To: Rising Calculus students
From: Your Math Teachers
Re: Optional Summer Work

Attached are some materials that you might find helpful in gearing up for your class next year. We want you to know two important things about these materials:

- Please know that we do not expect students to *have* to do this work in order to be prepared for class – if you had a good understanding (you didn't just memorize and mimic procedures but instead understood the concepts) of course content from last year, you should be in a good position for next year's class. But if you could use some additional understanding and/or practice, or if you want to refresh your memory and make sure your skills are sharp, this is meant to be helpful.
- Also, please understand that this packet is not, by any means, "the ticket" to strong achievement in math class next year. What's really important is to develop and use good study skills throughout each unit – adequately preparing nightly homework, sustaining your effort when initially stuck, participating and pushing yourself to understand during class, and thoughtfully studying for unit tests.

We've prepared a few handouts covering important content from last year. We will post solutions to these problems on the math department website by mid-summer. In addition, the Khan Academy website has helpful explanatory videos and problems sets:

To practice trig: https://www.khanacademy.org/math/trigonometry (in particular the Unit Circle and Trig Identities sections)

To practice parametric equations:
https://www.khanacademy.org/math/precalculus/parametric_equations

To practice exponential and logarithmic functions:
https://www.khanacademy.org/math/algebra2/exponential_and_logarithmic_func

We hope you make the time to do many non-math activities over the summer too – to relax, and to refresh and rejuvenate yourself. Enjoy!

Trigonometry Worksheet

Do the circled ones

1. (a) Convert 75° and 225° to radians.

(b) Convert $\dfrac{\pi}{15}$ and $\dfrac{7\pi}{9}$ to degrees.

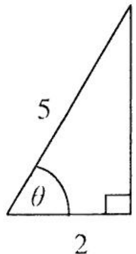
2. Fill in the table from memory.

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
$0 = 0^\circ$			
$\frac{\pi}{6} = 30^\circ$			
$\frac{\pi}{4} = 45^\circ$			
$\frac{\pi}{3} = 60^\circ$			
$\frac{\pi}{2} = 90^\circ$			
$\pi = 180^\circ$			
$\frac{3\pi}{2} = 270^\circ$			
$\frac{5\pi}{6} = 150^\circ$			
$\frac{4\pi}{3} = 240^\circ$			

3. Given that $\tan\theta = 3$, find the exact values of the remaining five trigonometric functions of θ. [Hint: draw the appropriate triangle.]

4. Find the cosine, sine, and tangent of θ.

(a)



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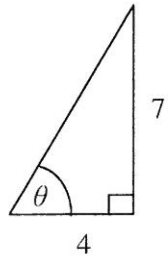


5. (a) Find all values of θ between 0 and 2π (in radians) such that $4\sin^2\theta - 2 = 0$.

(b) Find all values of θ between 0 and 2π (in radians) such that $\sin\theta = \cos\theta$.

6. Find the difference quotient of $f(x) = \sin x$.

2. (a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ ___________

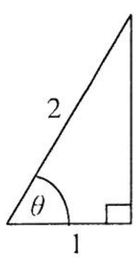
(b) $\sin^{-1}(1) =$ ___________

(c) $\tan^{-1}(1) =$ ___________

(d) $\sin^{-1}\left(-\frac{1}{2}\right) =$ ___________

3. Find θ.

(a)



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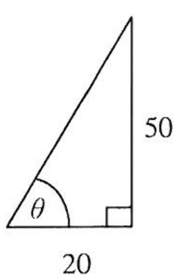


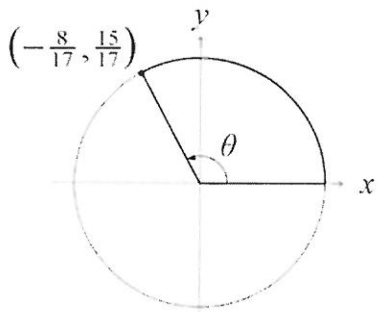
6. Find the length of the arc on a circle with a radius of 9 feet and a central angle of 60°.

7. A car is moving at a rate of 40 miles per hour, and the diameter of its wheels is 2.5 feet.
   (a) Find the linear speed of the tires in feet per minute.
   (b) Find the number of revolutions per minute the wheels are rotating.
   (c) Find the angular speed of the wheels in radians per minute.

Section 4.2 - Unit Circle

1. Determine the exact values of the six trigonometric functions of the angle θ.

(a)



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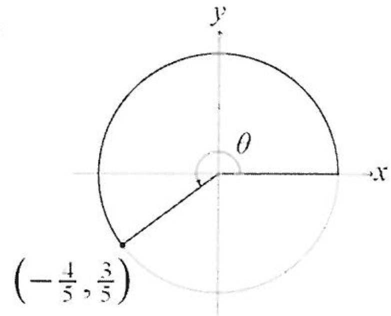


2. Find the point on the unit circle that corresponds to the angle θ.

(a) $\theta = \dfrac{\pi}{3}$ (b) $\theta = \dfrac{5\pi}{4}$ (c) $\theta = \pi$

3. Find the sine, cosine and tangent for the angle θ.

(a) $\theta = \dfrac{-3\pi}{4}$ (b) $\theta = \dfrac{-4\pi}{3}$ (c) $\theta = \dfrac{11\pi}{6}$

4. Find the six trigonometric functions for the angle θ.

(a) $\theta = \dfrac{5\pi}{6}$ (b) $\theta = \dfrac{3\pi}{2}$

5. Find the value of the given trigonometric function.

(a) $\cos 7\pi$ (b) $\sin \dfrac{9\pi}{4}$ (c) $\sin\left(-\dfrac{19\pi}{6}\right)$

6. For the given trigonometric function, find the value of the indicated function.

(a) Given $\cos\theta = -\dfrac{3}{4}$, find $\cos(-\theta)$ (b) Given $\sin(-\theta) = \dfrac{3}{8}$, find $\csc(\theta)$

7. A bocce ball suspended from a Slinky bobs up and down, but because of friction the ball moves up and down less with each cycle. This is called damped harmonic motion, and in this case the vertical position of the ball *y* (in feet) is given by the function $y(t) = \frac{1}{4}e^{-t}\cos 6t$, where *t* is the elapsed time (in seconds). Find the position of the ball at the following points in time.

(a) $t = 0$ (b) $t = \frac{1}{4}$ (c) $t = \frac{1}{2}$

For problems 4-9, sketch the graph of the function. Show two full periods.

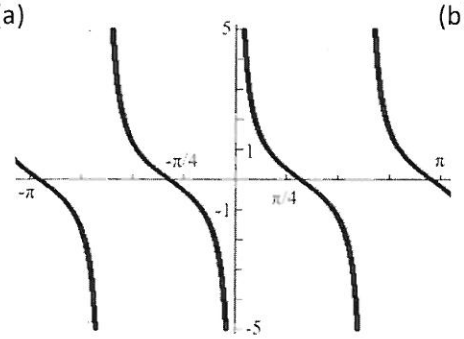
4. $y = \frac{1}{4}\sec x$ 5. $y = 2\csc \pi x$ 6. $y = -2\sec 4x + 2$ 7. $y = -\csc\frac{x}{3}$

8. $y = \sec(x+\pi)$ 9. $y = \csc(2x-\pi)$

Section 4.6 - Tangent and Cotangent Functions

For problems 1-3, match the function with its graph. Find the period of the function.

(a)



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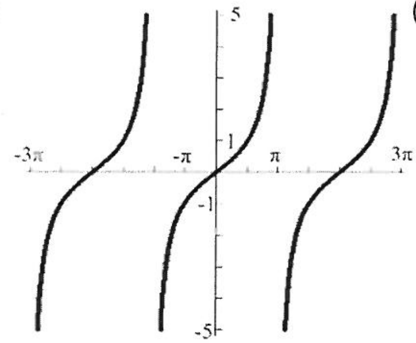

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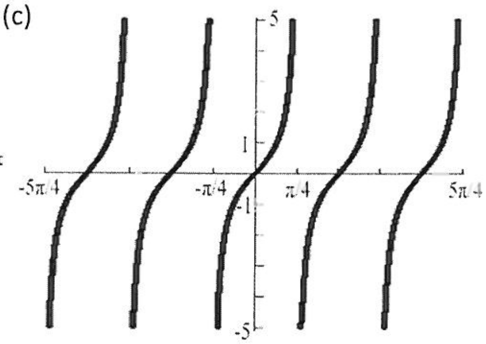


1. $y = \tan\frac{x}{2}$ 2. $y = \tan 2x$ 3. $y = \cot\frac{\pi x}{2}$

For problems 4-9, sketch the graph of the function. Show two full periods.

4. $y = \cot x$ 5. $y = -3\tan 4x$ 6. $y = 3\cot \pi x$ 7. $y = -\frac{1}{2}\tan \pi x$

8. $y = \frac{1}{4}\cot(x+\pi)$ 9. $y = \frac{1}{2}\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$

Chapter 4 Review

1. For a right triangle, $\tan\theta = \sqrt{3}$. Find the value of the other five trig functions.

2. For a right triangle, $\frac{1}{\csc\theta} = \frac{1}{2}$. Find the value of the six trig functions.

3. Name the quadrant that the angle *x* lies in given that sec *x* > 0, and sin *x* < 0.

4. Name the quadrant that the angle *x* lies in given that csc *x* > 0, and tan *x* < 0.

5. Suppose for an angle θ, cot θ = 7/24 and sec θ < 0. Find the exact value of sin θ.

6. Convert 75° to radian measure.

7. Convert -7π/4 to its exact degree measure and determine the quadrant of the terminal side of the angle.

8. Find the complement and supplement of π/5.

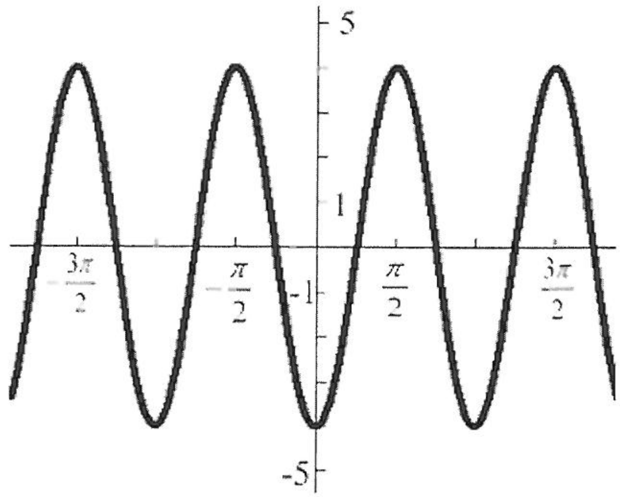
9. Find the reference angle for θ = -2π/3. Then find the exact values of the six trigonometric functions of θ=2π/3.

10. If tan θ = 1/2 and sin θ < 0, find the quadrant of θ and the exact values of the remaining five trigonometric functions of θ.

11. If sec θ = -5/4 and tan θ < 0, find the quadrant of θ and the exact values of the remaining five trigonometric functions of θ.

12. Identify the phase shift (horizontal shift), amplitude, and period of y = 4sin(3x – π).

13. Find the amplitude and period of the sinusoidal graph given, then write an equation of the graph.



14. Given that sec x = 3, find cot x.

15. An escalator 152 feet in length rises to a platform and makes a 37° angle with the ground. Find the height of the platform.

16. A man at the top of a ramp 30 feet in length looks down to the end of the ramp which rises to a loading platform 3 feet off the ground. For a safe ramp, the $\tan\theta \le 0.15$. Is this ramp safe?

17. cos 240° =

18. cot 13π/6 =

19. sin 120° =

20. tan π/4 =

21. sec 330° =

22. csc 5π/3 =

23. Use the fundamental identities to determine the simplified form of the expression.

a) cos θ csc θ =

b) tan θ cot θ =

c) sin θ cot θ =

24. Sketch one period of the graph of $f(x) = -\sin\,(x-\pi)+1$. Find the period, amplitude, and phase shift.

25. Sketch one period of the graph of $f(x) = 2\tan\,(x+\frac{\pi}{2})$. Find the period and phase shift.

26. Sketch one period of the graph of $f(x) = \sec\,(x-\frac{\pi}{4})-2$. Find the period and phase shift.

Chapter 5: Analytic Trigonometry

Essential Questions

How can we us different rules and relationships to represent the same function in different ways?

How can we represent the same angle in different ways mathematically?

Learning Targets

Simplify Trigonometric Functions

Develop Additional Trigonometric Identities

Solve Trigonometric Equations

Evaluate Trigonometric Functions

Homework

Section 5.1 – Using Fundamental Identities

1. Use the given values to evaluate all six trigonometric functions.

a. csc (-x) = - 5 b. $\sec\theta = -5$ c. $\tan\theta$ *is undefined*, $\sin\theta > 0$

2. Use the fundamental identities to simplify the expression.

a. $\cos\beta\tan\beta$ b. $\sec^2 x(1-\sin^2 x)$ c. $\dfrac{\sec\theta}{\csc\theta}$

d. $\dfrac{\tan^2\theta}{\sec^2\theta}$ e. $\cot\left(\dfrac{\pi}{2}-x\right)\cos x$ f. $\dfrac{1}{\tan^2 x+1}$

3. Verify the identity algebraically.

a. $(\sec\theta - \tan\theta)(\csc\theta + 1) = \cot\theta$

b. $\dfrac{1+\csc\theta}{\cot\theta + \cos\theta} = \sec\theta$

c. $\cos\theta\sec\theta - \cos^2\theta = \sin^2\theta$

d. $\dfrac{\tan\theta}{1+\sec\theta} + \dfrac{1+\sec\theta}{\tan\theta} = 2\csc\theta$

4. Use the fundamental identities to simplify.

a. $\sec^2 x\tan^2 x + \sec^2 x$

b. $\dfrac{\csc^2 x - 1}{\csc x - 1}$

c. $1 - 2\sin^2 x + \sin^4 x$

d. $\sec^4 x - \tan^4 x$

e. $(\sin x + \cos x)^2$

5.2 – Verifying Trigonometric Identities

Verify the Identity:

1. $\sec^6 x(\sec x\tan x) - \sec^4 x(\sec x\tan x) = \sec^5 x\tan^3 x$

2. $\dfrac{\cos\left[\frac{\pi}{2} - x\right]}{\sin\left[\frac{\pi}{2} - x\right]} = \tan x$

3. $(1+\sin y)\left[1+\sin(-y)\right] = \cos^2 y$

4. $\dfrac{\tan x + \tan y}{1 - \tan x\tan y} = \dfrac{\cot x + \cot y}{\cot x\cot y - 1}$

5. $\dfrac{\tan x + \cot y}{\tan x\cot y} = \tan y + \cot x$

6. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

7. $\dfrac{1+\csc\theta}{\sec\theta} - \cot\theta = \cos\theta$

8. $\sin x\left(1 - 2\cos^2 x + \cos^4 x\right) = \sin^5 x$

9. $\csc^4\theta - \cot^4\theta = 2\csc^2\theta - 1$

10. $\dfrac{\sin^3\beta + \cos^3\beta}{\sin\beta + \cos\beta} = 1 - \sin\beta\cos\beta$

5.3 – Solving Trigonometric Equations

Solve the equation

1. $\sqrt{2}\sin x + 1 = 0$
2. $\cot x + 1 = 0$
3. $3\cot^2 x - 1 = 0$
4. $\cos x(\cos x - 1) = 0$
5. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

Find all solutions of the equation in the interval [0, 2π) algebraically.

6. $\tan^2 x - 1 = 0$
7. $\sec x \csc x = 2\csc x$
8. $\sec x + \tan x = 1$
9. $\sin^2 x + \cos x + 1 = 0$
10. $2\sec^2 x + \tan^2 x - 3 = 0$
11. $\csc^2 x = 3\csc x + 4$

Solve the multiple angle equation.

12. $\sec 4x = 2$
13. $\tan^2 3x = 3$
14. $\tan\frac{x}{3} = 1$

Section 5.4 – Sum and Difference Formulas

1. Find the exact values of the sine, cosine, and tangent of the angle.

a. $165°$ b. $285°$ c. $\frac{17\pi}{12}$ d. $-\frac{19\pi}{12}$

e. $15°$ f. $-105°$ g. $\frac{5\pi}{12}$ h. $-\frac{13\pi}{12}$

2. Write the expression as the sine, cosine, or tangent of an angle.

a. $\sin 110° \cos 80° - \cos 110° \sin 80°$

b. $\dfrac{\tan 176° - \tan 86°}{1 + \tan 176° \tan 86°}$

c. $\cos 0.84\pi \cos 0.34\pi + \sin 0.84\pi \sin 0.34\pi$

d. $\sin\dfrac{4\pi}{9}\cos\dfrac{\pi}{8} + \cos\dfrac{4\pi}{9}\sin\dfrac{\pi}{8}$

3. Find the exact value of the trigonometric function given that $\sin u = \dfrac{5}{13}$ and $\cos v = -\dfrac{3}{5}$.

a. cos (v – u)

b. sin (u – v)

4. Find the exact value of the trigonometric function given that $\sin u = -\dfrac{7}{25}$ and $\cos v = -\dfrac{4}{5}$

a. tan (u + v)

b. cos (u – v)

5. Verify the identity.

a. cos(x+y) + cos (x – y) = 2cosxcosy

b. $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

5.5 – Multiple-Angle and Product –to – Sum Formulas

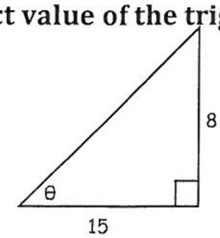
Find the exact values of sin2u, cos2u, and tan2u using the double-angle formulas.

1. $\cos u = -\dfrac{2}{7}, \ \dfrac{\pi}{2} < u < \pi$

2. $\cot u = -6, \ \dfrac{3\pi}{2} < u < 2\pi$

3. $\csc u = 3, \ \dfrac{\pi}{2} < u < \pi$

Use the figure below to find the exact value of the trigonometric function.



4. $\sin\dfrac{\theta}{2}$

5. $\sec\dfrac{\theta}{2}$

6. $\cot\dfrac{\theta}{2}$

7. $2\cos\dfrac{\theta}{2}\tan\dfrac{\theta}{2}$

Exponential & Logarithmic Functions

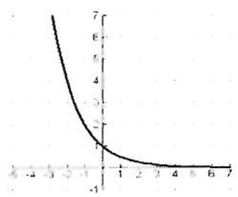
Exponential Functions & Graphs

Graph the function. Identify any asymptotes.

1. $f(x) = \left(\frac{3}{2}\right)^{x}$
2. $f(x) = \left(\frac{3}{2}\right)^{-x}$
3. $f(x) = \left(\frac{3}{2}\right)^{x+2}$
4. $f(x) = \left(\frac{3}{2}\right)^{-x} + 2$

For problems 5-8, match the function to the graph.

(a)



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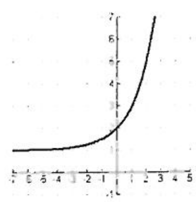

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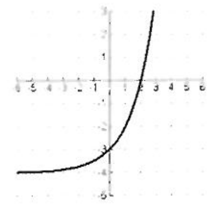

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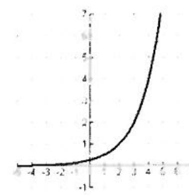


5. $f(x) = 2^{x-2}$
6. $f(x) = 2^{-x}$
7. $f(x) = 2^{x} - 4$
8. $f(x) = 2^{x} + 1$

Sketch the graph of the function.

9. $f(x) = \left(\frac{5}{2}\right)^{-x}$
10. $f(x) = 2^{x-1}$
11. $f(x) = e^{-x}$
12. $f(x) = 2e^{-0.5x}$
13. $f(x) = 4^{x-3} + 3$
14. $s(t) = 3e^{-0.2t}$
15. $g(x) = 1 + e^{-x}$

16. *Compound Interest*

Complete the table for balance *A* using the appropriate compound interest formula.

$P = \$1000$ $r = 6\%$ $t = 10$ years

n	1	2	4	12	365	Continuous
A						

17. *Radioactive Decay*

Let Q represent a mass of Carbon 14 (^{14}C), in grams, whose half-life is 5730 years. The quantity present after t years is given by $Q = 10\left(\frac{1}{2}\right)^{t/5730}$.

(a) Determine the initial quantity ($t = 0$)
(b) Determine the quantity present after 2000 years.
(c) Graph the function Q over the interval from $t = 0$ to $t = 10{,}000$
(d) When will this quantity of ^{14}C be 0 grams? Explain your answer.

18. *Population Growth*

The population of a town increases according to the model $P = 2500e^{0.0293t}$, where *t* is the time in years, with the year 2000 corresponding to *t* = 0 (e.g. This year is 2011).

(a) Graph the function for the years 2000 through 2025.
(b) Approximate the population in 2015 and 2025 using the graph.
(c) Verify your values in (b) using the model (i.e. evaluate the function at those times).

Logarithmic Functions & Graphs

Write the logarithmic equation in exponential form.

1. $\log_3 81 = 4$ 2. $\log_{10} \frac{1}{1000} = -3$ 3. $\log_{16} 8 = \frac{3}{4}$ 4. $\ln 4 = 1.386...$

Write the exponential equation in logarithmic form.

5. $8^2 = 64$ 6. $9^{3/2} = 27$ 7. $10^{-3} = 0.001$ 8. $e^x = 4$

Evaluate the function at the value of *x* without using a calculator.

9. $f(x) = \log_{16} x \;\; at \;\; x = \frac{1}{4}$ 10. $f(x) = \log_{10} x \;\; at \;\; x = 10$

Solve the equation.

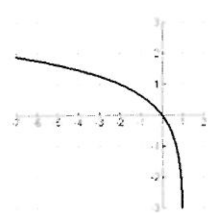
11. $\log_5 5 = x$ 12. $\log_2 2^{-1} = x$ 13. $\log_3 4^3 = x$

Sketch the graph of each function. Make sure to show the *x*-intercept and vertical asymptote.

14. $g(x) = \log_6 x$ 15. $g(x) = \log_2(-x)$ 16. $g(x) = -\log_6(x+2)$

17. $y = \log_{10}(x-1) + 4$ 18. $f(x) = -\log_3(x+2) - 4$

For problems 19-22, match the function to the graph.

(a)

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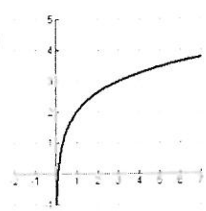
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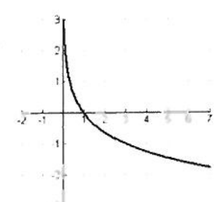
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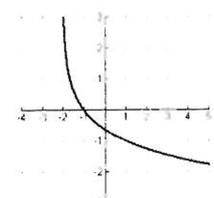


19. $f(x) = \log_3 x + 2$ 20. $f(x) = -\log_3 x$ 21. $f(x) = -\log_3(x+2)$

22. $f(x) = \log_3(1-x)$

Simplify using properties of natural logarithms.

23. $\ln e^2$ 24. $-\ln e$ 25. $e^{\ln 1.8}$ 26. $7 \ln e^0$

27. *Home Mortgage*: The model $t = 16.625\ln\left(\frac{x}{x-750}\right)$, if $x > 750$, approximates the length of a home mortgage of $150,000 at 6% in terms of monthly payments. In the model, t is the length of the mortgage in years and x is the monthly payment in dollars.

(a) Use the model to approximate the length of this mortgage when the monthly payment is $897.72. Approximate the total monthly payments over the term of this mortgage.
(b) Use the model to approximate the length of this mortgage when the monthly payment is $1659.24. Approximate the total monthly payments for this option.
(c) What can you conclude from (a) and (b) regarding these payment options?

Logarithmic Properties

Rewrite the logarithm as a ratio using the change of base formula. Use both the common and natural logarithm bases.

1. $\log_3 x$
2. $\log_a 15$
3. $\log_y x$

Use properties of logs to simplify the following expressions.

4. $\log_2\left(4^2 \cdot 3^4\right)$
5. $\ln\frac{6}{e^2}$
6. $\ln\left(5e^6\right)$

Use properties of logs to expand the following expressions.

7. $\log_{10} 10x$
8. $\log_{10}\frac{y}{2}$
9. $\log_6 n^{-3}$
10. $\ln\sqrt[3]{t}$
11. $\ln\frac{xy}{t}$
12. $\ln\left[g(g-1)^2\right]$ *when* $g > 1$
13. $\ln\sqrt{\frac{x^2}{y^3}}$
14. $\ln\frac{x}{\sqrt{x^2+1}}$
15. $\ln\sqrt{x^2(x+2)}$
16. $\log_b\frac{\sqrt{x}y^4}{z^4}$

Use properties of logarithms to condense the following expressions.

17. $\ln y + \ln x$
18. $\log_5 8 - \log_5 t$
19. $\frac{5}{2}\log_7(h-4)$
20. $-6\log_6 2x$
21. $2\ln 8 + 5\ln x$
22. $3\ln x + 2\ln y - 4\ln z$
23. $4\left[\ln x + \ln(x+5)\right] - 2\ln(x-5)$
24. $2\left[\ln x - \ln(x+1) - \ln(x-1)\right]$
25. $\frac{1}{2}\left[\ln(x+1) + 2\ln(x-1)\right] + 3\ln x$

Simplify to an exact value without a calculator.

26. $\log_6\sqrt[3]{6}$
27. $\log_5\left(\frac{1}{125}\right)$
28. $\log_4(-16)$
29. $\log_4 2 + \log_4 32$
30. $\ln e^6 - 2\ln e^5$
31. $\ln e^{4.5}$
32. $\ln\sqrt[5]{e^3}$

33. The relationship between the number of decibels *β* and the intensity of a sound *I* in watts per square meter is given by $\beta = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$. (a) Use properties of logarithms to simplify this formula.

(b) Complete the table for *β* given *I* without a calculator.

I	10^{-4}	10^{-6}	10^{-8}	10^{-10}	10^{-12}	10^{-14}
β						

34. Students participating in a psychology experiment attended several lectures and were given an exam. Every month for the next year, the students were retested to see how much of the material they remembered. The average scores for the group are given by the human memory model

$f(t) = 90 - 15\log_{10}(t+1)$ $\quad 0 \le t \le 12$ where *t* is the time in months from the first test.

(a) Graph the function over the given domain. You may use a calculator to help find points.
(b) Find the average score on the original exam (t = 0).
(c) Find the average score after 6 months. (d) Find the average score after 12 months.
(d) Find the time when the average score has dropped to 75.

Solving Exponential & Logarithmic Equations

Solve each equation. Leave each answer in exact form.

1. $3^x = 243$
2. $7^x = \frac{1}{49}$
3. $\left(\frac{1}{2}\right)^x = 32$
4. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$
5. $\ln x = -1$
6. $\ln x - \ln 2 = 0$
7. $\ln(3x+5) = 8$
8. $\log_x 25 = 2$
9. $\log_{10} x = -\frac{1}{2}$
10. $e^x = 0$

Solve each equation. Round each answer to three decimal places.

11. $6^{5x} = 3000$
12. $4e^{2x} = 40$
13. $1000e^{-4x} = 75$
14. $-14 + 3e^x = 11$
15. $4^{-3t} = 0.10$
16. $8^{-2-x} = 431$
17. $\frac{525}{1+e^{-x}} = 275$
18. $\left(16 + \frac{0.878}{26}\right)^{3t} = 30$
19. $\ln x = -2$
20. $\ln 2x = 1.5$
21. $3 + 2\ln x = 10$
22. $\log_{10} x^2 = 6$
23. $4\log_{10}(x-6) = 11$
24. $\ln\sqrt{x-8} = 5$
25. $\ln(x^2+1) = 8$
26. $\log_3 x + \log_3(x-8) = 2$
27. $\ln(x+1) - \ln(x-2) = \ln x$
28. $\log_{10} 4x^2 - \log_{10}(4+x) = 2$

29. The demand equation for an iPad is given by $p = 5000\left(1 - \frac{4}{4+e^{-0.0002x}}\right)$ where *x* is the number of iPads sold and *p* is the price per unit.

(a) Find *x* for *p* = \$600. (b) Find *x* for *p* = \$800.

30. The yield V (in millions of cubic feet of timber per acre) for a forest at age t years is given by

$V = 6.7e^{-48.1/t}$.

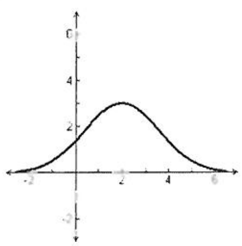
(a) Find the yield after 20 years.

(b) Find the time needed to obtain a yield of 1.3 million cubic feet.

Exponential and Logarithmic Models

For problems 1-6, match the function with its graph.

(a)



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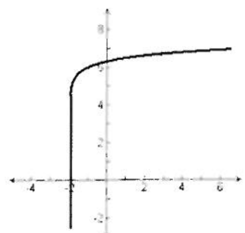

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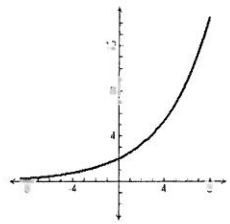

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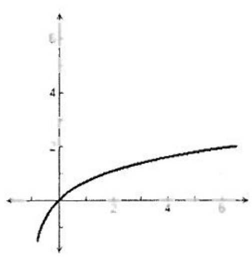

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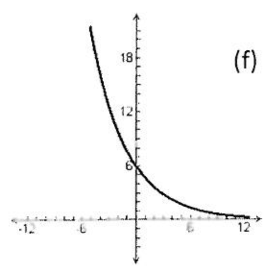
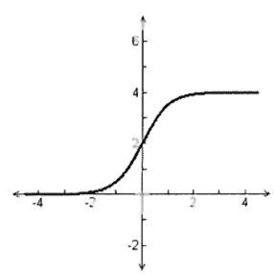


(f)



1. $y = 2e^{x/4}$
2. $y = 6e^{-x/4}$
3. $y = 6 + \log_{10}(x+2)$
4. $y = 3e^{-(x-2)^2/5}$
5. $y = \ln(x+1)$
6. $y = \dfrac{4}{1+e^{-2x}}$

7. Consider an initial investment of \$20,000 earning interest at 10.5% compounded continuously.

   (a) Find the amount of time needed to double the investment.

   (b) Find the value of the investment after 10 years.

8. Suppose an initial investment of \$600 was worth \$1,505 after 10 years of continuous compounding.

   (a) Find the interest rate required for this to occur.

   (b) Find the amount of time required to triple this investment.

9. The half-life of ^{14}C is 5,730 years. Consider a sample of 3 g. Find the amount of this sample remaining after 1,000 years.

10. The half-life of ^{226}Ra is 1,600 years. For a sample to have a mass of 1.5 g in 1,000 years, find the initial quantity of this sample.

11. Find the exponential model $y = ae^{bx}$ that fits the following points.

(a) (0,1) and (3,10) (b) (0, ½) and (4,5) (c) (0, 1) and (3, 0.25)

12. The number N of bacteria in a culture is given by the model $N = 250e^{kt}$, where t is the time (hours). If N = 280 and t = 10, estimate the time required for the population to double in size.

13. The time x (hours per week) a student uses a math lab roughly follows the normal distribution $y = 0.7979e^{-(x-5.4)^2/0.5}$ where $4 \le x \le 7$.

(a) Make a table of values and graph this function over the given domain.
(b) Estimate the average time a student spends per week in the math lab.

14. The amount Y of yeast in a culture is given by the model

$$Y = \frac{663}{1+72e^{-0.547t}}, \quad 0 \le t \le 18$$ where t represents time (hours).

(a) Make a table of values and graph this function over the given domain.
(b) Use the model to predict the population for the 19th hour (e.g. t =19) and the 30th hour.
(c) Find the limiting value of the population described by this model.
(d) Explain why the population of yeast follows a logistic growth model instead of an exponential decay model.

15. The Richter scale measures the magnitude R of an earthquake of intensity I using the model $R = \log_{10} \frac{I}{I_0}$, where $I_0 = 1$.

(a) Find the intensity of the March 11, 2011 earthquake off the coast of Japan (R = 9.0).
(b) Find the intensity of the February 7, 1812 earthquake near New Madrid, Missouri ($R \approx 7.0$).
(c) Find the magnitude of an earthquake with an intensity of 251, 200.

16. The level of sound β (in decibels) is related to sound intensity I using the model $\beta = 10\log_{10}(I/I_0)$, where $I_0 = 10^{-12}$ watts per m^2, which is roughly the faintest sound that can be heard by the human ear.

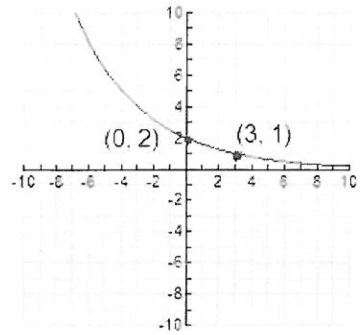
(a) Calculate the level of sound (a.k.a. decibel level) of a quiet room ($I = 10^{-10}$ watts per m^2).
(b) Calculate the decibel level of a loud car horn ($I = 10^{-3}$ watts per m^2).
(c) An F-16 fighter jet has a sound level of about 90 decibels (dB) when landing, while the newer F-35 jet lands with a sound level of 105 dB. Calculate the difference in sound intensity represented by this 15 dB difference in sound level.

17. The acidity of a solution is represented by pH, and uses the model $\text{pH} = -\log_{10}\left[\text{H}^+\right]$, where pH is a measure of the hydrogen ion concentration $\left[\text{H}^+\right]$ of a solution. $\left[\text{H}^+\right]$ is measured in moles of hydrogen per liter.

(a) Find the pH of a solution is $\left[\text{H}^+\right] = 2.3\times10^{-5}$.
(b) Find $\left[\text{H}^+\right]$ for a solution for which pH = 5.8
(c) Find the pH of gastric acid (which is inside your stomach), if $\left[\text{H}^+\right] = 0.1$ moles per liter.

Review

1. Determine the exponential function whose graph is shown in the figure.



$y = ae^{bx}$

Determine if the following are an exponential growth function or an exponential decay function.

2. $y = e^{4-x} + 2$

3. $y = 2(5^{x-3}) + 1$

Graph the function and analyze it for domain, range, and asymptotes.

4. $y = \log(x-3) + 2$

5. $y = 3^{x+2} - 5$

Evaluate the logarithmic expression without using a calculator.

6. $\log_2 32$

7. $\log_3 81$

8. $\log \sqrt[3]{10}$

9. $\ln \frac{1}{\sqrt{e^7}}$

Rewrite the equation in exponential form.

10. $\log_3 x = 5$

11. $\log_2 x = y$

Expand the following:

12. $\ln \frac{xy}{z} =$

13. $\log_3 \frac{\sqrt{x}y^4}{z^4} =$

Condense the following:

14. $2\ln 8 + 5\ln z =$

15. $3[\ln(x-1) + 2\ln(x+1)] - 2\ln(x^2-1) =$

Solve the equation.

16. $10^x = 4$

17. $e^x = 0.25$

18. $1.05^x = 3$

19. $\ln x = 5.4$

20. $\log x = -6$

21. $3^{x-3} = 5$

22. $3\log_2 x + 1 = 7$

23. $2\log_3 x - 3 = 4$

Solve the equation.

24. $\dfrac{50}{4+e^{2x}} = 11$

25. $\log(x+2)+\log(x-1)=4$

26. $\ln(3x+5)-\ln(2x+1)=\ln 4$

27. Find the amount A accumulated after investing \$450 for 3 years at an interest rate of 4.6% compounded annually.

28. Find the amount A accumulated after investing \$4800 for 17 years at an interest of rate 6.2% compouned quarterly.

29. How long would it take for your investment to double if it is compounded continuously at 8.5% interest rate?

30. If Jane invests \$1500 in a savings account with a 6% interest rate compounded monthly, how long will it take until Jane's amount has a balance of \$5200?

31. The time *t* in years for the world populations to double if it is increasing at a continuous rate of *r* is given by $t = \dfrac{\ln 2}{r}$. Efficiently complete the table below and interpret your results. Use a graphing utility to graph the function.

r	0.005	0.010	0.015	0.020	0.025	0.030
t						

32. The number of bacteria in a culture is given by the model $N = 250e^{kt}$ where *t* is the time (in hours). If *N* = 280 when *t* = 10, estimate the time required for the population to double in size. Verify your estimate graphically.

33. The sales *S* (in thousands of units) of a cleaning solution after *x* hundred dollars is spent on advertising are given by $S = 10(1-e^{kx})$. When \$500 is spent on advertising, 2,500 units are sold. Complete the model by solving for *k*. Estimate the number of units that will be sold if advertising expenditures are raised to \$700.

34. The time (in hours per week) a student uses a math lab roughly follows the normal distribution

$y = 0.7979e^{-(x-5.4)^2/0.5}$, $4 \le x \le 7$, where *x* is the time spent in the lab. Use a graphing utility to graph the function. From the graph, estimate the average time a student spends per week in the math lab.

35. The amount *Y* of yeast in a culture is given by the model $Y = \dfrac{663}{1+72e^{-0.547t}}$, $0 \le t \le 18$, *t* in hours.

Use a graphing utility to graph the model. Use the model to predict the population for the 19th hour and the 30th hour. According to this model, what is the limiting value of the population? Why do you think the population of yeast follows a logistic growth model instead of an exponential growth model?

36. $(3x^3)^3$ is equivalent to:

a. x　　b. $9x^6$　　c. $9x^9$　　d. $27x^6$　　e. $27x^9$

37. In the real numbers, what is the solution of the equation $8^{2x+1} = 4^{1-x}$?

a. -1/3　　b. -1/4　　c. -1/8　　d. 0　　e. 1/7

38. If $\log_3 2 = p$ and $\log_3 5 = q$, which of the following expressions is equal to 10?

a. 3^{p+q}　　b. $3^p + 3^q$　　c. 9^{p+q}　　d. pq　　e. p + q

39. Whenever x, y, and z are positive real numbers, which of the following expressions is equivalent to $2 \log_3 x + \frac{1}{2} \log_6 y - \log_3 z$?

a. $\log_3(\frac{x^2 y}{z})$　　b. $\log_3\left(\frac{x^2}{z}\right) + \log_6\left(\sqrt{y}\right)$　　c. $\log_3\left(\frac{z}{x^2}\right) + \log_6\left(\frac{y}{2}\right)$

d. $\log_3(x - z) + \log_6\left(\sqrt{y}\right)$　　e. $2\log_3(x - z) + \log_6\left(\frac{y}{2}\right)$

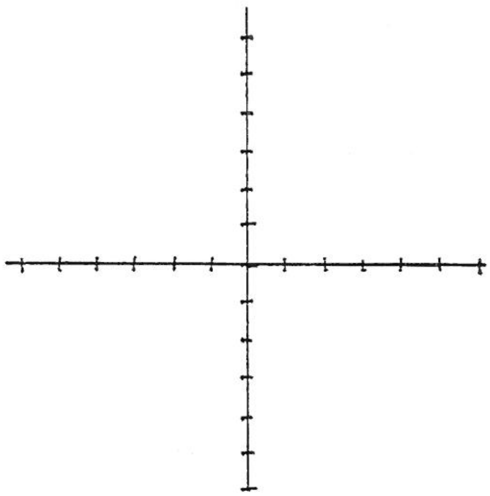
MORE EQUATION PRACTICE:

Any of these.

1. $\log_4(x+2) = \log_4 8$
2. $\log_5(2x+3) = \log_5 3$
3. $\frac{1}{2}\log_3 x = 2\log_3 2$
4. $-2\log_4 x = \log_4 9$
5. $2\log_5 x = 3\log_5 4$
6. $3\log_2 x = -\log_2 27$
7. $3\log_2(x-1) + \log_2 4 = 5$
8. $2\log_3(x+4) - \log_3 9 = 2$
9. $\log x + \log(x+15) = 2$
10. $\log_4 x + \log_4(x-3) = 1$
11. $\ln(x+1) - \ln x = 2$
12. $2^x = 10$
13. $3^x = 14$
14. $8^{-x} = 1.2$
15. $2^{-x} = 1.5$
16. $3^{1-2x} = 4^x$
17. $\left(\frac{3}{5}\right)^x = 7^{1-x}$
18. $\left(\frac{4}{3}\right)^{1-x} = 5^x$
19. $1.2^x = (0.5)^{-x}$
20. $(0.3)^{1+x} = 1.7^{2x-1}$
21. $\pi^{1-x} = e^x$
22. $\pi^x = e^{x+3}$
23. $5(2^{3x}) = 8$
24. $0.3(4^{0.2x}) = 0.2$

Parametrics

1. During the time period from $t = 0$ to $t = 6$ seconds, a particle moves along the path given by $x(t) = 3\cos(\pi t)$ and $y(t) = 5\sin(\pi t)$.
   a. Find the position of the particle when $t = 2.5$.
   b. On the axes provided, sketch the graph of the path of the particle from $t = 0$ to $t = 6$. Indicate the direction of the particle along its path.
   c. How many times does the particle pass through the point found in part (a)?



2. Consider the curve given by the parametric equations $x(t) = 2t^3 - 3t^2$ and $y(t) = -t^3 + 12t$ for $-2 \le t \le 2$.

   Find window dimensions suitable for viewing a complete graph. Label the window dimensions you used, sketch the graph, and indicate the direction of motion of a particle moving on this curve for $t \in [-2, 2]$.

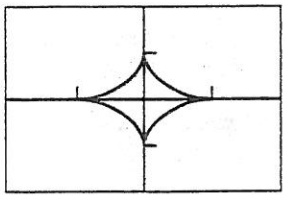
In Exercises 1–4, match the parametric equations with their graph. State the approximate dimensions of the viewing window. Give a parameter interval that traces the curve exactly once.

1. $x = 3 \sin(2t), \quad y = 1.5 \cos t$

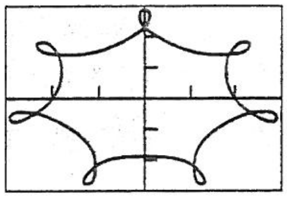
2. $x = \sin^3 t, \quad y = \cos^3 t$

3. $x = 7 \sin t - \sin(7t), \quad y = 7 \cos t - \cos(7t)$

4. $x = 12 \sin t - 3 \sin(6t), \quad y = 12 \cos t + 3 \cos(6t)$



(a)



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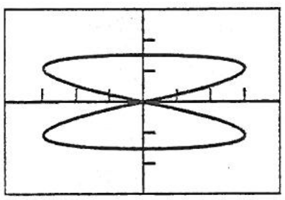

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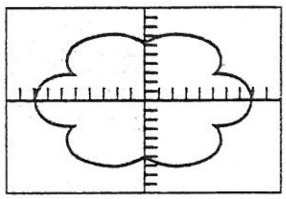


(d)

In Exercises 7–26, a parametrization is given for a curve.

(a) Graph the curve. What are the initial and terminal points, if any? Indicate the direction in which the curve is traced.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

7. $x = \cos t, \quad y = \sin t, \quad 0 \leq t \leq \pi$

8. $x = \sin(2\pi t), \quad y = \cos(2\pi t), \quad 0 \leq t \leq 1$

9. $x = \cos(\pi - t), \quad y = \sin(\pi - t), \quad 0 \leq t \leq \pi$

10. $x = 4 \cos t, \quad y = 2 \sin t, \quad 0 \leq t \leq 2\pi$

11. $x = 4 \sin t, \quad y = 2 \cos t, \quad 0 \leq t \leq \pi$

12. $x = 4 \sin t, \quad y = 5 \cos t, \quad 0 \leq t \leq 2\pi$

13. $x = 3t, \quad y = 9t^2, \quad -\infty < t < \infty$

14. $x = -\sqrt{t}, \quad y = t, \quad t \geq 0$

15. $x = t, \quad y = \sqrt{t}, \quad t \geq 0$

16. $x = (\sec^2 t) - 1, \quad y = \tan t, \quad -\pi/2 < t < \pi/2$

17. $x = -\sec t, \quad y = \tan t, \quad -\pi/2 < t < \pi/2$

18. $x = \tan t, \quad y = -2 \sec t, \quad -\pi/2 < t < \pi/2$

19. $x = 2t - 5, \quad y = 4t - 7, \quad -\infty < t < \infty$

20. $x = 1 - t, \quad y = 1 + t, \quad -\infty < t < \infty$

21. $x = t, \quad y = 1 - t, \quad 0 \leq t \leq 1$

22. $x = 3 - 3t, \quad y = 2t, \quad 0 \leq t \leq 1$

23. $x = 4 - \sqrt{t}, \quad y = \sqrt{t}, \quad 0 \leq t$

24. $x = t^2, \quad y = \sqrt{4 - t^2}, \quad 0 \leq t \leq 2$

25. $x = \sin t, \quad y = \cos 2t, \quad -\infty < t < \infty$

26. $x = t^2 - 3, \quad y = t, \quad t \leq 0$

In Exercises 27–32, find a parametrization for the curve.

27. the line segment with endpoints $(-1, -3)$ and $(4, 1)$

28. the line segment with endpoints $(-1, 3)$ and $(3, -2)$

29. the lower half of the parabola $x - 1 = y^2$

30. the left half of the parabola $y = x^2 + 2x$

31. the ray (half line) with initial point $(2, 3)$ that passes through the point $(-1, -1)$

32. the ray (half line) with initial point $(-1, 2)$ that passes through the point $(0, 0)$

In Exercises 33–36, refer to the graph of

$$x = 3 - |t|, \quad y = t - 1, \quad -5 \leq t \leq 5,$$

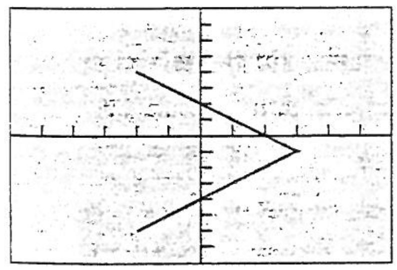
shown in the figure. *Work in groups of two or three* to find the values of t that produce the graph in the given quadrant.

33. Quadrant I

34. Quadrant II

35. Quadrant III

36. Quadrant IV



[–6, 6] by [–8, 8]

In Exercises 37 and 38, find a parametrization for the part of the graph that lies in Quadrant I.

37. $y = x^2 + 2x + 2$

38. $y = \sqrt{x + 3}$

39. *Circles* Find parametrizations to model the motion of a particle that starts at $(a, 0)$ and traces the circle $x^2 + y^2 = a^2$, $a > 0$, as indicated.

(a) once clockwise

(b) once counterclockwise

(c) twice clockwise

(d) twice counterclockwise