BC Calculus Summer Assignment

Hello Gentlemen,

I look forward to working with you this fall. In preparation for that, I have created a summer assignment that addresses two of the major topics limits and derivatives.

Included in this packet are some definitions and formulas. Hopefully, these along with your class notes, and any tutorials you might find at Kahn Academy or elsewhere on the internet will help you complete the summer assignment.

Directions:

Except for the last problem (skydiver), you must write the original problem and then develop complete supporting analytical work. This means do not find limits or derivatives using your calculator. Your calculator should be used only as an aid in completing arithmetic work.

It is expected that you will show your supporting work and answers neatly organized on notebook paper. You will submit your assignment to me at our first class meeting.

Good Luck!

Ms. Bohan

Definitions that may be helpful in Chapter 2

DEFINITION **Continuity at a Point**

A function f is **continuous** at a if $\lim_{x \to a} f(x) = f(a)$. If f is not continuous at a, then a is a point of discontinuity.

There is more to this definition than first appears. If $\lim_{x \to a} f(x) = f(a)$, then $f(a)$ and $\lim_{x \to a} f(x)$ must both exist, and they must be equal. The following checklist is helpful in determining whether a function is continuous at a.

Continuity Checklist

In order for f to be continuous at a, the following three conditions must hold.

1. $f(a)$ is defined (a is in the domain of f).
2. $\lim_{x \to a} f(x)$ exists.
3. $\lim_{x \to a} f(x) = f(a)$ (the value of f equals the limit of f at a).

Derivative Formulas of trig Functions

$$\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$$

Derivatives of exponential/logarithmic functions

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\frac{du}{dx}$$

Derivatives Formulas of Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \qquad |u| > 1$$

$$\frac{d}{dx}\csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \qquad |u| > 1$$

Derivative Formula for Absolute Value

$$\frac{d}{dx}|u| = \frac{|u|}{u}\frac{du}{dx} \qquad \text{when } u \neq 0$$

8–21. Evaluating limits *Evaluate the following limits analytically.*

8. $\lim_{x \to 1000} 18\pi^2$

9. $\lim_{x \to 1} \sqrt{5x+6}$

10. $\lim_{h \to 0} \frac{\sqrt{5x+5h}-\sqrt{5x}}{h}$, where x is constant omit

11. $\lim_{x \to 1} \frac{x^3-7x^2+12x}{4-x}$

12. $\lim_{x \to 4} \frac{x^3-7x^2+12x}{4-x}$

*Hint: Try mulitplying by the conjugate

13. $\lim_{x \to 1} \frac{1-x^2}{x^2-8x+7}$

*14.** $\lim_{x \to 3} \frac{\sqrt{3x+16}-5}{x-3}$

43–46. Continuity at a point *Determine whether the following functions are continuous at a using the continuity checklist to justify your answers.*

43. $f(x) = \frac{1}{x-5}; a = 5$

44. $g(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ 9 & \text{if } x = 4 \end{cases}; a = 4$

15–36. Evaluating derivatives *Evaluate and simplify the following derivatives.*

15. $\frac{d}{dx}\left(\frac{2}{3}x^3 + \pi x^2 + 7x + 1\right)$

16. $\frac{d}{dx}\left(2x\sqrt{x^2 - 2x + 2}\right)$

17. $\frac{d}{dt}\left(5t^2 \sin t\right)$

18. $\frac{d}{dx}\left(5x + \sin^3 x + \sin x^3\right)$

19. $\frac{d}{d\theta}\left(4 \tan\left(\theta^2 + 3\theta + 2\right)\right)$

20. $\frac{d}{dx}\left(\csc^5 3x\right)$

21. $\frac{d}{du}\left(\frac{4u^2+u}{8u+1}\right)$

22. $\frac{d}{dt}\left(\frac{3t^2-1}{3t^2+1}\right)^{-3}$ omit

23. $\frac{d}{d\theta}\left(\tan\left(\sin \theta\right)\right)$

24. $\frac{d}{dv}\left(\frac{v}{3v^2+2v+1}\right)^{1/3}$ omit

27. $\frac{d}{dx}\left(x \ln^2 x\right)$

28. $\frac{d}{dw}\left(e^{-w} \ln w\right)$

29. $\frac{d}{dx}\left(2^{x^2-x}\right)$

30. $\frac{d}{dx}\left(\log_3 (x+8)\right)$

31. $\frac{d}{dx}\left[\sin^{-1} \frac{1}{x}\right]$ omit

41–44. Tangent lines *Find an equation of the line tangent to the following curves at the given point.*

41. $y = 3x^3 + \sin\, x;\ x = 0$

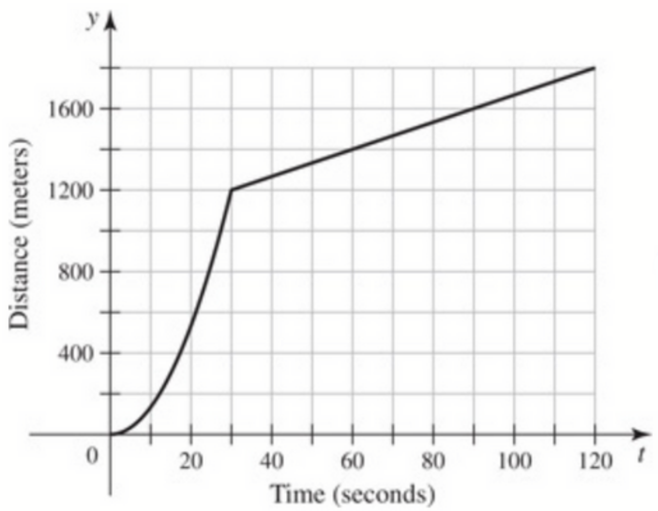
42. $y = \frac{4x}{x^2+3};\ x = 3$

T 65. Population growth Suppose $p(t) = -1.7t^3 + 72t^2 + 7200t + 80,000$ is the population of a city *t* years after 1950.

a. Determine the average rate of growth of the city from 1950 to 2000.

b. What was the rate of growth of the city in 1990?

9. Velocity of a skydiver Assume the graph represents the distance (in m) fallen by a skydiver *t* seconds after jumping out of a plane.



a. Estimate the velocity of the skydiver at $t = 15$.

b. Estimate the velocity of the skydiver at $t = 70$.

c. Estimate the average velocity of the skydiver between $t = 20$ and $t = 90$.

d. Sketch a graph of the velocity function for, $0 \leq t \leq 120$.

e. What significant event do you think occurred at $t = 30$?