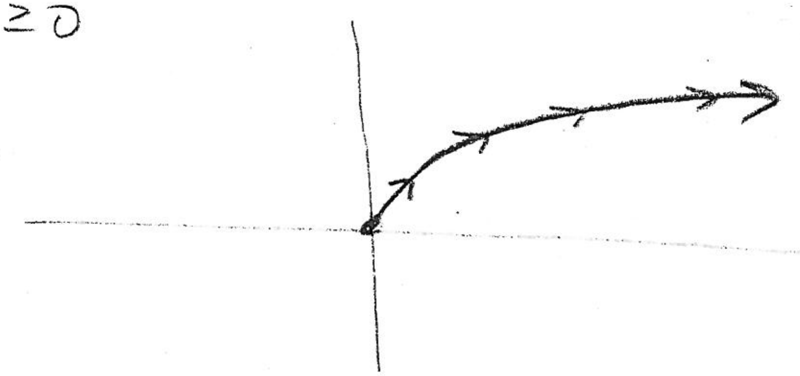
(14.) $x = -\sqrt{t}$ $t \geq 0$

$y = t$ $y = x^2$



(15.) $x = t$ $t \geq 0$

$y = \sqrt{t}$

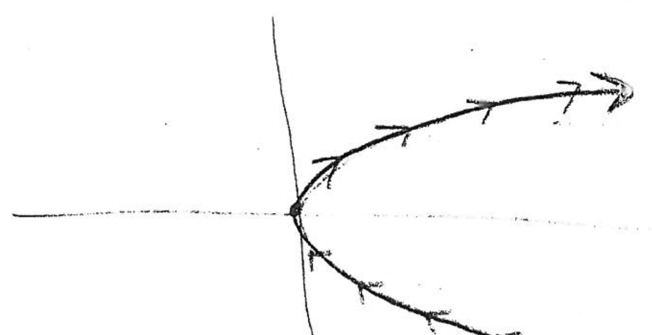


$y = \sqrt{x}$

(16.) $x = \sec^2 t - 1$

$y = \tan t$

$-\pi/2 < t < \pi/2$



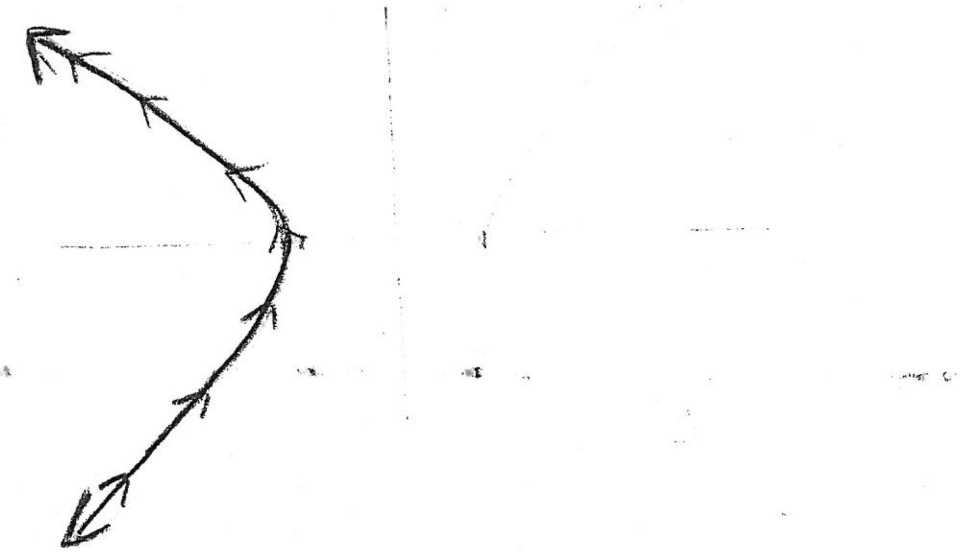
$y^2 = x$

Handout FDWK

(17.) $x = -\sec t$

$y = \tan t$

$-\pi/2 < t < \pi/2$



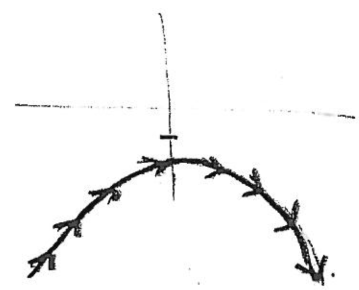
$x^2 - y^2 = 1$

(18.) $x = \tan t$

$y = -2\sec t$

$-\pi/2 < t < \pi/2$

$\frac{y^2}{4} - x^2 = 1$



(19.) $x = 2t - 5$

$y = 4t - 7$

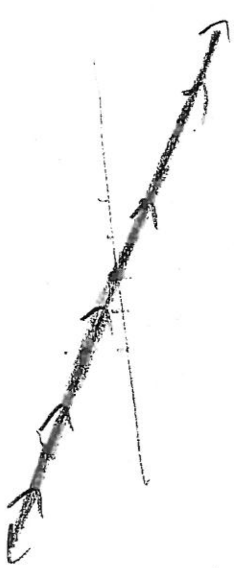
$-\infty < t < \infty$

$2t = x + 5$

$t = \frac{x+5}{2}$

$y = 4\left(\frac{x+5}{2}\right) - 7$

$\boxed{y = 2x + 3}$



(20.) $x = 1 - t$

$y = 1 + t$

$-\infty < t < \infty$

$x + y = 2$

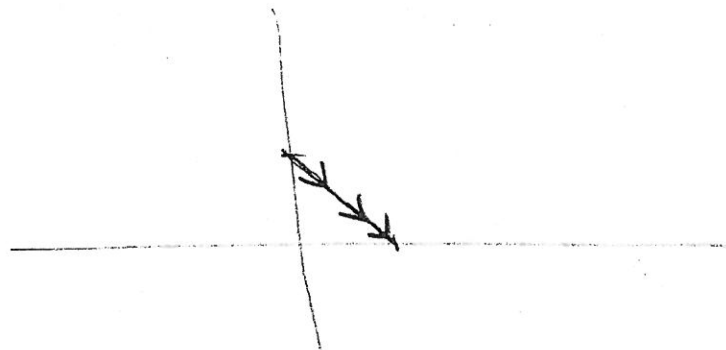
$y = -x + 2$

Handout
FDWK

21.
$x = t$ $\quad 0 \le x \le 1$
$y = 1 - t$ $\quad 0 \le y \le 1$

$0 \le t \le 1$

$\boxed{y = 1 - x}$

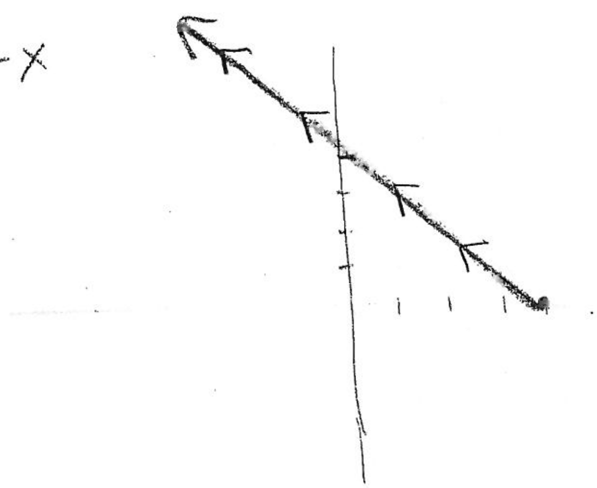


22.

23.
$x = 4 - \sqrt{t}$ $\quad t \ge 0 \Longrightarrow R_x : x \le 4$
$y = \sqrt{t}$ $\quad R_y : y \ge 0$

good

$y = 4 - x$



22.
$x = 3 - 3t$
$y = 2t$

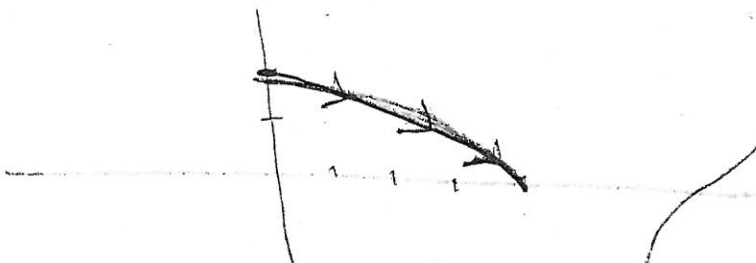
$0 \le t \le 1$

$x = 3 -$

24. $x = t^2$

$y = \sqrt{4 - t^2}$ $\qquad$ $y = \sqrt{4 - x}$

$0 \le t \le 2$



25. $x = \sin t$

$y = \cos 2t$

$t \in \mathbb{R}$

$R_x: [-1, 1]$

$R_y: [-1, 1]$

$y = 1 - 2\sin^2 t$

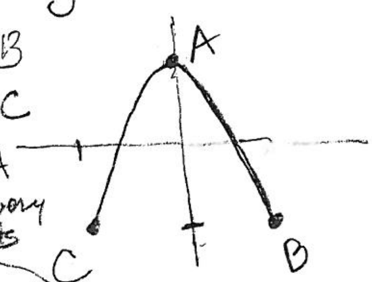
$y = 1 - 2x^2$

$[0, \pi/2]$ $A \to B$

$[\pi/2, \frac{3\pi}{2}]$ $B \to C$

$[\frac{3\pi}{2}, 2\pi]$ $C \to A$

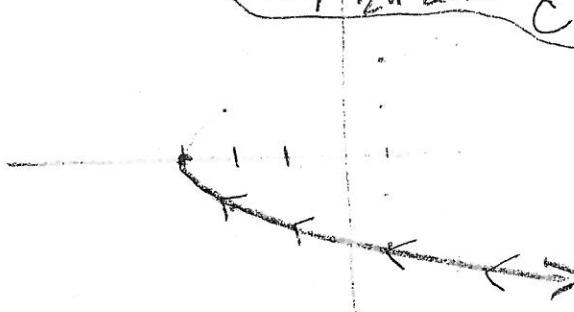
Repeats every 2π units



26. $x = t^2 - 3$

$y = t$

$t \le 0$



$x = y^2 + 3$

$y^2 = x - 3$

How does FOWK

(25.)

$x = \sin t$

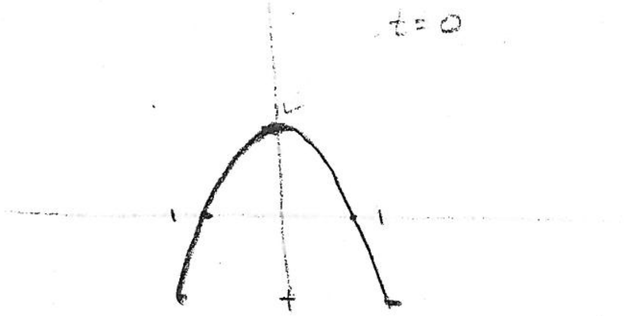
$y = \cos 2t$

$-\infty < t < \infty$

$y = 1 - 2\sin^2 t$

$y = 1 - 2x^2$

$y = -2x^2 + 1$



$-1 \le x \le 1$

$-1 \le y \le 1$

For what value(s) of t does particle cross x-axis?

$\cos 2t = 0$?

$2t = \pi/2, 3\pi/2$

$t = \pi/4, 3\pi/4$

Variation:

$x = \cos t$

$y = \cos 2t$

$y = 2x^2 - 1$

(27.) Line Segment: Endpts $(-1,-3)$, $(4,1)$

$x = -1 + at$

$y = -3 + bt$

Want $t=1$ to determine $(4,1)$,

So $4 = -1 + a$ and $1 = -3 + b$

$a = 5$. $b = 4$

$$\boxed{\begin{aligned} x &= -1 + 5t \\ y &= -3 + 4t \quad 0 \le t \le 1 \end{aligned}}$$

OR $m = \frac{4}{5}$

$x = t - 1$

$y = \frac{4}{5}t - 3$

$0 \le t \le 5$

$x = 2t - 1$

$y = \frac{8}{5}t - 3$

$0 \le t \le 5/2$

Handout
FRWK

29. $x - 1 = y^2$ — Bottom half

$x = t$

$y = -\sqrt{t-1}$

$t \geq 1$

30. $y = x^2 + 2x$ — Left half

$y = x(x+2)$

3 ways

i.

$x = t$

$y = t^2 + 2t$

$t \leq -1$

ii.

$x = t - 1$

$y = (t-1)^2 + 2(t-1) = t^2 - 2t + 1 + 2t - 2$

$y = t^2 - 1$

$t \leq 0$

iii.

$x = -1 - t$

$y = (-1-t)^2 + 2(-1-t)$

$y = t^2 - 1$

$t \geq 0$

38 Line Eq. : Endpts. (-1, 3) + (3, -2)

FDWK handout

$\text{slope} = \frac{5}{-4}$

$0 \le t \le 4$

$$\boxed{\begin{aligned} x &= -1 + t \\ y &= 3 - \tfrac{5}{4}t \end{aligned}}$$

$y - 3 = -\frac{5}{4}(x+1)$

$y = -\frac{5}{4}x - \frac{5}{4} + \frac{12}{4}$

$y = -\frac{5}{4}x + \frac{7}{4}$

$$\boxed{\begin{aligned} x &= t \\ y &= -\tfrac{5}{4}t + \tfrac{7}{4} \end{aligned}}$$

$x = -1 + at$

$y = 3 + bt$

$3 = -1 + a, \quad a = 4$

$-2 = 3 + b, \quad b = -5$

$$\boxed{\begin{aligned} x &= -1 + 4t \\ y &= 3 - 5t \end{aligned}}$$

$$\boxed{\begin{aligned} x &= 3 + 4t \\ y &= -2 - 5t \end{aligned}}$$

Many More.

FDWK

31. Ray
Initial pt. (2, 3)
Passing thru (−1, −1)

$x = 2 + at$
$y = 3 + bt$

To go thru (−1, −1) at $t = 1$:

$-1 = 2 + a$ $\quad$ $-1 = 3 + b$
$a = -3$ $\quad$ $b = -4$

$$\boxed{\begin{aligned} x &= 2 - 3t \\ y &= 3 - 4t \\ t &\geq 0 \end{aligned}}$$

32. Ray
Initial pt. (−1, 2)
passing through (0, 0)

$x = -1 + at$
$y = 2 + bt$

$0 = -1 + a$
$a = 1$
$0 = 2 + b$

$$\boxed{\begin{aligned} x &= -1 + t \\ y &= 2 - 2t \\ t &\geq 0 \end{aligned}}$$

33-36

$x = 3 - |t|$ $\quad -5 \le t \le 5$

$y = t - 1$

t	-5	-4	-3	-2	-1	0	1	2	3	4	5
x	-2	-1	0	1	2	3	2	1	0	-1	-2
y	-6	-5	-4	-3	-2	-1	0	1	2	3	4

Quad. III: $-5 \le t < -3$

Quad. IV: $-3 < t < 1$

Quad. I: $1 < t < 3$

Quad. II: $3 < t \le 5$