DAVIS ESSENTIAL SKILLS AND KNOWLEDGE

Secondary Math II

Essential Skills and Knowledge
Refer to the Utah State Mathematics Standards for more detail

Mathematical Practice Standards

Students will be able to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Number and Quantity – The Real Number System

Students will be able to:

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of* 5 *because we want* $(5^{1/3})^3 = (5^{(1/3)3}$ *to hold so* $(5^{1/3})^3$ *must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
3. Explain why sums and products of rational numbers are rational, the sum of a rational number and an irrational number is irrational, and that the product of a non-zero rational number and an irrational number is irrational. Connect to concrete representations such as finding the perimeter of a square of area 2.

Number and Quantity – The Complex Number System

Students will be able to:

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ where a and b are real numbers.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Limit to multiplication that involves i^2 as the highest power of i.
3. Solve quadratic equations with real coefficients that have complex solutions.
4. Extend polynomial identities to the complex numbers, limit to quadratics with real coefficients.

5. Know the Fundamental Theorem of Algebra and show that it is true for quadratic polynomials.

Algebra – Seeing Structure in Expression

Students will be able to:

1. Interpret quadratic and exponential expressions that represent a quantity in terms of its context including parts of an expression (terms, factors, and coefficients). Interpret increasingly more complex expressions by viewing one or more of their parts as a single entity. Exponents are extended from integer exponents to rational exponents with special focus on square or cube roots.
2. Use the structure of an expression to identify ways to rewrite it. *For example, see that* $x^4 + x^4$ *as* $(x^2)^2 - (y^2)^2$ *and recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression including factoring a quadratic function and identifying the zeros, completing the square to reveal the maximum or minimum values and using the properties of exponents to transform expressions for exponential functions.

Algebra – Arithmetic with Polynomials and Rational Expressions

Students will be able to:

1. Understand that polynomials form a system analogous to the integers—specifically polynomials are closed under the operations of addition, subtraction, and multiplication and gain experience adding, subtracting, and multiplying polynomials.

Algebra – Reasoning with Equations and Inequalities

Students will be able to:

1. Solve quadratic equations in one variable using inspection, taking square roots, completing the square, the quadratic formula and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them in the form $a \pm bi$ where a and b are real numbers.
2. Solve a simple system consisting of a linear and quadratic equation in two variables, algebraically and graphically.

Functions – Building Functions

Students will be able to:

1. Write a quadratic or exponential function that describes a relationship between two quantities using an explicit expression, a recursive process, or from context.
2. Identify the effect on the parent graph by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k where k is both positive and negative and find the value of k given the graph. Special focus should be on quadratic and absolute value functions both with

and without technology and including odd and even functions in graphical and algebraic representations.

Functions – Interpret Functions

Students will be able to:

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on quadratic functions and compare with linear and exponential functions.
3. Calculate and interpret the average rate of change of a function (presented symbolically or extrapolated from a table of values) over a specified interval. Estimate the rate of change from a graph.
4. Graph functions expressed symbolically and show key features of the graph, by hand and using technology for more complicated cases with special attention on linear, quadratic, piecewise, and absolute value functions. Compare and contrast each function family with one another.
5. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. For quadratic functions, use the process of completing the square to show zeros, extrema, and symmetry and use these terms to interpret in context. Use the properties of exponents to interpret expressions for exponential functions.
6. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.

Functions – Trigonometric Functions

Students will be able to:

1. Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Geometry - Congruence

Students will be able to:

1. Prove theorems about lines and angles in context of a transversal crossing parallel lines including: vertical angle congruence, alternate interior angle congruence, alternate interior

angles congruence, corresponding angle congruence, and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
2. Prove theorems about triangles including: the sum of the interior angles of a triangle is $180°$, congruence of base angles of isosceles triangles, the segment joining midpoints of two sides of a triangle is parallel to the third side and half of the length, and medians of a triangle meet at a point.
3. Prove theorems about parallelograms including: the congruence of opposite sides, the congruence of opposite angles, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.

Geometry – Similarity, Right Triangles, and Trigonometry

Students will be able to:

1. Verify the properties of dilations given by a center and a scale factor, specifically a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged, and the dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar, explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
4. Prove theorems about triangles including: a line parallel to one side of a triangle divides the other two proportionally and conversely and the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complimentary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Geometry - Circles

Students will be able to:

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords including the relationship between central, inscribed, and circumscribed angles, inscribed angles on a diameter are right angles, the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

3. Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
4. Construct a tangent line from a point outside a given circle to the circle.
5. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality, and derive the formula for the area of a sector.

Geometry – Expressing Geometric Properties with Equations

Students will be able to:

1. Derive the equation of a circle given the center and radius using the Pythagorean Theorem and complete the square to find the center and radius of a circle given by an equation.
2. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle and prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.
3. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Geometry – Geometric Measurement and Dimension

Students will be able to:

1. Give an informal argument for the formulas for the circumference and area of a circle, and the volume of a cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments.
2. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems with attention to scale factors and transformations.

Statistics – Interpreting Categorical and Quantitative Data

Students will be able to:

1. Summarize categorical data for two categories in two-way frequency tables and interpret relative frequencies in the context of the data including joint, marginal, and condition relative frequencies. Recognize possible associations and trends in the data.

Statistics – Conditional Probability and the Rules of Probability

Students will be able to:

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or compliments of other events ("or," "and," "not").
2. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified and use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

3. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
4. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Literacy Standards

Students will be able to:

1. Acquire, interpret, and accurately use grade level appropriate mathematical words and terms.
2. Engage in collaborative discussions with diverse partners on grade level concepts.