Welcome to

Calculus 1 Honors

2019-2020

Welcome to Calculus 1 Honors! This September, you will begin your study of a first semester college course in calculus. Our Calculus 1 Honors course is the only math course offered as a dual enrollment course with Quincy College. To be successful in Calculus 1 Honors, you need to have excellent algebra skills and a strong understanding of Precalculus concepts.

About Your Summer Assignment

This summer you will have a chance to sharpen your algebra skills and review rational functions so that we can immediately begin working on calculus when you return in September. During this summer, you will be expected to complete the REQUIRED summer assignment (see reverse).

To complete your summer assignment:

- **READ** (*yes, READ*!) each section to make sure you understand the concepts.
- **STUDY** the **PowerPoint** on the website that further reviews the topics from the book.
- **COMPLETE** ALL assigned problems (see reverse.)

This assignment is DUE the first day of classes. You will have one full class period to ask questions about this assignment.

You will have TEST on this material during the first week of classes.

TO PURCHASE YOUR BOOK:

1. Go to www.mbsdirect.net and purchase an access code.
2. Go to www.mymathlab.com to register for Calculus 1 H using course ID: **curley67315**

If you have any problems, contact Mr. Curley at pcurley@awhs.org.

To access your book on your iPad, go to the App Store and download the Pearson eText app, which looks like:

SUMMER ASSIGNMENT:

Complete the following worksheets (attached):

- Exercise Set 2.6 Worksheet: #1-8 odds, 9-20 all, 21-70 odds, 71-78 all
- Algebra That Occurs in Calculus Problems Worksheet:
    - I: #35-42 all
    - II: #121-130 all
    - III: #59-78, 87-100 odds, 101-110 all

Solutions to both of these worksheets are posted on Schoology in the "Summer Math Assignment" course.

Algebra That Occurs in Calculus Problems

I

In Problems 35–42, expressions that occur in calculus are given. Reduce each expression to lowest terms.

35. $\dfrac{(2x+3)\cdot 3-(3x-5)\cdot 2}{(3x-5)^2}$

36. $\dfrac{(4x+1)\cdot 5-(5x-2)\cdot 4}{(5x-2)^2}$

37. $\dfrac{x\cdot 2x-(x^2+1)\cdot 1}{(x^2+1)^2}$

38. $\dfrac{x\cdot 2x-(x^2-4)\cdot 1}{(x^2-4)^2}$

39. $\dfrac{(3x+1)\cdot 2x-x^2\cdot 3}{(3x+1)^2}$

40. $\dfrac{(2x-5)\cdot 3x^2-x^3\cdot 2}{(2x-5)^2}$

41. $\dfrac{(x^2+1)\cdot 3-(3x+4)\cdot 2x}{(x^2+1)^2}$

42. $\dfrac{(x^2+9)\cdot 2-(2x-5)\cdot 2x}{(x^2+9)^2}$

II

In Problems 121–130, expressions that occur in calculus are given. Factor completely each expression.

121. $2(3x+4)^2+(2x+3)\cdot 2(3x+4)\cdot 3$

122. $5(2x+1)^2+(5x-6)\cdot 2(2x+1)\cdot 2$

123. $2x(2x+5)+x^2\cdot 2$

124. $3x^2(8x-3)+x^3\cdot 8$

125. $2(x+3)(x-2)^3+(x+3)^2\cdot 3(x-2)^2$

126. $4(x+5)^3(x-1)^2+(x+5)^4\cdot 2(x-1)$

127. $(4x-3)^2+x\cdot 2(4x-3)\cdot 4$

128. $3x^2(3x+4)^2+x^3\cdot 2(3x+4)\cdot 3$

129. $2(3x-5)\cdot 3(2x+1)^3+(3x-5)^2\cdot 3(2x+1)^2\cdot 2$

130. $3(4x+5)^2\cdot 4(5x+1)^2+(4x+5)^3\cdot 2(5x+1)\cdot 5$

III

In Problems 59–70, simplify each expression.

59. $8^{2/3}$ **60.** $4^{3/2}$ **61.** $(-27)^{1/3}$ **62.** $16^{3/4}$ **63.** $16^{3/2}$ **64.** $25^{3/2}$

65. $9^{-3/2}$ **66.** $16^{-3/2}$ **67.** $\left(\dfrac{9}{8}\right)^{3/2}$ **68.** $\left(\dfrac{27}{8}\right)^{2/3}$ **69.** $\left(\dfrac{8}{9}\right)^{-3/2}$ **70.** $\left(\dfrac{8}{27}\right)^{-2/3}$

In Problems 71–78, simplify each expression. Express your answer so that only positive exponents occur. Assume that the variables are positive.

71. $x^{3/4}x^{1/3}x^{-1/2}$

72. $x^{2/3}x^{1/2}x^{-1/4}$

73. $(x^3y^6)^{1/3}$

74. $(x^4y^8)^{3/4}$

75. $\dfrac{(x^2y)^{1/3}(xy^2)^{2/3}}{x^{2/3}y^{2/3}}$

76. $\dfrac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}}$

77. $\dfrac{(16x^2y^{-1/3})^{3/4}}{(xy^2)^{1/4}}$

78. $\dfrac{(4x^{-1}y^{1/3})^{3/2}}{(xy)^{3/2}}$

In Problems 87–100, expressions that occur in calculus are given. Write each expression as a single quotient in which only positive exponents and/or radicals appear.

87. $\dfrac{x}{(1+x)^{1/2}} + 2(1+x)^{1/2} \quad x > -1$

88. $\dfrac{1+x}{2x^{1/2}} + x^{1/2} \quad x > 0$

89. $2x(x^2+1)^{1/2} + x^2 \cdot \dfrac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

90. $(x+1)^{1/3} + x \cdot \dfrac{1}{3}(x+1)^{-2/3} \quad x \neq -1$

91. $\sqrt{4x+3} \cdot \dfrac{1}{2\sqrt{x-5}} + \sqrt{x-5} \cdot \dfrac{1}{5\sqrt{4x+3}} \quad x > 5$

92. $\dfrac{\sqrt[3]{8x+1}}{3\sqrt[3]{(x-2)^2}} + \dfrac{\sqrt[3]{x-2}}{24\sqrt[3]{(8x+1)^2}} \quad x \neq 2, x \neq -\dfrac{1}{8}$

93. $\dfrac{\sqrt{1+x} - x \cdot \dfrac{1}{2\sqrt{1+x}}}{1+x} \quad x > -1$

94. $\dfrac{\sqrt{x^2+1} - x \cdot \dfrac{2x}{2\sqrt{x^2+1}}}{x^2+1}$

95. $\dfrac{(x+4)^{1/2} - 2x(x+4)^{-1/2}}{x+4} \quad x > -4$

96. $\dfrac{(9-x^2)^{1/2} + x^2(9-x^2)^{-1/2}}{9-x^2} \quad -3 < x < 3$

97. $\dfrac{\dfrac{x^2}{(x^2-1)^{1/2}} - (x^2-1)^{1/2}}{x^2} \quad x < -1 \text{ or } x > 1$

98. $\dfrac{(x^2+4)^{1/2} - x^2(x^2+4)^{-1/2}}{x^2+4}$

99. $\dfrac{\dfrac{1+x^2}{2\sqrt{x}} - 2x\sqrt{x}}{(1+x^2)^2} \quad x > 0$

100. $\dfrac{2x(1-x^2)^{1/3} + \dfrac{2}{3}x^3(1-x^2)^{-2/3}}{(1-x^2)^{2/3}} \quad x \neq -1, x \neq 1$

In Problems 101–110, expressions that occur in calculus are given. Factor each expression. Express your answer so that only positive exponents occur.

101. $(x+1)^{3/2} + x \cdot \dfrac{3}{2}(x+1)^{1/2} \quad x \geq -1$

102. $(x^2+4)^{4/3} + x \cdot \dfrac{4}{3}(x^2+4)^{1/3} \cdot 2x$

103. $6x^{1/2}(x^2+x) - 8x^{3/2} - 8x^{1/2} \quad x \geq 0$

104. $6x^{1/2}(2x+3) + x^{3/2} \cdot 8 \quad x \geq 0$

105. $3(x^2+4)^{4/3} + x \cdot 4(x^2+4)^{1/3} \cdot 2x$

106. $2x(3x+4)^{4/3} + x^2 \cdot 4(3x+4)^{1/3}$

107. $4(3x+5)^{1/3}(2x+3)^{3/2} + 3(3x+5)^{4/3}(2x+3)^{1/2} \quad x \geq -\dfrac{3}{2}$

108. $6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2} \quad x \geq \dfrac{3}{4}$

109. $3x^{-1/2} + \dfrac{3}{2}x^{1/2} \quad x > 0$

110. $8x^{1/3} - 4x^{-2/3} \quad x \neq 0$

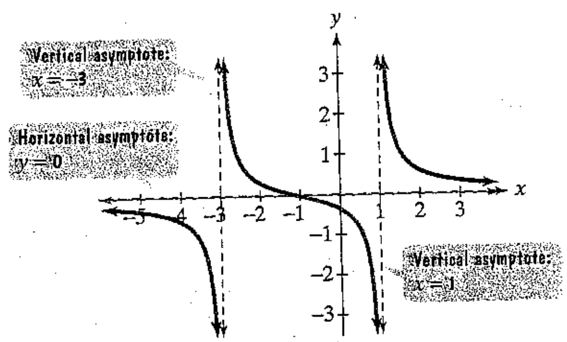
EXERCISE SET 2.6

Practice Exercises

In Exercises 1–8, find the domain of each rational function.

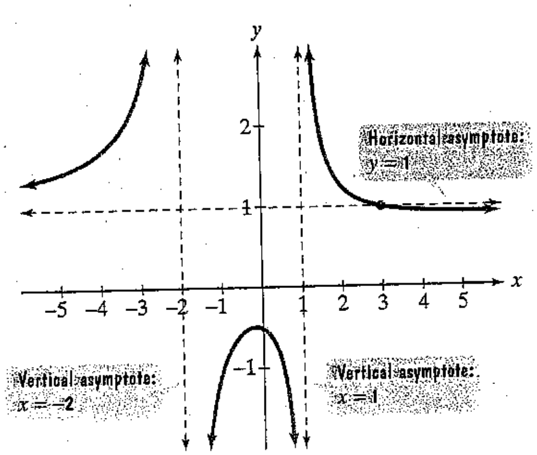
1. $f(x) = \frac{5x}{x-4}$
2. $f(x) = \frac{7x}{x-8}$
3. $g(x) = \frac{3x^2}{(x-5)(x+4)}$
4. $g(x) = \frac{2x^2}{(x-2)(x+6)}$
5. $h(x) = \frac{x+7}{x^2-49}$
6. $h(x) = \frac{x+8}{x^2-64}$
7. $f(x) = \frac{x+7}{x^2+49}$
8. $f(x) = \frac{x+8}{x^2+64}$

Use the graph of the rational function in the figure shown to complete each statement in Exercises 9–14.



9. As $x \to -3^-$, $f(x) \to$ _____.
10. As $x \to -3^+$, $f(x) \to$ _____.
11. As $x \to 1^-$, $f(x) \to$ _____.
12. As $x \to 1^+$, $f(x) \to$ _____.
13. As $x \to -\infty$, $f(x) \to$ _____.
14. As $x \to \infty$, $f(x) \to$ _____.

Use the graph of the rational function in the figure shown to complete each statement in Exercises 15–20.



15. As $x \to 1^+$, $f(x) \to$ _____.
16. As $x \to 1^-$, $f(x) \to$ _____.
17. As $x \to -2^+$, $f(x) \to$ _____.
18. As $x \to -2^-$, $f(x) \to$ _____.
19. As $x \to \infty$, $f(x) \to$ _____.
20. As $x \to -\infty$, $f(x) \to$ _____.

In Exercises 21–28, find the vertical asymptotes, if any, of the graph of each rational function.

21. $f(x) = \frac{x}{x+4}$
22. $f(x) = \frac{x}{x-3}$
23. $g(x) = \frac{x+3}{x(x+4)}$
24. $g(x) = \frac{x+3}{x(x-3)}$
25. $h(x) = \frac{x}{x(x+4)}$
26. $h(x) = \frac{x}{x(x-3)}$
27. $r(x) = \frac{x}{x^2+4}$
28. $r(x) = \frac{x}{x^2+3}$

In Exercises 29–36, find the horizontal asymptote, if any, of the graph of each rational function.

29. $f(x) = \frac{12x}{3x^2+1}$
30. $f(x) = \frac{15x}{3x^2+1}$
31. $g(x) = \frac{12x^2}{3x^2+1}$
32. $g(x) = \frac{15x^2}{3x^2+1}$
33. $h(x) = \frac{12x^3}{3x^2+1}$
34. $h(x) = \frac{15x^3}{3x^2+1}$
35. $f(x) = \frac{-2x+1}{3x+5}$
36. $f(x) = \frac{-3x+7}{5x-2}$

In Exercises 37–48, use transformations of $f(x) = \frac{1}{x}$ or $f(x) = \frac{1}{x^2}$ to graph each rational function.

37. $g(x) = \frac{1}{x-1}$
38. $g(x) = \frac{1}{x-2}$
39. $h(x) = \frac{1}{x} + 2$
40. $h(x) = \frac{1}{x} + 1$
41. $g(x) = \frac{1}{x+1} - 2$
42. $g(x) = \frac{1}{x+2} - 2$
43. $g(x) = \frac{1}{(x+2)^2}$
44. $g(x) = \frac{1}{(x+1)^2}$
45. $h(x) = \frac{1}{x^2} - 4$
46. $h(x) = \frac{1}{x^2} - 3$
47. $h(x) = \frac{1}{(x-3)^2} + 1$
48. $h(x) = \frac{1}{(x-3)^2} + 2$

In Exercises 49–70, follow the seven steps on page 334 to graph each rational function.

49. $f(x) = \frac{4x}{x-2}$
50. $f(x) = \frac{3x}{x-1}$
51. $f(x) = \frac{2x}{x^2-4}$
52. $f(x) = \frac{4x}{x^2-1}$
53. $f(x) = \frac{2x^2}{x^2-1}$
54. $f(x) = \frac{4x^2}{x^2-9}$

55. $f(x) = \frac{-x}{x+1}$ 56. $f(x) = \frac{-3x}{x+2}$

57. $f(x) = -\frac{1}{x^2-4}$ 58. $f(x) = -\frac{2}{x^2-1}$

59. $f(x) = \frac{2}{x^2+x-2}$ 60. $f(x) = \frac{-2}{x^2-x-2}$

61. $f(x) = \frac{2x^2}{x^2+4}$ 62. $f(x) = \frac{4x^2}{x^2+1}$

63. $f(x) = \frac{x+2}{x^2+x-6}$ 64. $f(x) = \frac{x-4}{x^2-x-6}$

65. $f(x) = \frac{x^4}{x^2+2}$ 66. $f(x) = \frac{2x^4}{x^2+1}$

67. $f(x) = \frac{x^2+x-12}{x^2-4}$ 68. $f(x) = \frac{x^2}{x^2+x-6}$

69. $f(x) = \frac{3x^2+x-4}{2x^2-5x}$ 70. $f(x) = \frac{x^2-4x+3}{(x+1)^2}$

In Exercises 71–78, **a.** *Find the slant asymptote of the graph of each rational function and* **b.** *Follow the seven-step strategy and use the slant asymptote to graph each rational function.*

71. $f(x) = \frac{x^2-1}{x}$ 72. $f(x) = \frac{x^2-4}{x}$

73. $f(x) = \frac{x^2+1}{x}$ 74. $f(x) = \frac{x^2+4}{x}$

75. $f(x) = \frac{x^2+x-6}{x-3}$ 76. $f(x) = \frac{x^2-x+1}{x-1}$

77. $f(x) = \frac{x^3+1}{x^2+2x}$ 78. $f(x) = \frac{x^3-1}{x^2-9}$

Practice Plus

In Exercises 79–84, the equation for f is given by the simplified expression that results after performing the indicated operation. Write the equation for f and then graph the function.

79. $\frac{5x^2}{x^2-4} \cdot \frac{x^2+4x+4}{10x^3}$ 80. $\frac{x-5}{10x-2} \div \frac{x^2-10x+25}{25x^2-1}$

81. $\frac{x}{2x+6} - \frac{9}{x^2-9}$ 82. $\frac{2}{x^2+3x+2} - \frac{4}{x^2+4x+3}$

83. $\frac{1-\frac{3}{x+2}}{1+\frac{1}{x-2}}$ 84. $\frac{x-\frac{1}{x}}{x+\frac{1}{x}}$

In Exercises 85–88, use long division to rewrite the equation for g in the form quotient, plus remainder divided by divisor. Then use this form of the function's equation and transformations of $f(x) = \frac{1}{x}$ to graph g.

85. $g(x) = \frac{2x+7}{x+3}$ 86. $g(x) = \frac{3x+7}{x+2}$

87. $g(x) = \frac{3x-7}{x-2}$ 88. $g(x) = \frac{2x-9}{x-4}$

Application Exercises

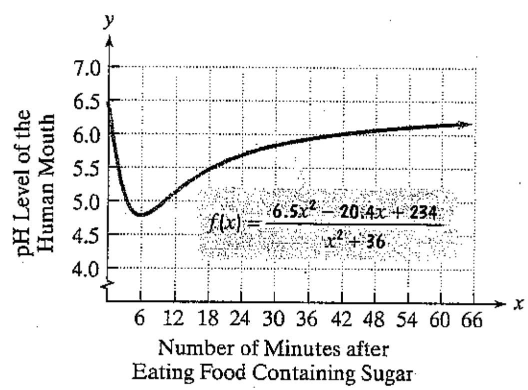
89. A company is planning to manufacture mountain bikes. The fixed monthly cost will be \$100,000 and it will cost \$100 to produce each bicycle.
   a. Write the cost function, C, of producing x mountain bikes.
   b. Write the average cost function, $\overline{C}$, of producing x mountain bikes.
   c. Find and interpret $\overline{C}(500)$, $\overline{C}(1000)$, $\overline{C}(2000)$, and $\overline{C}(4000)$.
   d. What is the horizontal asymptote for the graph of the average cost function, $\overline{C}$? Describe what this means in practical terms.

90. A company that manufactures running shoes has a fixed monthly cost of \$300,000. It costs \$30 to produce each pair of shoes.
   a. Write the cost function, C, of producing x pairs of shoes.
   b. Write the average cost function, $\overline{C}$, of producing x pairs of shoes.
   c. Find and interpret $\overline{C}(1000)$, $\overline{C}(10{,}000)$, and $\overline{C}(100{,}000)$.
   d. What is the horizontal asymptote for the graph of the average cost function, $\overline{C}$? Describe what this represents for the company.

91. The function

$$f(x) = \frac{6.5x^2 - 20.4x + 234}{x^2 + 36}$$

models the pH level, $f(x)$, of the human mouth x minutes after a person eats food containing sugar. The graph of this function is shown in the figure.



   a. Use the graph to obtain a reasonable estimate, to the nearest tenth, of the pH level of the human mouth 42 minutes after a person eats food containing sugar.
   b. After eating sugar, when is the pH level the lowest? Use the function's equation to determine the pH level, to the nearest tenth, at this time.
   c. According to the graph, what is the normal pH level of the human mouth?
   d. What is the equation of the horizontal asymptote associated with this function? Describe what this means in terms of the mouth's pH level over time.
   e. Use the graph to describe what happens to the pH level during the first hour.

92. A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug's concentration, $C(t)$, in milligrams per liter, after t hours is modeled by

$$C(t) = \frac{5t}{t^2+1}.$$

Rational Functions

Rational Function

- A rational function is a function whose rule is the quotient of two polynomials.

Domain of Rational Functions

The domain of a rational function is the set of all real numbers that are NOT zeros of the denominator.

An Example or two…

- Find the domain:

1. $g(x) = \dfrac{x^2 + 3x + 1}{x^2 - x - 6}$

2. $f(x) = \dfrac{1}{x}$

Vertical Asymptote

The line $x = a$ is a vertical asymptote of the graph of f if

$$f(x) \to \infty \text{ or } f(x) \to -\infty$$

as $x \to a$, either from the right or from the left.

Vertical Asymptotes

A rational function has a vertical asymptote at $x = a$, provided

- c is a zero of the denominator
- c is NOT a zero of the numerator

Horizontal Asymptote

The line $y = b$ is a horizontal asymptote of the graph of f if

$$f(x) \to b$$

as $x \to \infty$ or $x \to -\infty$.

Horizontal Asymptotes-The Shortcut

- If the degree of the numerator is less than the degree of the denominator, there is horizontal asymptote at $y = 0.$
- If the degree of the numerator equals the degree of the denominator, there is a horizontal asymptote at the ratio of their highest degree terms.
- If the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

More Examples

Find all horizontal and vertical asymptotes of the graph of each rational function.

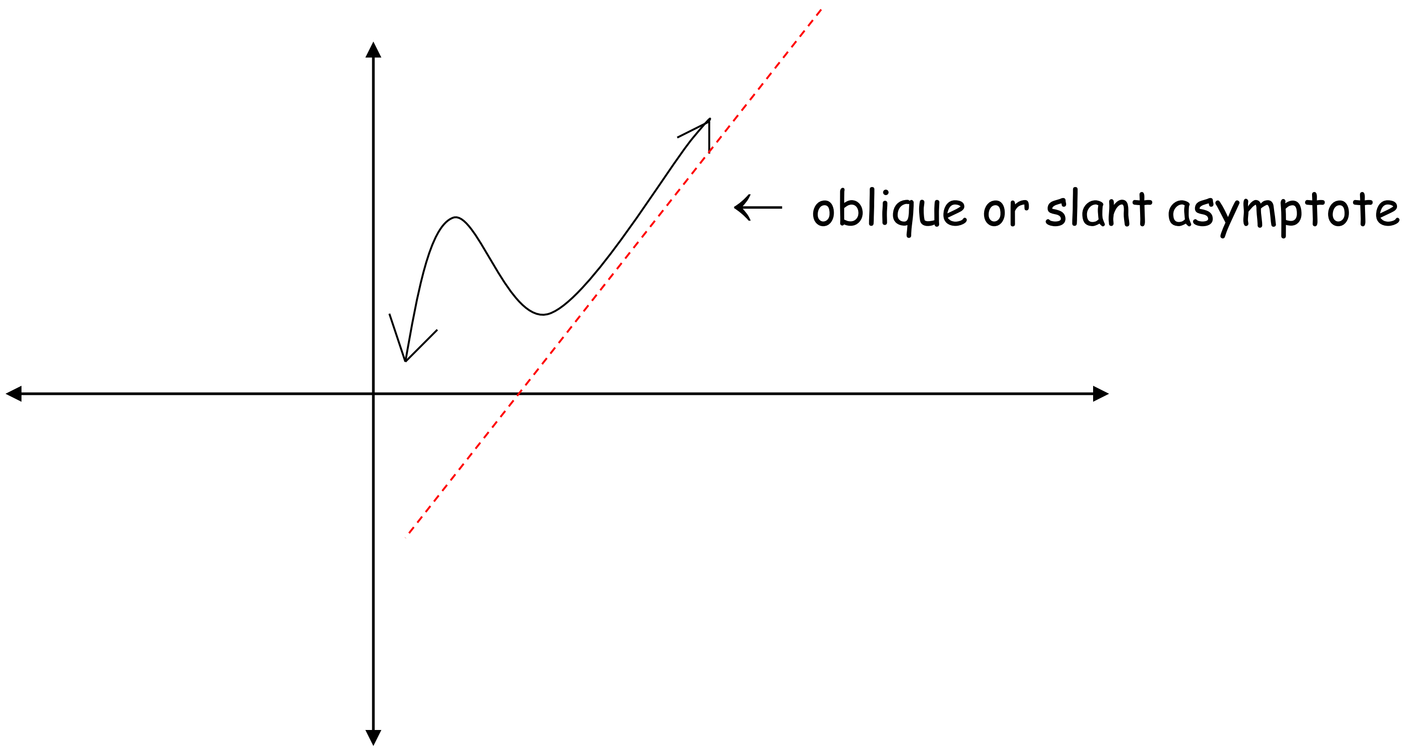
1. $\dfrac{2x}{x^4 + 2x^2 + 1}$

2. $\dfrac{2x^2}{x^2 - 1}$

Oblique or Slant Asymptote

If, as $x \to -\infty$ or as $x \to \infty$, the value of the rational function approaches a linear expression $ax + b$, $a \neq 0$, then the line $y = ax + b$, $a \neq 0$, is an oblique or slant asymptote of R.

What does it look like?



An oblique asymptote, when it occurs, describes the end behavior of the graph.

- **The graph of a function may intersect an oblique asymptote.**

Find the vertical, horizontal and oblique asymptotes, if any, of each rational function.

1. $R(x)=\dfrac{6x^2+7x-5}{3x+5}$

Oblique Asymptote: $y=2x-1$

2. $H(x)=\dfrac{x^3-8}{x^2-5x+6}$

VA: $x=3$

Oblique: $y=x+5$

3. $Q(x)=\dfrac{2x^2-5x-12}{3x^2-11x-4}$

VA: $x=\dfrac{-1}{3}$

HA: $y=\dfrac{2}{3}$

Graphing Rational Functions

Help! What do I need to know to make this easy?

1. Find the domain.
2. Find any vertical asymptotes.
3. Find any horizontal asymptotes.

   (You already know how to find these!)

4. Find any “holes”, if they exist .

   (How do I do that?)

“Holes”

If a rational function has a factor that is common to both the numerator and denominator, then there is a “hole” at that value instead of a vertical asymptote.

Example: $f(x) = \dfrac{x^2 - 4}{x + 2}$

5. Find the *x and y* intercepts, if they exist.

(OK, I used to know how to do this…refresh my memory…)

Intercepts of Rational Functions

- To find the y-intercept substitute 0 for x and solve for y.
- To find the x-intercept set the numerator equal to zero and solve for x.
- Example: Find the intercepts of $f(x) = \dfrac{x^2 - x - 2}{x - 1}$

- 6. Sketch what you know.
- 7. Plot at least one point between and one point beyond each x-intercept and vertical asymptote.
- 8. Use smooth curves to complete the graph between and beyond the vertical asymptotes.

O.T.L.

- Sketch the graph and state its domain.

1. $f(x) = \dfrac{3}{x-2}$ This one's easy!

2. $f(x) = \dfrac{2x-1}{x}$ First , rewrite this function…

3. $g(x) = \dfrac{x}{x^2 - x - 2}$

4. $y = \dfrac{x^2 - x - 2}{x+1}$

How-To for Skill©
Graphing Rational Functions

- Analyze the function, algebraically to determine its vertical asymptotes, holes, intercepts.
- Determine the end behavior of the graph

1. If the degree of the numerator is less than or equal to the degree of the denominator, find the horizontal asymptote.
2. Otherwise, divide the numerator by the denominator. The quotient is the nonvertical asymptote of the graph.