DAVIS ESSENTIAL SKILLS AND KNOWLEDGE

Secondary Math III Honors

Essential Skills and Knowledge
Refer to the Utah State Mathematics Standards for more detail

Mathematical Practice Standards

Students will be able to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Number and Quantity – The Complex Number System

Students will be able to:

1. Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
2. Know the Fundamental Theorem of Algebra and show that it is true for quadratic polynomials limited to real coefficients.

Algebra – Seeing Structure in Expression

Students will be able to:

1. Interpret polynomial and rational expressions that represent a quantity in terms of its context. Interpret the parts of an expression—terms, factors, and coefficients and interpret complex expressions by viewing one or more of their parts as a single entity. *For example, examine the behavior of* $P(1 + r/n)^{nt)}$ *as* n *becomes large.*
2. Use the structure of an expression to identify different ways to rewrite it. *For example, see* $x^4 + x^4$ *as* $(x^2)^2 - (y^2)^2$ *and recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
3. Understand the formula for the sum of a series and use the formula to solve problems, deriving the formula for the sum of an arithmetic and geometric series and using the formula to solve problems.

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Algebra – Arithmetic with Polynomials and Rational Expressions

Students will be able to:

1. Understand that all polynomials form a system analogous to the integers—specifically polynomials are closed under the operations of addition, subtraction, and multiplication. Add, subtract, and multiply polynomials.
2. Know and apply the Remainder Theorem for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial by hand.
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, with coefficients determined by Pascal's Triangle.*
6. Rewrite simple rational expressions in different forms—write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with degree of $r(x)$ less than the degree of $b(x)$ using inspection, long division or technology for more complicated examples.
7. Understand that rational expressions form a system analogous to the rational numbers that are closed under addition, subtraction, multiplication, and division by a non-zero rational expression. Add, subtract, multiply, and divide rational expressions.

Algebra – Creating Equations

Students will be able to:

1. Create linear, quadratic, simple rational, and exponential equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with appropriate labels and scale.
3. Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. *For example, maximizing the volume of a box for a given surface area with proper domain constraints.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1 + r/n)^{nt}$.

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Algebra – Reasoning with Equations and Inequalities

Students will be able to:

1. Solve simple rational and radical equations in one variable and give examples showing extraneous solutions may arise.
2. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$, find the solutions approximately (using a graph) and exactly (using a table of values) incorporating technology where appropriate. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Functions – Interpret Functions

Students will be able to:

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior; and periodicity.*
2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
3. Calculate and interpret the average rate of change of a function (presented symbolically or extrapolated from a table of values) over a specified interval and estimate the rate of change from a graph.
4. Graph functions expressed symbolically and show key features of the graph, by hand and using technology for more complicated cases with special attention on square root, cube root, piecewise-defined, step, absolute value, polynomial, rational, exponential, logarithmic, and trigonometric functions. Compare and contrast each function family with one another and identify end-behavior, factorizations, period, midline, and amplitude as they apply to the function family.
5. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
6. Compare properties of two functions, each represented in a different way such as algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression of a quadratic function, interpret which has the larger maximum.*

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Functions – Building Functions

Students will be able to:

1. Write a function that describes a relationship between two quantities including combining standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model.*
2. Identify the effect on the graph by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(k + x)$ for specific values of k that are both positive and negative and find the value of k given the graphs. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph and experiment with cases that illustrate an explanation of the effects on the graph using technology. Recognize odd and even functions from their graphs and their algebraic expressions.
3. Find an expression that represents the inverse of linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions.

Functions – Trigonometric Functions

Students will be able to:

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers when interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. Use special triangles to geometrically determine the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$ and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
5. Use inverse functions to solve trigonometric equations that arise in modeling context and evaluate the solutions using technology—interpreting them in context and limiting solutions to a given interval.

Functions – Linear, Quadratic, and Exponential Models

Students will be able to:

1. Observe, using graphs and tables, that a quantity that increases exponentially eventually exceeds a quantity that increases linearly, quadratically, or as a polynomial function.
2. Express exponential models as a logarithm to arrive at the solution using $ab^{ct} = d$ where a, c, and d are numbers and the base, b, is $2, 10$, or e and using technology to evaluate the logarithm. Include the relationship between properties of logarithms and exponents, such as

the connection between the properties of exponents and the basic logarithm property that $\log(xy) = \log x + \log y$.

3. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Geometry – Similarity, Right Triangles, and Trigonometry

Students will be able to:

1. Derive the formula $\frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex that is perpendicular to the opposite side.
2. Prove the Law of Sines and Cosines and use them to solve problems in context.
3. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles in context.

Geometry – Geometric Measurement and Dimension

Students will be able to:

1. Identify the shapes of 2-dimensional cross sections of 3-dimensional objects and identify 3-dimensional objects generated by rotations of 2-dimensional objects.

Geometry – Modeling with Geometry

Students will be able to:

1. Use geometric shapes, their measures, and their properties to describe objects.
2. Apply concepts of density based on area and volume in modeling situations.
3. Apply geometric methods to solve design problems such as designing an object or structure to satisfy physical constraints or to minimize cost.

Statistics – Interpreting Categorical and Quantitative Data

Students will be able to:

1. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages recognizing that that are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Statistics – Making Inferences and Justifying Conclusions

Students will be able to:

1. Understand that statistics allow inferences to be made about population parameters based on a random sample from that population.
2. Recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each.
3. Use data from a sample survey to estimate a population mean or proportion and develop a margin of error through the use of simulation models for random sampling.
4. Evaluate reports based on data.

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Number and Quantity – Complex Number Systems – HONORS TOPIC

Students will be able to:

1. Find the conjugate of a complex number and use conjugates to find moduli and quotients of complex numbers.
2. Represent complex numbers on the complex plane in rectangular form and in polar form (including real and imaginary numbers) and explain why the rectangular form of a given complex number represents the same number.
3. Represent addition, subtraction, and multiplication, and conjugation of complex numbers geometrically on the same complex plane and use properties of this representation for computation. *For example,* $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus* 2 *and argument* $120°$.
4. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.
5. Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Functions – Interpreting Functions – HONORS TOPIC

Students will be able to:

1. Graph functions expressed symbolically and show key features of the graph by hand and using technology for more complicated cases. Including rational functions and indentifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior. Defining a curve parametrically and draw its graph.

Functions – Building Functions – HONORS TOPIC

Students will be able to:

1. Write a function that describes a relationship between two quantities with a focus on the composition of functions in context.
2. Find inverse functions and verify, by composition, that one function is the inverse of another, reading values of an inverse function from a graph or a table given that the inverse of the function exists, and produce an invertible function from a non-invertible function by restricting the domain.
3. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Functions—Trigonometric Functions—HONORS TOPIC

Students will be able to:

1. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
2. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

3. Use inverse functions to solve trigonometric equations that arise in the modeling contexts, evaluate the solutions using technology, and interpret them in terms of the context.
4. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Geometry – Geometric Measurement and Dimension – HONORS TOPIC

Students will be able to:

1. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Statistics – Conditional Probability and the Rules of Probability – HONORS TOPIC

Students will be able to:

1. Use permutations and combinations to compute probabilities of compound events and to solve problems.

Literacy Standards

Students will be able to:

1. Acquire, interpret, and accurately use grade level appropriate mathematical words and terms.
2. Engage in collaborative discussions with diverse partners on grade level concepts.