



AP Calculus AB Summer Assignment

(for students who have completed Honors Precalculus and are entering AP Calculus AB)

Reviewing key concepts from Algebra 2 and Precalculus is an excellent way to be fully prepared for learning new concepts in Calculus that require prerequisite skills. The following packet will help you practice and also self-assess any concepts that you may want to spend extra time on before the start of school.

You should expect to submit this completed packet on the first day of school and plan to be assessed on these skills at the beginning of the school year. An answer key is provided; however, ensure your work is shown to receive full credit.

A breakdown of the skills covered in the packet by item number is as follows:

- #1-3 Transformations using function notation
- #4-6 Graphing piece-wise functions
- #7-12 Evaluating composite functions
- #13-14 End behavior and zeros of polynomial functions
- #15-16 Rational functions, holes, intercepts, limit behavior at vertical & horizontal asymptotes
- #17-18 Properties of logarithms
- #19-20 Solving trigonometric equations
- #21-22 Evaluating inverse trigonometric expressions
- #23-29 Evaluating limits using direct substitution and simplification techniques

If you would like additional resources to support your practice, we recommend Khan Academy as a great first step. For in-person support, consider a peer tutor or a more structured option such as Mathnasium. For a list of peer tutors who are willing to tutor over the summer for community service hours, please contact Linda Graham, Math Department Chair.

Name: _____

You are to be able to recite these following facts from memory:

1. To find x-intercepts, set $y = 0$ and solve the equation for x .
2. To find y-intercepts, set $x = 0$ and solve the equation for y .
3. Tests for symmetry
 - a. The graph of an equation in x and y is symmetric with respect to the y -axis if replacing x by $-x$ yields an equivalent equation.
 - b. The graph of an equation in x and y is symmetric with respect to the x -axis if replacing y by $-y$ yields an equivalent equation.
 - c. The graph of an equation in x and y is symmetric with respect to the origin if replacing x by $-x$ and y by $-y$ yields an equivalent equation.
4. The slope m of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$
5. Summary of Equations of Lines
 - a. General form: $Ax + By + C = 0$
 - b. Vertical line: $x = a$
 - c. Horizontal line: $y = b$
 - d. Point-slope form: $y - y_1 = m(x - x_1)$
 - e. Slope-intercept form: $y = mx + b$
6. Functions can be specified in a variety of ways:
 - a. Implicitly: $x^2 + 2y = 2$
 - b. Explicitly: (y by itself) $y = (2 - x^2)/2$
 - c. Function notation: $f(x) = (2 - x^2)/2$
7. A function is one-to-one if to each y -value in the range there corresponds exactly one x -value in the domain.
8. Basic Types of Transformations ($c > 0$)

Original graph: $y = f(x)$

 - a. Horizontal shift c units to the right: $y = f(x - c)$
 - b. Horizontal shift c units to the left: $y = f(x + c)$
 - c. Vertical shift c units downward: $y = f(x) - c$
 - d. Vertical shift c units upward: $y = f(x) + c$
 - e. Reflection about the x -axis: $y = -f(x)$
 - f. Reflection about the y -axis: $y = f(-x)$
 - g. Reflection about the origin: $y = -f(-x)$
9. Terms: polynomial, degree of a polynomial, coefficients, leading coefficient, constant term, rational function, algebraic function, transcendental function
10. Definition of a Composite Function: Let f and g be functions. The function given by $(f \circ g)(x) = f(g(x))$ is called the composite of f with g . The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f .
11. Even and odd functions
 - a. A function is even if its graph is symmetric with respect to the y -axis. The function $y = f(x)$ is even if $f(-x) = f(x)$.
 - b. A function is odd if its graph is symmetric with respect to the origin. The function $y = f(x)$ is odd if $f(-x) = -f(x)$.
12. Definition of a Limit: Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement $\lim_{x \rightarrow c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.
13. Some Basic Limits: Let b and c be real numbers and let n be a positive integer.
 - a. $\lim_{x \rightarrow c} b = b$
 - b. $\lim_{x \rightarrow c} x = c$
 - c. $\lim_{x \rightarrow c} x^n = c^n$
14. Properties of Limits: Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = K$

 - a. Scalar multiple: $\lim_{x \rightarrow c} [bf(x)] = bL$
 - b. Sum or difference: $\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$
 - c. Product: $\lim_{x \rightarrow c} [f(x)g(x)] = LK$
 - d. Quotient: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
 - e. Power: $\lim_{x \rightarrow c} [f(x)]^n = L^n$

15. Limits of Polynomial and Rational Functions

- a. If p is a polynomial function and c is a real number, then $\lim_{x \rightarrow c} p(x) = p(c)$.
- b. If r is a rational function given by $r(x) = p(x)/q(x)$ and c is a real number such that $q(c) \neq 0$, then $\lim_{x \rightarrow c} r(x) = r(c) = \frac{p(c)}{q(c)}$.

16. The Limit of a Function Involving a Radical: Let n be a positive integer

The following limit is valid for all c if n is odd, and is valid for $c > 0$ if n is even. $\lim_{x \rightarrow c} \sqrt[n]{x} = \sqrt[n]{c}$

17. The Limit of a Composite Function. If f and g are functions such that $\lim_{x \rightarrow c} g(x) = L$ and $\lim_{x \rightarrow L} f(x) = f(L)$, then $\lim_{x \rightarrow c} f(g(x)) = f(L)$.

18. Limits of Trigonometric Functions. Let c be a real number in the domain of the given trigonometric function.

- a. $\lim_{x \rightarrow c} \sin x = \sin c$ b. $\lim_{x \rightarrow c} \cos x = \cos c$ c. $\lim_{x \rightarrow c} \tan x = \tan c$
- d. $\lim_{x \rightarrow c} \cot x = \cot c$ e. $\lim_{x \rightarrow c} \sec x = \sec c$ f. $\lim_{x \rightarrow c} \csc x = \csc c$

19. Functions that agree at All But One Point

Let c be a real number and let $f(x) = g(x)$ for all $x \neq c$ in an open interval containing c . If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ also exists and $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)$.

20. Intervals on the real line

- a. Bounded open interval: (a, b) $\{x : a < x < b\}$
- b. Bounded closed interval: $[a, b]$ $\{x : a \leq x \leq b\}$
- c. Unbounded open interval: $(-\infty, a)$ $\{x : x < a\}$
- d. Unbounded closed interval: $[a, \infty)$ $\{x : x \geq a\}$ etc.

21. The distance between points (x_1, y_1) and (x_2, y_2) in a plane is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

22. Midpoint formula for points (x_1, y_1) and (x_2, y_2) in a plane is given by $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.

23. Standard Form of the Equation of a Circle. The point (x, y) lies on the circle of radius r and center (h, k) if and only if $(x - h)^2 + (y - k)^2 = r^2$.

24. General Form of the Equation of a Circle. $Ax^2 + Ay^2 + Dx + Ey + F = 0$, $A \neq 0$

25. Definitions of the Six Trigonometric Functions.

- a. For an acute angle in a right triangle: SohCahToa

- b. Circular functions where θ is any angle $r = \sqrt{x^2 + y^2}$
- $\sin \theta = y/r$ $\cos \theta = x/r$ $\tan \theta = y/x$ $\csc \theta = r/y$ $\sec \theta = r/x$ $\cot \theta = x/y$

26. Pythagorean Identities:

$\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $\cot^2 \theta + 1 = \csc^2 \theta$

27. Reduction Formulas:

$\sin(-\theta) = -\sin \theta$ $\cos(-\theta) = \cos \theta$ $\tan(-\theta) = -\tan \theta$ $\sin(\pi - \theta) = \sin \theta$ $\cos(\pi - \theta) = -\cos \theta$ $\tan(\pi - \theta) = -\tan \theta$

28. Sum of Difference of Two Angles:

$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$ $\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

29. Double Angles:

$\sin 2\theta = 2\sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\cos 2\theta = 2\cos^2 \theta - 1$ $\cos 2\theta = 1 - 2\sin^2 \theta$

30. Reciprocal Identities:

$\csc \theta = 1/\sin \theta$ $\sec \theta = 1/\cos \theta$ $\cot \theta = 1/\tan \theta$

31. Quotient Identities:

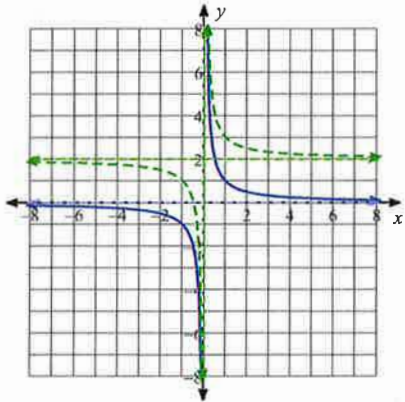
$\tan \theta = \sin \theta / \cos \theta$ $\cot \theta = \cos \theta / \sin \theta$

32. Derivative of a function (slope function): $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

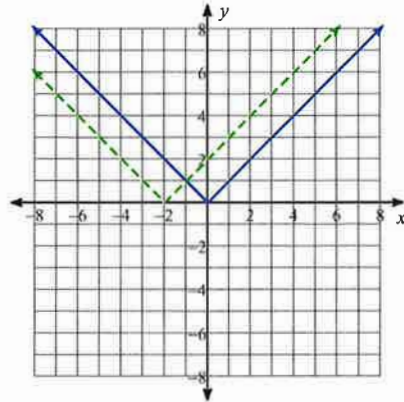
Summer Review

Write $g(x)$ (dashed line) in terms of $f(x)$ (solid line).

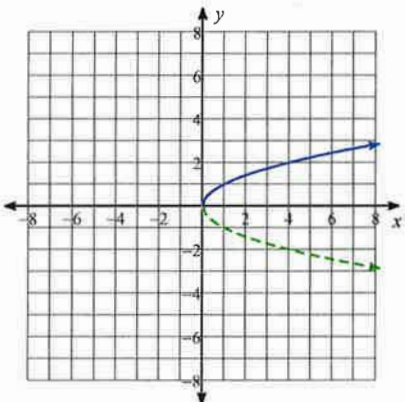
1)



2)

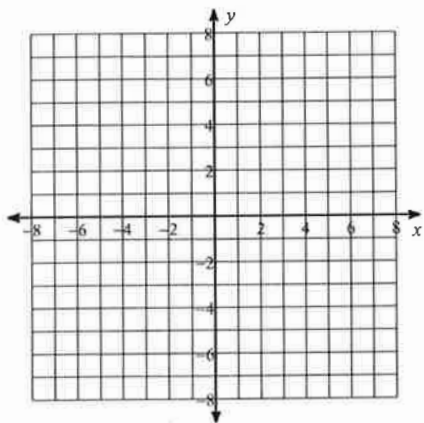


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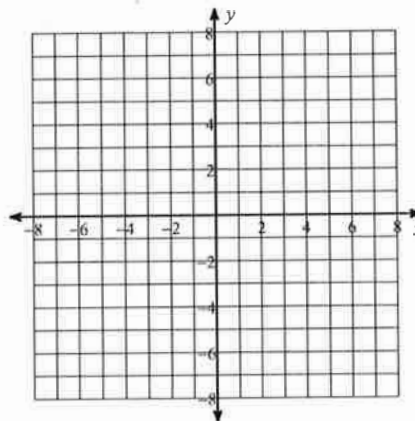


Sketch the graph of each function.

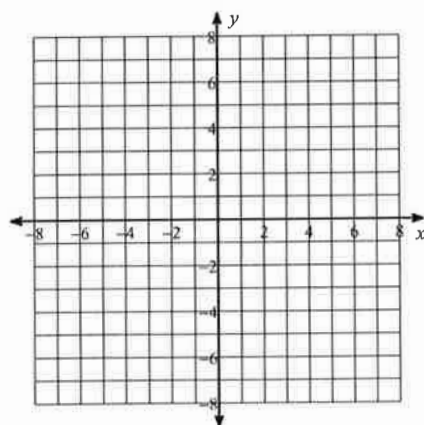
$$4) g(x) = \begin{cases} (x+2)^2, & x \leq -2 \\ -4 + \sqrt{x}, & -2 < x \leq 4 \\ \sqrt{3x}, & x > 4 \end{cases}$$



$$5) f(x) = \begin{cases} \frac{|x|}{2}, & x \leq -3 \\ -2|x|, & x > -3 \end{cases}$$



$$6) f(x) = \begin{cases} (x+1)^2, & x \leq -1 \\ 4 - x^2, & x > -1 \end{cases}$$



Perform the indicated operation.

$$7) \begin{aligned} g(x) &= x^2 - 4 \\ f(x) &= x - 1 \\ \text{Find } (g + f)(x) \end{aligned}$$

$$8) \begin{aligned} g(n) &= 3n - 4 \\ f(n) &= -3n \\ \text{Find } (g \circ f)(n) \end{aligned}$$

$$9) \begin{aligned} g(x) &= 4x + 2 \\ \text{Find } (g \circ g)(x) \end{aligned}$$

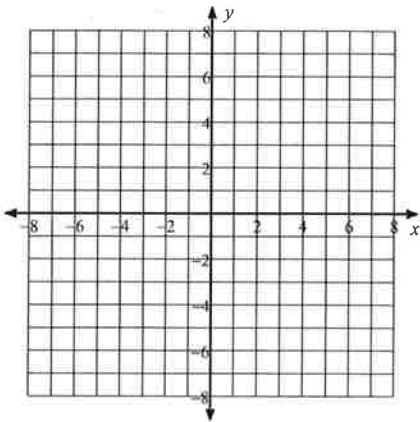
$$10) \begin{aligned} h(t) &= 4t + 2 \\ g(t) &= t^2 + 4t \\ \text{Find } (h - g)(-6) \end{aligned}$$

$$11) \begin{aligned} h(n) &= -2n + 3 \\ g(n) &= n^2 + 2n \\ \text{Find } h(-3) + g(-3) \end{aligned}$$

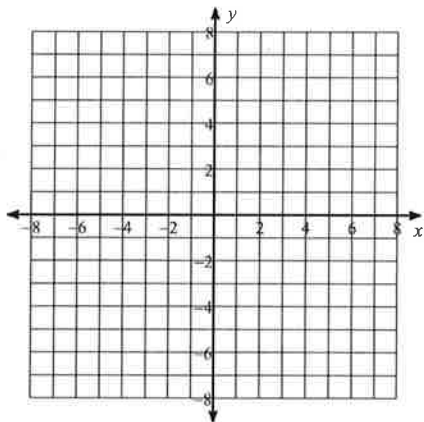
$$12) \begin{aligned} g(x) &= x - 3 \\ f(x) &= x + 5 \\ \text{Find } g(f(10)) \end{aligned}$$

For each function: (1) determine the real zeros and state the multiplicity of any repeated zeros, (2) describe the end behavior, and (3) sketch the graph.

13) $f(x) = -x^2 + 2x - 1$

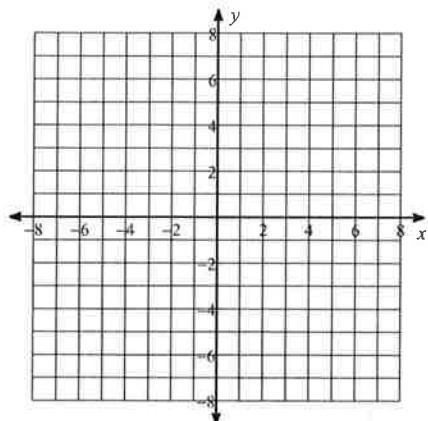


14) $f(x) = x^3 + 4x^2 + 4x$

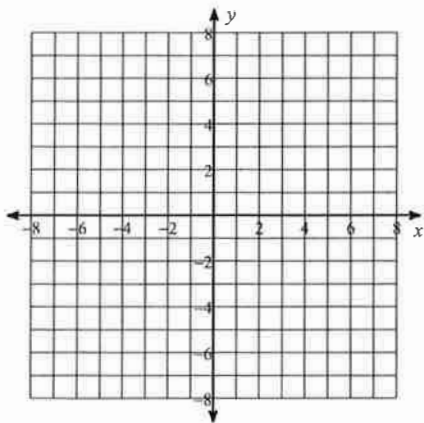


For each function, identify the holes, intercepts, horizontal asymptote, limit behavior at all vertical asymptotes, and end behavior asymptote. Then sketch the graph.

15) $f(x) = \frac{x^2 + 4x}{4x^2 - 36}$



$$16) f(x) = \frac{x^3 - x^2 - 12x}{4x^2 - 4x - 8}$$



Condense each expression to a single logarithm.

$$17) \ln u - 2 \ln v$$

$$18) 5 \ln u - \ln v$$

Solve each equation for $0 \leq \theta < 2\pi$.

$$19) -2 + \tan \theta = -3$$

$$20) \frac{-10 - \sqrt{2}}{2} = -5 + \cos \theta$$

Find the exact value of each expression.

$$21) \cos^{-1} \frac{\sqrt{2}}{2}$$

$$22) \sin^{-1} -\frac{1}{2}$$

Evaluate each limit.

$$23) \lim_{x \rightarrow -4} 1$$

$$24) \lim_{x \rightarrow 2} \left(\frac{x^2}{2} - 3 \right)$$

$$25) \lim_{x \rightarrow 2} \frac{x-2}{x^2-6x+8}$$

$$26) \lim_{x \rightarrow 1} \frac{x^2-5x+4}{x-1}$$

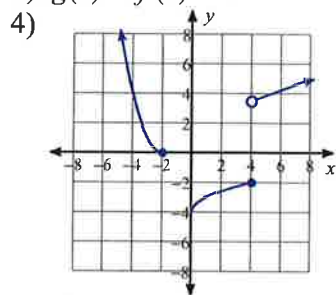
$$27) \lim_{x \rightarrow -\infty} \frac{2x}{x+2}$$

$$28) \lim_{x \rightarrow -\infty} \frac{x^4}{2x^2-1}$$

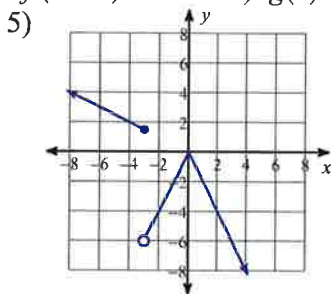
$$29) \lim_{x \rightarrow \infty} \frac{3x^3}{2x^2+1}$$

Answers to Summer Review

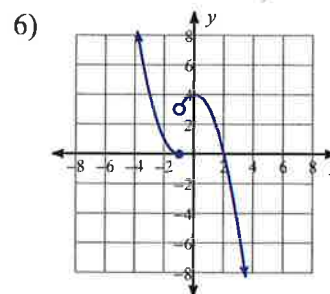
1) $g(x) = f(x) + 2$



2) $g(x) = f(x + 2)$



3) $g(x) = -f(x)$



7) $x^2 + x - 5$

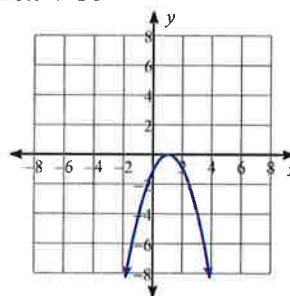
11) 12

8) $-9n - 4$

12) 12

9) $16x + 10$

13)



10) -34

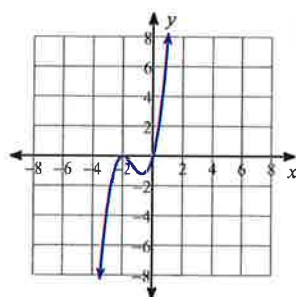
Real zeros: {1 mult. 2}

End behavior:

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = -\infty$$

14)



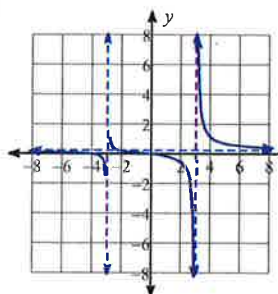
Real zeros: {0, -2 mult. 2}

End behavior:

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

15)



Holes: None

Horz. Asym.: $y = \frac{1}{4}$

x-intercepts: 0, -4, y-intercept: 0

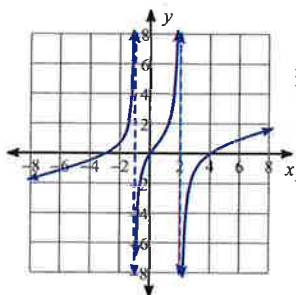
Vert. Asym. behavior:

$$\lim_{x \rightarrow -3^-} f(x) = -\infty, \lim_{x \rightarrow -3^+} f(x) = \infty$$

$$\lim_{x \rightarrow 3^-} f(x) = -\infty, \lim_{x \rightarrow 3^+} f(x) = \infty$$

End behavior asym.: $y = \frac{1}{4}$

16)



Holes: None

Horz. Asym.: None

x-intercepts: 0, 4, -3, y-intercept: 0

Vert. Asym. behavior:

$$\lim_{x \rightarrow -1^-} f(x) = \infty, \lim_{x \rightarrow -1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = \infty, \lim_{x \rightarrow 2^+} f(x) = -\infty$$

End behavior asym.: $y = \frac{x}{4}$

17) $\ln \frac{u}{v^2}$

18) $\ln \frac{u^5}{v}$

19) $\left\{ \frac{3\pi}{4}, \frac{7\pi}{4} \right\}$

23) 1

27) 2

20) $\left\{ \frac{3\pi}{4}, \frac{5\pi}{4} \right\}$

24) -1

28) $-\infty$

21) $\frac{\pi}{4}$

25) $\frac{1}{2}$

29) $-\infty$

22) $-\frac{\pi}{6}$

26) -3