Summer Work for Students Entering Precalculus

The work in this packet is designed to take about 45-60 minutes per week, it is suggested not to leave it all for the last couple weeks of August, you won't be happy if you do! (Do only the circled problems)

Both the problems and the answers are here. You will turn this work in the first day you have class. Please do the problems and check your answers (marking which are correct and looking for your mistakes in the incorrect ones). Please use your textbooks both from Algebra 2 and for the coming year as references for any trouble you find. In addition Miss Hedges will be checking email over the summer so you can email questions that you have. All topics in this packet should be review!

Enjoy the summer but keep your mind and skills sharp with this work!

Lines

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

The Slope of a Line

1. A line with positive slope ($m > 0$) *rises* from left to right.
2. A line with negative slope ($m < 0$) *falls* from left to right.
3. A line with zero slope ($m = 0$) is *horizontal*.
4. A line with undefined slope is *vertical*.

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

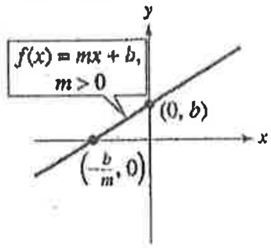
Library of Functions: Linear Function

In the next section, you will be introduced to the precise meaning of the term *function*. The simplest type of function is a *linear function* of the form

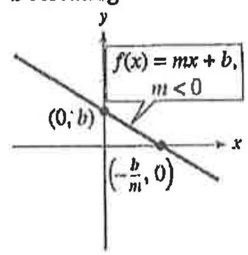
$$f(x) = mx + b.$$

As its name implies, the graph of a linear function is a line that has a slope of m and a y-intercept at $(0, b)$. The basic characteristics of a linear function are summarized below. (Note that some of the terms below will be defined later in the text.)

Graph of $f(x) = mx + b, m > 0$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing



Graph of $f(x) = mx + b, m < 0$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Decreasing



When $m = 0$, the function $f(x) = b$ is called a *constant function* and its graph is a horizontal line.

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is,

$$m_1 = m_2.$$

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

1.1 Exercises

Vocabulary Check

1. Match each equation with its form.

(a) $Ax + By + C = 0$	(i) vertical line
(b) $x = a$	(ii) slope-intercept form
(c) $y = b$	(iii) general form
(d) $y = mx + b$	(iv) point-slope form
(e) $y - y_1 = m(x - x_1)$	(v) horizontal line

In Exercises 2–5, fill in the blanks.

2. For a line, the ratio of the change in y to the change in x is called the _______ of the line.
3. Two lines are _______ if and only if their slopes are equal.
4. Two lines are _______ if and only if their slopes are negative reciprocals of each other.
5. The prediction method _______ is the method used to estimate a point on a line that does not lie between the given points.

In Exercises 1 and 2, identify the line that has the indicated slope.

1. (a) $m = \frac{2}{3}$ (b) m is undefined. (c) $m = -2$
2. (a) $m = 0$ (b) $m = -\frac{3}{4}$ (c) $m = 1$

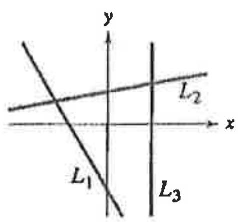


Figure for 1

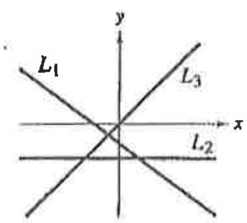
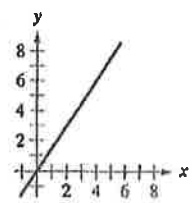


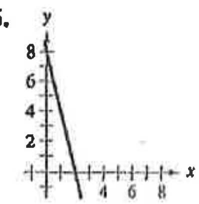
Figure for 2

In Exercises 3 and 4, sketch the lines through the point with the indicated slopes on the same set of coordinate axes.

Point	*Slopes*	
3. (2, 3)	(a) 0	(b) 1
	(c) 2	(d) -3
4. $(-4, 1)$	(a) 3	(b) -3
	(c) $\frac{1}{2}$	(d) Undefined

In Exercises 5 and 6, estimate the slope of the line.

5. 

6. 

In Exercises 7–10, use a graphing utility to plot the points and use the *draw* feature to graph the line segment connecting the two points. (Use a square setting.) Then find the slope of the line passing through the pair of points.

7. $(0, -10), (-4, 0)$
8. $(2, 4), (4, -4)$
9. $(-6, -1), (-6, 4)$
10. $(-3, -2), (1, 6)$

In Exercises 11–18, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)

Point	*Slope*
11. (2, 1)	$m = 0$
12. $(3, -2)$	$m = 0$
13. (1, 5)	m is undefined.
14. $(-4, 1)$	m is undefined.

	Point	*Slope*
15.	$(0, -9)$	$m = -2$
16.	$(-5, 4)$	$m = 2$
17.	$(7, -2)$	$m = \frac{1}{2}$
18.	$(-1, -6)$	$m = -\frac{1}{2}$

In Exercises 19–24, (a) find the slope and *y*-intercept (if possible) of the equation of the line algebraically, (b) sketch the line by hand, and (c) use a graphing utility to verify your answers to parts (a) and (b).

19. $5x - y + 3 = 0$
20. $2x + 3y - 9 = 0$
21. $5x - 2 = 0$
22. $3x + 7 = 0$
23. $3y + 5 = 0$
24. $-11 - 8y = 0$

In Exercises 25–32, find the general form of the equation of the line that passes through the given point and has the indicated slope. Sketch the line by hand. Use a graphing utility to verify your sketch, if possible.

	Point	*Slope*
25.	$(0, -2)$	$m = 3$
26.	$(-3, 6)$	$m = -2$
27.	$(0, 0)$	$m = 4$
28.	$(-2, -5)$	$m = \frac{3}{4}$
29.	$(6, -1)$	m is undefined.
30.	$(-10, 4)$	m is undefined.
31.	$\left(-\frac{1}{2}, \frac{3}{2}\right)$	$m = 0$
32.	$(2.3, -8.5)$	$m = 0$

In Exercises 33–42, find the slope-intercept form of the equation of the line that passes through the points. Use a graphing utility to graph the line.

33. $(5, -1), (-5, 5)$
34. $(4, 3), (-4, -4)$
35. $(-8, 1), (-8, 7)$
36. $(-1, 4), (6, 4)$
37. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$
38. $(1, 1), \left(6, -\frac{2}{3}\right)$
39. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$
40. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$
41. $(1, 0.6), (-2, -0.6)$
42. $(-8, 0.6), (2, -2.4)$

43. *Annual Salary* A jeweler's salary was \$28,500 in 2000 and \$32,900 in 2002. The jeweler's salary follows a linear growth pattern. What will the jeweler's salary be in 2006?

44. *Annual Salary* A librarian's salary was \$25,000 in 2000 and \$27,500 in 2002. The librarian's salary follows a linear growth pattern. What will the librarian's salary be in 2006?

In Exercises 45–48, determine the slope and *y*-intercept of the linear equation. Then describe its graph.

45. $x - 2y = 4$
46. $3x + 4y = 1$
47. $x = -6$
48. $y = 12$

In Exercises 49 and 50, use a graphing utility to graph the equation using each of the suggested viewing windows. Describe the difference between the two graphs.

49. $y = 0.5x - 3$

Xmin = -5	Xmin = -2
Xmax = 10	Xmax = 10
Xscl = 1	Xscl = 1
Ymin = -1	Ymin = -4
Ymax = 10	Ymax = 1
Yscl = 1	Yscl = 1

50. $y = -8x + 5$

Xmin = -5	Xmin = -5
Xmax = 5	Xmax = 10
Xscl = 1	Xscl = 1
Ymin = -10	Ymin = -80
Ymax = 10	Ymax = 80
Yscl = 1	Yscl = 20

In Exercises 51–54, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.

51. L_1: $(0, -1), (5, 9)$
    L_2: $(0, 3), (4, 1)$
52. L_1: $(-2, -1), (1, 5)$
    L_2: $(1, 3), (5, -5)$
53. L_1: $(3, 6), (-6, 0)$
    L_2: $(0, -1), \left(5, \frac{7}{3}\right)$
54. L_1: $(4, 8), (-4, 2)$
    L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

In Exercises 55–60, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	*Line*
55.	$(2, 1)$	$4x - 2y = 3$
56.	$(-3, 2)$	$x + y = 7$
57.	$\left(-\frac{2}{3}, \frac{7}{8}\right)$	$3x + 4y = 7$
58.	$(-3.9, -1.4)$	$6x + 2y = 9$
59.	$(3, -2)$	$x - 4 = 0$
60.	$(-4, 1)$	$y + 2 = 0$

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Graphical Analysis **In Exercises 61–64, identify any relationships that exist among the lines, and then use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that each slope appears visually correct. Use the slopes of the lines to verify your results.**

61. (a) $y = 2x$ (b) $y = -2x$ (c) $y = \frac{1}{2}x$

62. (a) $y = \frac{2}{3}x$ (b) $y = -\frac{3}{2}x$ (c) $y = \frac{2}{3}x + 2$

63. (a) $y = -\frac{1}{2}x$ (b) $y = -\frac{1}{2}x + 3$ (c) $y = 2x - 4$

64. (a) $y = x - 8$ (b) $y = x + 1$ (c) $y = -x + 3$

65. ***Sales*** The following are the slopes of lines representing annual sales y in terms of time x in years. Use each slope to interpret any change in annual sales for a one-year incfease in time.

(a) The line has a slope of $m = 135$.

(b) The line has a slope of $m = 0$.

(c) The line has a slope of $m = -40$.

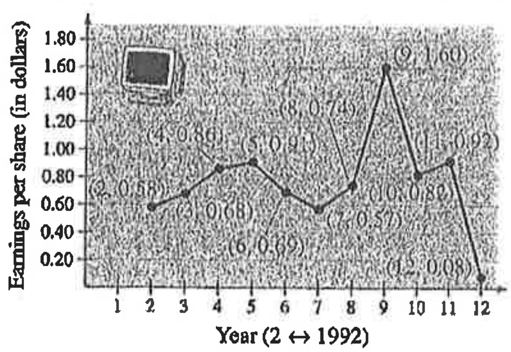
66. ***Revenue*** The following are the slopes of lines representing daily revenues y in terms of time x in days. Use each slope to interpret any change in daily revenues for a one-day increase in time.

(a) The line has a slope of $m = 400$.

(b) The line has a slope of $m = 100$.

(c) The line has a slope of $m = 0$.

67. ***Earnings per Share*** The graph shows the earnings per share of stock for Circuit City for the years 1992 through 2002. (Source: Circuit City Stores, Inc.)



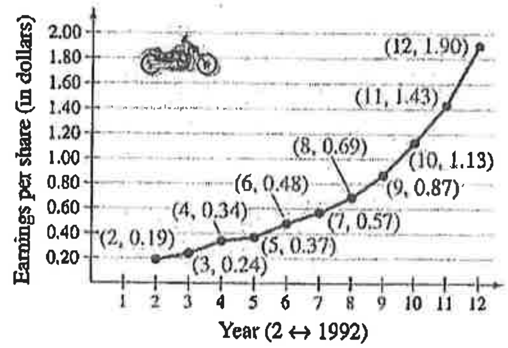
(a) Use the slopes to determine the year(s) in which the earnings per share of stock showed the greatest increase and decrease.

(b) Find the equation of the line between the years 1992 and 2002.

(c) Interpret the meaning of the slope of the equation from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the earnings per share of stock for the year 2006. Do you think this is an accurate estimation? Explain.

68. ***Earnings per Share*** The graph shows the earnings per share of stock for Harley-Davidson, Inc. for the years 1992 through 2002. (Source: Harley-Davidson, Inc.)



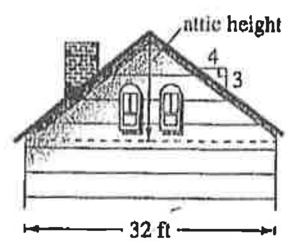
(a) Use the slopes to determine the years in which the earnings per share of stock showed the greatest increase and the smallest increase.

(b) Find the equation of the line between the years 1992 and 2002.

(c) Interpret the meaning of the slope of the equation from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the earnings per share of stock for the year 2006. Do you think this is an accurate estimation? Explain.

69. ***Height*** The "rise to run" ratio of the roof of a house determines the steepness of the roof. The rise to run ratio of a roof is 3 to 4. Determine the maximum height in the attic of the house if the house is 32 feet wide.



Functions

Definition of a Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

Characteristics of a Function from Set A to Set B

1. Each element of A must be matched with an element of B.
2. Some elements of B may not be matched with any element of A.
3. Two or more elements of A may be matched with the same element of B.
4. An element of A (the domain) cannot be matched with two different elements of B.

Library of Functions: Data Defined Function

Many functions do not have simple mathematical formulas, but are defined by real-life data. Such functions arise when you are using collections of data to model real-life applications. Functions can be represented in four ways.

1. *Verbally* by a sentence that describes how the input variable is related to the output variable

   Example: The input value x is the election year from 1952 to 2004 and the output value y is the elected president of the United States.

2. *Numerically* by a table or a list of ordered pairs that matches input values with output values

   Example: In the set of ordered pairs $\{(2, 34), (4, 40), (6, 45), (8, 50), (10, 54)\}$, the input value is the age of a male child in years and the output value is the height of the child in inches.

3. *Graphically* by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis

   Example: See Figure 1.15.

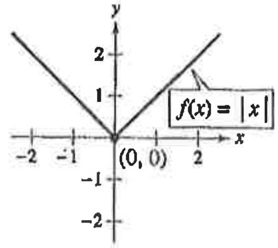
4. *Algebraically* by an equation in two variables

   Example: The formula for temperature, $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius, is an equation that represents a function. You will see that it is often convenient to approximate data using a mathematical model or formula.

Library of Functions: Piecewise-Defined Function

A *piecewise-defined function* is a function that is defined by two or more equations over a specified domain. The *absolute value function* given by $f(x) = |x|$ can be written as a piecewise-defined function. The basic characteristics of the absolute value function are summarized below.

Graph of $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

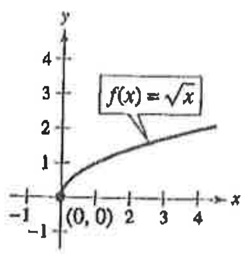


Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$

Library of Functions: Radical Function

Radical functions arise from the use of rational exponents. The most common radical function is the *square root function* given by $f(x) = \sqrt{x}$. The basic characteristics of the square root function are summarized below.

Graph of $f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Difference Quotients

One of the basic definitions in calculus employs the ratio

$$\frac{f(x + h) - f(x)}{h}, \qquad h \neq 0.$$

Summary of Function Terminology

Function: A **function** is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

Function Notation: $y = f(x)$

f is the *name* of the function.
y is the **dependent variable,** or output value.
x is the **independent variable,** or input value.
$f(x)$ is the *value of the function at x.*

Domain: The **domain** of a function is the set of all values (inputs) of the independent variable for which the function is defined. If x is in the domain of f, f is said to be *defined* at x. If x is not in the domain of f, f is said to be *undefined* at x.

Range: The **range** of a function is the set of all values (outputs) assumed by the dependent variable (that is, the set of all function values).

Implied Domain: If f is defined by an algebraic expression and the domain is not specified, the **implied domain** consists of all real numbers for which the expression is defined.

1.2 Exercises

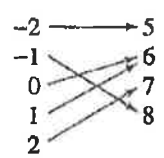
Vocabulary Check

Fill in the blanks.

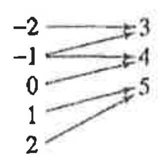
1. A relation that assigns to each element x from a set of inputs, or _______ , exactly one element y in a set of outputs, or _______ , is called a _______ .
2. For an equation that represents y as a function of x, the _______ variable is the set of all x in the domain, and the _______ variable is the set of all y in the range.
3. The function $f(x) = \begin{cases} x^2 - 4, x \le 0 \\ 2x + 1, x > 0 \end{cases}$ is an example of a _______ function.
4. If the domain of the function f is not given, then the set of values of the independent variable for which the expression is defined is called the _______ .
5. In calculus, one of the basic definitions is that of a _______ , given by $\frac{f(x + h) - f(x)}{h}, h \neq 0$.

In Exercises 1–4, does the relationship describe a function? Explain your reasoning.

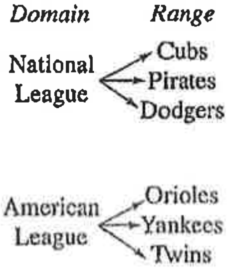
1. *Domain* *Range*



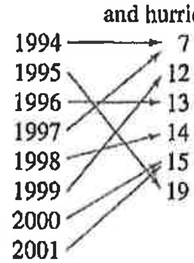
2. *Domain* *Range*



3. *Domain* *Range*



4. *Domain* (Year) *Range* (Number of North Atlantic tropical storms and hurricanes)



In Exercises 5–8, does the table describe a function? Explain your reasoning.

5.

Input Value	−2	−1	0	1	2
Output Value	−8	−1	0	1	8

6.

Input Value	0	1	2	1	0
Output Value	−4	−2	0	2	4

7.

Input Value	10	7	4	7	10
Output Value	3	6	9	12	15

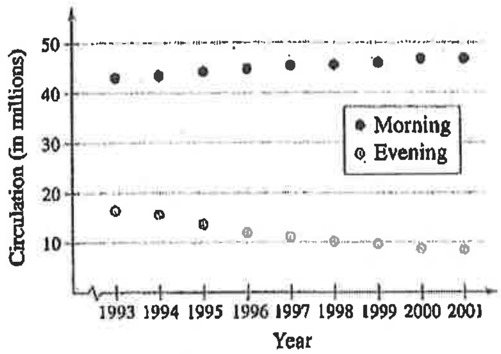
8.

Input Value	0	3	9	12	15
Output Value	3	3	3	3	3

In Exercises 9 and 10, which sets of ordered pairs represent functions from A to B? Explain.

9. $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2\}$
   (a) $\{(0, 1), (1, -2), (2, 0), (3, 2)\}$
   (b) $\{(0, -1), (2, 2), (1, -2), (3, 0), (1, 1)\}$
   (c) $\{(0, 0), (1, 0), (2, 0), (3, 0)\}$
   (d) $\{(0, 2), (3, 0), (1, 1)\}$
10. $A = \{a, b, c\}$ and $B = \{0, 1, 2, 3\}$
    (a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$
    (b) $\{(a, 1), (b, 2), (c, 3)\}$
    (c) $\{(1, a), (0, a), (2, c), (3, b)\}$
    (d) $\{(c, 0), (b, 0), (a, 3)\}$

Circulation of Newspapers **In Exercises 11 and 12, use the graph, which shows the circulation (in millions) of daily newspapers in the United States.** (Source: Editor & Publisher Company)



11. Is the circulation of morning newspapers a function of the year? Is the circulation of evening newspapers a function of the year? Explain.

12. Let $f(x)$ represent the circulation of evening newspapers in year x. Find $f(2000)$.

In Exercises 13–24, determine whether the equation represents y as a function of x.

13. $x^2 + y^2 = 4$	**14.** $x = y^2$
15. $x^2 + y = -1$	**16.** $y = \sqrt{x + 5}$
17. $2x + 3y = 4$	**18.** $x = -y + 5$
19. $y^2 = x^2 - 1$	**20.** $x + y^2 = 3$
21. $y = \lvert 4 - x \rvert$	**22.** $\lvert y \rvert = 4 - x$
23. $x = -7$	**24.** $y = 8$

In Exercises 25 and 26, fill in the blanks using the specified function and the given values of the independent variable. Simplify the result.

25. $f(x) = \dfrac{1}{x + 1}$

(a) $f(4) = \dfrac{1}{(\quad) + 1}$

(b) $f(0) = \dfrac{1}{(\quad) + 1}$

(c) $f(4t) = \dfrac{1}{(\quad) + 1}$

(d) $f(x + c) = \dfrac{1}{(\quad) + 1}$

26. $g(x) = x^2 - 2x$

(a) $g(2) = (\quad)^2 - 2(\quad)$

(b) $g(-3) = (\quad)^2 - 2(\quad)$

(c) $g(t + 1) = (\quad)^2 - 2(\quad)$

(d) $g(x + c) = (\quad)^2 - 2(\quad)$

In Exercises 27–38, evaluate the function at each specified value of the independent variable and simplify.

27. $f(x) = 2x - 3$
(a) $f(1)$ (b) $f(-3)$ (c) $f(x - 1)$

28. $g(y) = 7 - 3y$
(a) $g(0)$ (b) $g\left(\frac{7}{3}\right)$ (c) $g(s + 2)$

29. $h(t) = t^2 - 2t$
(a) $h(2)$ (b) $h(1.5)$ (c) $h(x + 2)$

30. $V(r) = \frac{4}{3}\pi r^3$
(a) $V(3)$ (b) $V\left(\frac{3}{2}\right)$ (c) $V(2r)$

31. $f(y) = 3 - \sqrt{y}$
(a) $f(4)$ (b) $f(0.25)$ (c) $f(4x^2)$

32. $f(x) = \sqrt{x + 8} + 2$
(a) $f(-8)$ (b) $f(1)$ (c) $f(x - 8)$

33. $q(x) = \dfrac{1}{x^2 - 9}$
(a) $q(0)$ (b) $q(3)$ (c) $q(y + 3)$

34. $q(t) = \dfrac{2t^2 + 3}{t^2}$
(a) $q(2)$ (b) $q(0)$ (c) $q(-x)$

35. $f(x) = \dfrac{\lvert x \rvert}{x}$
(a) $f(2)$ (b) $f(-2)$ (c) $f(x^2)$

36. $f(x) = \lvert x \rvert + 4$
(a) $f(2)$ (b) $f(-2)$ (c) $f(x^2)$

37. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$
(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$

38. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$
(a) $f(-2)$ (b) $f(1)$ (c) $f(2)$

In Exercises 39–42, complete the table.

39. $h(t) = \frac{1}{2}|t + 3|$

t	-5	-4	-3	-2	-1
$h(t)$					

40. $f(s) = \dfrac{|s - 2|}{s - 2}$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$					

41. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x - 2)^2, & x > 0 \end{cases}$

x	-2	-1	0	1	2
$f(x)$					

42. $h(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

x	1	2	3	4	5
$h(x)$					

In Exercises 43–46, find all real values of x such that $f(x) = 0$.

43. $f(x) = 15 - 3x$
44. $f(x) = 5x + 1$
45. $f(x) = \dfrac{3x - 4}{5}$
46. $f(x) = \dfrac{12 - x^2}{5}$

In Exercises 47 and 48, find the value(s) of x for which $f(x) = g(x)$.

47. $f(x) = x^2, \quad g(x) = x + 2$
48. $f(x) = x^2 + 2x + 1, \quad g(x) = 3x + 3$

In Exercises 49–58, find the domain of the function.

49. $f(x) = 5x^2 + 2x - 1$
50. $g(x) = 1 - 2x^2$
51. $h(t) = \dfrac{4}{t}$
52. $s(y) = \dfrac{3y}{y + 5}$
53. $f(x) = \sqrt[3]{x - 4}$
54. $f(x) = \sqrt[4]{x^2 + 3x}$
55. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$
56. $h(x) = \dfrac{10}{x^2 - 2x}$
57. $g(y) = \dfrac{y + 2}{\sqrt{y - 10}}$
58. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$

In Exercises 59–62, use a graphing utility to graph the function. Find the domain and range of the function.

59. $f(x) = \sqrt{4 - x^2}$
60. $f(x) = \sqrt{x^2 + 1}$
61. $g(x) = |2x + 3|$
62. $g(x) = |x - 5|$

In Exercises 63–66, assume that the domain of f is the set $A = \{-2, -1, 0, 1, 2\}$. Determine the set of ordered pairs representing the function f.

63. $f(x) = x^2$
64. $f(x) = x^2 - 3$
65. $f(x) = |x| + 2$
66. $f(x) = |x + 1|$

67. ***Geometry*** Write the area A of a circle as a function of its circumference C.

68. ***Geometry*** Write the area A of an equilateral triangle as a function of the length s of its sides.

69. ***Exploration*** The cost per unit to produce a radio model is \$60. The manufacturer charges \$90 per unit for orders of 100 or less. To encourage large orders, the manufacturer reduces the charge by \$0.15 per radio for each unit ordered in excess of 100 (for example, there would be a charge of \$87 per radio for an order size of 120).

(a) The table shows the profit P (in dollars) for various numbers of units ordered, x. Use the table to estimate the maximum profit.

Units, x	Profit, P
110	3135
120	3240
130	3315
140	3360
150	3375
160	3360
170	3315

(b) Plot the points (x, P) from the table in part (a). Does the relation defined by the ordered pairs represent P as a function of x?

(c) If P is a function of x, write the function and determine its domain.

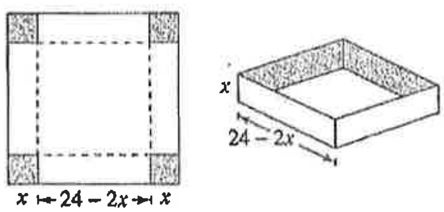
70. ***Exploration*** An open box of maximum volume is to be made from a square piece of material, 24 centimeters on a side, by cutting equal squares from the corners and turning up the sides (see figure).

(a) The table shows the volume V (in cubic centimeters) of the box for various heights x (in centimeters). Use the table to estimate the maximum volume.

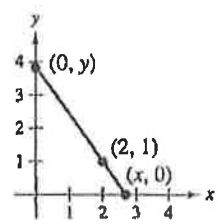
Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

(b) Plot the points (x, V) from the table in part (a). Does the relation defined by the ordered pairs represent V as a function of x?

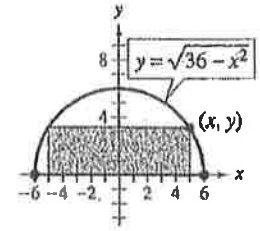
(c) If V is a function of x, write the function and determine its domain.



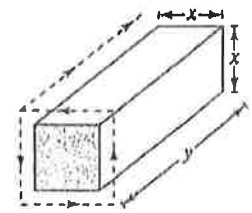
71. ***Geometry*** A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(2, 1)$ (see figure). Write the area A of the triangle as a function of x, and determine the domain of the function.



72. ***Geometry*** A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{36 - x^2}$ (see figure). Write the area A of the rectangle as a function of x, and determine the domain of the function.



73. ***Postal Regulations*** A rectangular package to be sent by the U.S. Postal Service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches (see figure).



(a) Write the volume V of the package as a function of x.

(b) What is the domain of the function?

(c) Use a graphing utility to graph the function. Be sure to use the appropriate viewing window.

(d) What dimensions will maximize the volume of the package? Explain.

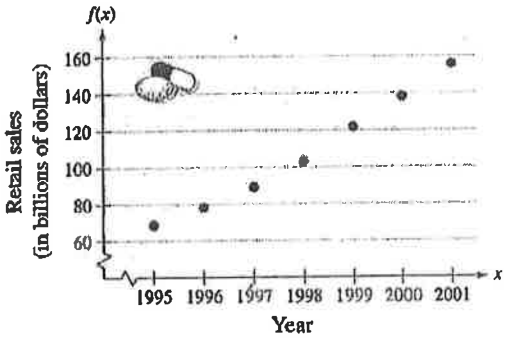
74. ***Cost, Revenue, and Profit*** A company produces a toy for which the variable cost is \$12.30 per unit and the fixed costs are \$98,000. The toy sells for \$17.98. Let x be the number of units produced and sold.

(a) The total cost for a business is the sum of the variable cost and the fixed costs. Write the total cost C as a function of the number of units produced.

(b) Write the revenue R as a function of the number of units sold.

(c) Write the profit P as a function of the number of units sold. (*Note:* $P = R - C$.)

82. ***Data Analysis*** The graph shows the retail sales (in billions of dollars) of prescription drugs in the United States from 1995 through 2001. Let $f(x)$ represent the retail sales in year x. (Source: National Association of Chain Drug Stores)



(a) Find $f(1998)$.

(b) Find $\dfrac{f(2001) - f(1995)}{2001 - 1995}$

and interpret the result in the context of the problem.

(c) An approximate model for the function is

$P(t) = -0.1556t^3 + 4.657t^2 - 28.75t + 115.7,$
$5 \le t \le 11$

where P is the retail sales (in billions of dollars) and t represents the year, with $t = 5$ corresponding to 1995. Complete the table and compare the results with the data.

t	5	6	7	8	9	10	11
$P(t)$							

(d) Use a graphing utility to graph the model and data in the same viewing window. Comment on the validity of the model.

In Exercises 83–88, find the difference quotient and simplify your answer.

83. $f(x) = 2x, \quad \dfrac{f(x + c) - f(x)}{c}, \quad c \ne 0$

84. $g(x) = 3x - 1, \quad \dfrac{g(x + h) - g(x)}{h}, \quad h \ne 0$

85. $f(x) = x^2 - x + 1, \quad \dfrac{f(2 + h) - f(2)}{h}, \quad h \ne 0$

86. $f(x) = x^3 + x, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \ne 0$

87. $f(t) = \dfrac{1}{t}, \quad \dfrac{f(t) - f(1)}{t - 1}, \quad t \ne 1$

88. $f(x) = \dfrac{4}{x + 1}, \quad \dfrac{f(x) - f(7)}{x - 7}, \quad x \ne 7$

Synthesis

True or False? **In Exercises 89 and 90, determine whether the statement is true or false. Justify your answer.**

89. The domain of the function $f(x) = x^4 - 1$ is $(-\infty, \infty)$, and the range of $f(x)$ is $(0, \infty)$.

90. The set of ordered pairs $\{(-8, -2), (-6, 0), (-4, 0), (-2, 2), (0, 4), (2, -2)\}$ represents a function.

Exploration **In Exercises 91 and 92, match the data with one of the functions $g(x) = cx^2$ or $r(x) = c/x$ and determine the value of the constant c such that the function fits the data given in the table.**

91.

x	-4	-1	0	1	4
y	-8	-32	Undef.	32	8

92.

x	-4	-1	0	1	4
y	-32	-2	0	-2	-32

93. ***Writing*** In your own words, explain the meanings of *domain* and *range*.

94. ***Think About It*** Describe an advantage of function notation.

Review

In Exercises 95–98, perform the operations and simplify.

95. $12 - \dfrac{4}{x + 2}$

96. $\dfrac{3}{x^2 + x - 20} + \dfrac{x}{x^2 + 4x - 5}$

97. $\dfrac{2x^3 + 11x^2 - 6x}{5x} \cdot \dfrac{x + 10}{2x^2 + 5x - 3}$

98. $\dfrac{x + 7}{2(x - 9)} + \dfrac{x - 7}{2(x - 9)}$

The symbol indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

Graphs of Functions

Vertical Line Test for Functions

A set of points in a coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval,

$x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval,

$x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval,

$f(x_1) = f(x_2)$.

Definition of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f if there exists an interval (x_1, x_2) that contains a such that

$x_1 < x < x_2$ implies $f(a) \le f(x)$.

A function value $f(a)$ is called a **relative maximum** of f if there exists an interval (x_1, x_2) that contains a such that

$x_1 < x < x_2$ implies $f(a) \ge f(x)$.

Library of Functions: Greatest Integer Function

The *greatest integer function*, denoted by $[\![x]\!]$ and defined as the greatest integer less than or equal to x, has an infinite number of breaks or steps—one at each integer value in its domain. The basic characteristics of the greatest integer function are summarized below.

Graph of $f(x) = [\![x]\!]$

Domain: $(-\infty, \infty)$

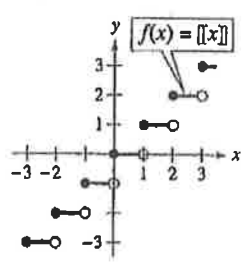
Range: the set of integers

x-intercepts: in the interval $[0, 1)$

y-intercept: $(0, 0)$

Constant between each pair of consecutive integers

Jumps vertically one unit at each integer value



Could you describe the greatest integer function using a piecewise-defined function? How does the graph of the greatest integer function differ from the graph of a line with a slope of zero?

Test for Even and Odd Functions

A function f is even if, for each x in the domain of f, $f(-x) = f(x)$.
A function f is odd if, for each x in the domain of f, $f(-x) = -f(x)$.

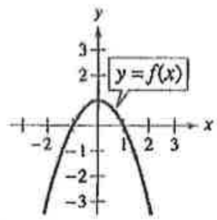
1.3 Exercises

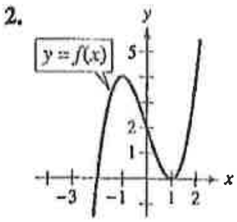
Vocabulary Check

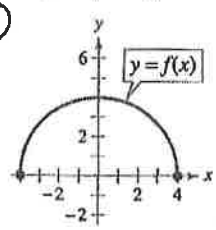
Fill in the blanks.

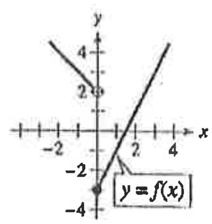
1. The graph of a function f is a collection of ______ (x, y) such that x is in the domain of f.
2. The ______ is used to determine whether the graph of an equation is a function of y in terms of x.
3. A function f is ______ on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
4. A function value $f(a)$ is a relative ______ of f if there exists an interval (x_1, x_2) containing a such that $x_1 < x < x_2$ implies $f(a) \le f(x)$.
5. The function $f(x) = [\![x]\!]$ is called the ______ function, and is an example of a step function.
6. A function f is ______ if, for each x in the domain of f, $f(-x) = f(x)$.

In Exercises 1–4, use the graph of the function to find the domain and range of f. Then find $f(0)$.

1.


2.


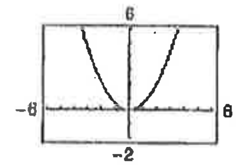
3.


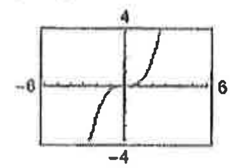
4.


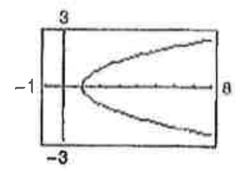
In Exercises 5–10, use a graphing utility to graph the function and estimate its domain and range. Then find the domain and range algebraically.

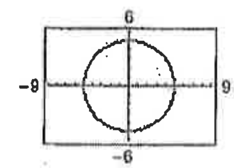
5. $f(x) = 2x^2 + 3$
6. $f(x) = -x^2 - 1$
7. $f(x) = \sqrt{x - 1}$
8. $h(t) = \sqrt{4 - t^2}$
9. $f(x) = |x + 3|$
10. $f(x) = -\frac{1}{4}|x - 5|$

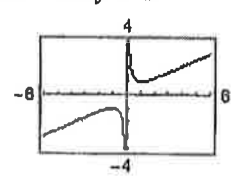
In Exercises 11–16, use the Vertical Line Test to determine whether y is a function of x. Describe how you can use a graphing utility to produce the given graph.

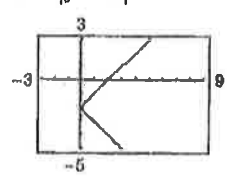
11. $y = \frac{1}{2}x^2$


12. $y = \frac{1}{4}x^3$


13. $x - y^2 = 1$


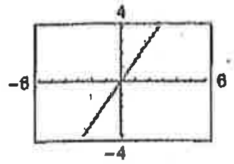
14. $x^2 + y^2 = 25$


15. $x^2 = 2xy - 1$


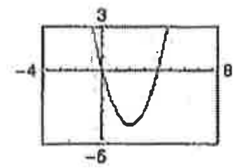
16. $x = |y + 2|$


In Exercises 17–20, determine the intervals over which the function is increasing, decreasing, or constant.

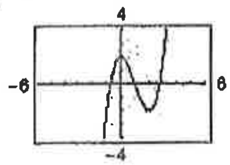
17. $f(x) = \frac{3}{2}x$



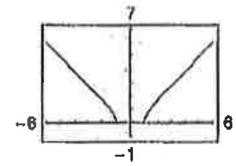
18. $f(x) = x^2 - 4x$



19. $f(x) = x^3 - 3x^2 + 2$



20. $f(x) = \sqrt{x^2 - 1}$



In Exercises 21–28, (a) use a graphing utility to graph the function and (b) determine the open intervals on which the function is increasing, decreasing, or constant.

21. $f(x) = 3$
22. $f(x) = x$
23. $f(x) = x^{2/3}$
24. $f(x) = -x^{3/4}$
25. $f(x) = x\sqrt{x + 3}$
26. $f(x) = \sqrt{1 - x}$
27. $f(x) = |x + 1| + |x - 1|$
28. $f(x) = -|x + 4| - |x + 1|$

In Exercises 29–34, use a graphing utility to approximate (to two decimal places) any relative minimum or maximum values of the function.

29. $f(x) = x^2 - 6x$
30. $f(x) = 3x^2 - 2x - 5$
31. $y = 2x^3 + 3x^2 - 12x$
32. $y = x^3 - 6x^2 + 15$
33. $h(x) = (x - 1)\sqrt{x}$
34. $g(x) = x\sqrt{4 - x}$

In Exercises 35–40, (a) approximate the relative minimum or maximum values of the function by sketching its graph using the point-plotting method, (b) use a graphing utility to approximate (to two decimal places) any relative minimum or maximum values, and (c) compare your answers from parts (a) and (b).

35. $f(x) = x^2 - 4x - 5$
36. $f(x) = 3x^2 - 12$
37. $f(x) = x^3 - 8x$
38. $f(x) = -x^3 + 7x$
39. $f(x) = (x - 4)^{2/3}$
40. $f(x) = \sqrt{4x^2 + 1}$

In Exercises 41–48, sketch the graph of the piecewise-defined function by hand.

41. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$

42. $f(x) = \begin{cases} x + 6, & x \leq -4 \\ 2x - 4, & x > -4 \end{cases}$

43. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$

45. $f(x) = \begin{cases} x + 3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x - 1, & x > 2 \end{cases}$

46. $g(x) = \begin{cases} x + 5, & x \leq -3 \\ -2, & -3 < x < 1 \\ 5x - 4, & x \geq 1 \end{cases}$

47. $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$

48. $h(x) = \begin{cases} 3 + x, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

In Exercises 49–56, algebraically determine whether the function is even, odd, or neither. Verify your answer using a graphing utility.

49. $f(t) = t^2 + 2t - 3$
50. $f(x) = x^6 - 2x^2 + 3$
51. $g(x) = x^3 - 5x$
52. $h(x) = x^3 - 5$
53. $f(x) = x\sqrt{1 - x^2}$
54. $f(x) = x\sqrt{x + 5}$
55. $g(s) = 4s^{2/3}$
56. $f(s) = 4s^{3/2}$

Think About It **In Exercises 57–62, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is (a) even and (b) odd.**

57. $\left(-\frac{3}{2}, 4\right)$
58. $\left(-\frac{5}{3}, -7\right)$
59. $(4, 9)$
60. $(5, -1)$
61. $(x, -y)$
62. $(2a, 2c)$

In Exercises 63–72, use a graphing utility to graph the function and determine whether it is even, odd, or neither. Verify your answer algebraically.

63. $f(x) = 5$
64. $f(x) = -9$
65. $f(x) = 3x - 2$
66. $f(x) = 5 - 3x$
67. $h(x) = x^2 - 4$
68. $f(x) = -x^2 - 8$

69. $f(x) = \sqrt{1 - x}$ 70. $g(t) = \sqrt[3]{t - 1}$

71. $f(x) = |x + 2|$ 72. $f(x) = -|x - 5|$

In Exercises 73–76, graph the function and determine the interval(s) (if any) on the real axis for which $f(x) \geq 0$. Use a graphing utility to verify your results.

73. $f(x) = 4 - x$ 74. $f(x) = 4x + 2$

75. $f(x) = x^2 - 9$ 76. $f(x) = x^2 - 4x$

In Exercises 77 and 78, use a graphing utility to graph the function. State the domain and range of the function. Describe the pattern of the graph.

77. $s(x) = 2(\frac{1}{4}x - [\![\frac{1}{4}x]\!])$ 78. $g(x) = 2(\frac{1}{4}x - [\![\frac{1}{4}x]\!])^2$

79. ***Geometry*** The perimeter of a rectangle is 100 meters.

(a) Show that the area of the rectangle is given by $A = x(50 - x)$, where x is its length.

(b) Use a graphing utility to graph the area function.

(c) Use a graphing utility to approximate the maximum area of the rectangle and the dimensions that yield the maximum area.

80. ***Cost, Revenue, and Profit*** The marketing department of a company estimates that the demand for a color scanner is $p = 100 - 0.0001x$, where p is the price per scanner and x is the number of scanners. The cost of producing x scanners is $C = 350{,}000 + 30x$ and the profit for producing and selling x scanners is

$$P = R - C = xp - C.$$

Use a graphing utility to graph the profit function and estimate the number of scanners that would produce a maximum profit.

81. ***Communications*** The cost of using a telephone calling card is \$1.05 for the first minute and \$0.38 for each additional minute or portion of a minute.

(a) A customer needs a model for the cost C of using the calling card for a call lasting t minutes. Which of the following is the appropriate model?

$$C_1(t) = 1.05 + 0.38[\![t - 1]\!]$$

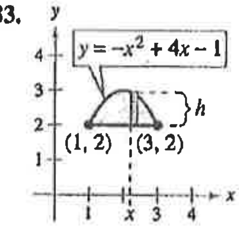
$$C_2(t) = 1.05 - 0.38[\![-(t - 1)]\!]$$

(b) Use a graphing utility to graph the appropriate model. Use the *value* feature or the *zoom* and *trace* features to estimate the cost of a call lasting 18 minutes and 45 seconds.

82. ***Delivery Charges*** The cost of sending a package from New York to Atlanta is $ package weighing up to but not includi and \$2.50 for each additional pound or p pound. Use the greatest integer function model for the cost C of overnight deliver age weighing x pounds, where $x > 0$. graph of the function.

In Exercises 83 and 84, write the heigh rectangle as a function of x.

83.



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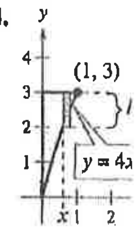
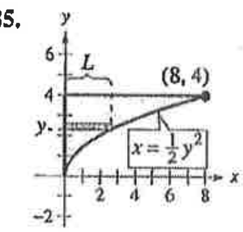


In Exercises 85 and 86, write the length rectangle as a function of y.

85.



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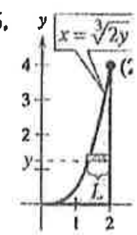


87. ***Population*** During a seven-year period lation P (in thousands) of North Dakot and then decreased according to the mod

$$P = -0.76t^2 + 9.9t + 618, \quad 5 \leq t \leq 1$$

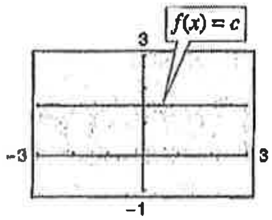
where t represents the year, with $t = 5$ ding to 1995. (Source: U.S. Census Bure

(a) Use a graphing utility to graph the r the appropriate domain.

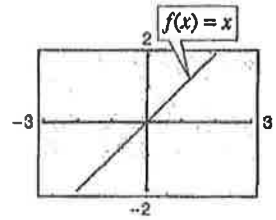
(b) Use the graph from part (a) to determ which years the population was i During which years was the p decreasing?

(c) Use the *zoom* and *trace* features or the feature of a graphing utility to appro maximum population between 1995 a

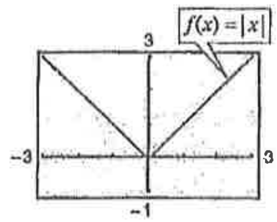
Translations



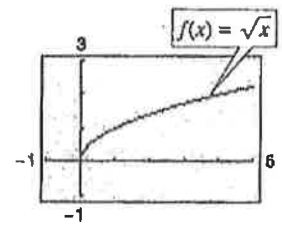
(a) Constant Function



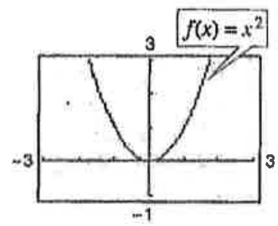
(b) Identity Function



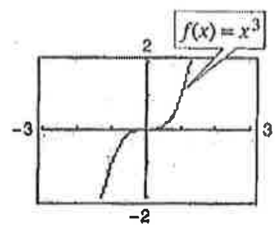
(c) Absolute Value Function



(d) Square Root Function



(e) Quadratic Function



(f) Cubic Function

Figure 1.40

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward*: $h(x) = f(x) + c$
2. Vertical shift c units *downward*: $h(x) = f(x) - c$
3. Horizontal shift c units to the *right*: $h(x) = f(x - c)$
4. Horizontal shift c units to the *left*: $h(x) = f(x + c)$

Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$
2. Reflection in the y-axis: $h(x) = f(-x)$

1.4 Exercises

Vocabulary Check

In Exercises 1–5, fill in the blanks.

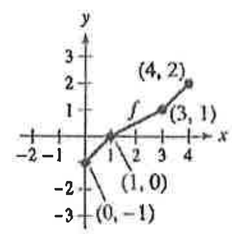
1. The graph of a _______ is U-shaped.
2. The graph of an _______ is V-shaped.
3. Horizontal shifts, vertical shifts, and reflections are called _______ .
4. A reflection in the x-axis of $y = f(x)$ is represented by $h(x) =$ _______, while a reflection in the y-axis of $y = f(x)$ is represented by $h(x) =$ _______ .
5. A nonrigid transformation of $y = f(x)$ represented by $cf(x)$ is a vertical stretch if _______ and a vertical shrink if _______ .
6. Match the rigid transformation of $y = f(x)$ with the correct representation, where $c > 0$.

(a) $h(x) = f(x) + c$	(i) horizontal shift c units to the left
(b) $h(x) = f(x) - c$	(ii) vertical shift c units upward
(c) $h(x) = f(x - c)$	(iii) horizontal shift c units to the right
(d) $h(x) = f(x + c)$	(iv) vertical shift c units downward

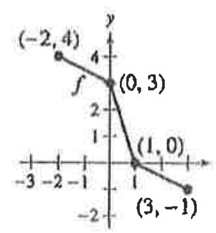
In Exercises 1–12, sketch the graphs of the three functions by hand on the same rectangular coordinate system. Verify your result with a graphing utility.

1. $f(x) = x$
   $g(x) = x - 4$
   $h(x) = 3x$
2. $f(x) = \frac{1}{2}x$
   $g(x) = \frac{1}{2}x + 2$
   $h(x) = \frac{1}{2}(x - 2)$
3. $f(x) = x^2$
   $g(x) = x^2 + 2$
   $h(x) = (x - 2)^2$
4. $f(x) = x^2$
   $g(x) = x^2 - 4$
   $h(x) = (x + 2)^2 + 1$
5. $f(x) = -x^2$
   $g(x) = -x^2 + 1$
   $h(x) = -(x - 2)^2$
6. $f(x) = (x - 2)^2$
   $g(x) = (x - 2)^2 + 2$
   $h(x) = -(x - 2)^2 + 4$
7. $f(x) = x^2$
   $g(x) = \frac{1}{2}x^2$
   $h(x) = (2x)^2$
8. $f(x) = x^2$
   $g(x) = \frac{1}{4}x^2 + 2$
   $h(x) = -\frac{1}{4}x^2$
9. $f(x) = |x|$
   $g(x) = |x| - 1$
   $h(x) = |x - 3|$
10. $f(x) = |x|$
    $g(x) = |2x|$
    $h(x) = -2|x + 2| - 1$
11. $f(x) = \sqrt{x}$
    $g(x) = \sqrt{x + 1}$
    $h(x) = \sqrt{x - 2} + 1$
12. $f(x) = \sqrt{x}$
    $g(x) = \frac{1}{2}\sqrt{x}$
    $h(x) = -\frac{1}{2}\sqrt{x + 4}$

13. Use the graph of f to sketch each graph. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com.*
    (a) $y = f(x) + 2$
    (b) $y = -f(x)$
    (c) $y = f(x - 2)$
    (d) $y = f(x + 3)$
    (e) $y = 2f(x)$
    (f) $y = f(-x)$
    (g) $y = f(\frac{1}{2}x)$

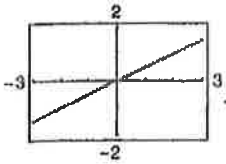


14. Use the graph of f to sketch each graph. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com.*
    (a) $y = f(x) - 1$
    (b) $y = f(x + 1)$
    (c) $y = f(x - 1)$
    (d) $y = -f(x - 2)$
    (e) $y = f(-x)$
    (f) $y = \frac{1}{2}f(x)$
    (g) $y = f(2x)$



In Exercises 15–26, identify the common function and describe the transformation shown in the graph. Write an equation for the graphed function.

15.



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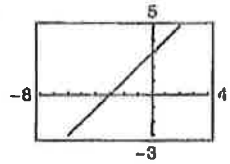

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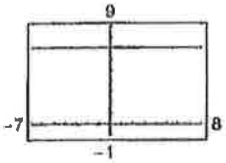

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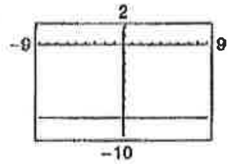

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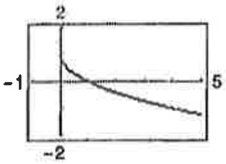

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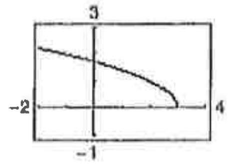

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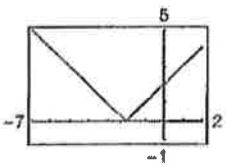

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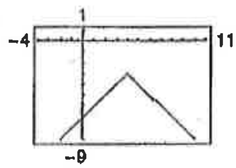

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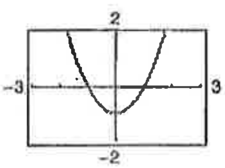

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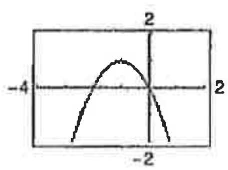

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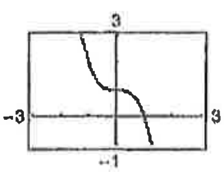

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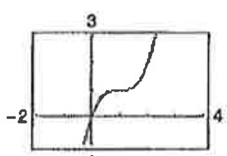


In Exercises 27–32, compare the graph of the function with the graph of $f(x) = \sqrt{x}$.

27. $y = -\sqrt{x} - 1$ **28.** $y = \sqrt{x} + 2$

29. $y = \sqrt{x - 2}$ **30.** $y = \sqrt{x + 4}$

31. $y = \sqrt{2x}$ **32.** $y = \sqrt{-x + 3}$

In Exercises 33–38, compare the graph of the function with the graph of $f(x) = |x|$.

33. $y = |x + 5|$ **34.** $y = |x| - 3$

35. $y = -|x|$ **36.** $y = |-x|$

37. $y = 4|x|$ **38.** $y = |\frac{1}{2}x|$

In Exercises 39–44, compare the graph of the function with the graph of $f(x) = x^3$.

39. $g(x) = 4 - x^3$ **40.** $g(x) = -(x - 1)^3$

41. $h(x) = \frac{1}{4}(x + 2)^3$ **42.** $h(x) = -2(x - 1)^3 + 3$

43. $p(x) = \left(\frac{1}{3}x\right)^3 + 2$ **44.** $p(x) = [3(x - 2)]^3$

In Exercises 45–48, use a graphing utility to graph the three functions in the same viewing window. Describe the graphs of g and h relative to the graph of f.

45. $f(x) = x^3 - 3x^2$
$g(x) = f(x + 2)$
$h(x) = \frac{1}{2}f(x)$

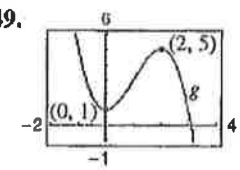
46. $f(x) = x^3 - 3x^2 + 2$
$g(x) = f(x - 1)$
$h(x) = f(3x)$

47. $f(x) = x^3 - 3x^2$
$g(x) = -\frac{1}{3}f(x)$
$h(x) = f(-x)$

48. $f(x) = x^3 - 3x^2 + 2$
$g(x) = -f(x)$
$h(x) = f(2x)$

In Exercises 49 and 50, use the graph of $f(x) = x^3 - 3x^2$ (see Exercise 45) to write a formula for the function g shown in the graph.

49.



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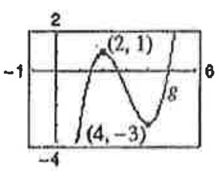


In Exercises 51–64, g is related to one of the six common functions on page 42. (a) Identify the common function f. (b) Describe the sequence of transformations from f to g. (c) Sketch the graph of g by hand. (d) Use function notation to write g in terms of the common function f.

51. $g(x) = 2 - (x + 5)^2$

52. $g(x) = -(x + 10)^2 + 5$

53. $g(x) = 3 + 2(x - 4)^2$

54. $g(x) = -\frac{1}{4}(x + 2)^2 - 2$

55. $g(x) = 3(x - 2)^3$ **56.** $g(x) = -\frac{1}{2}(x + 1)^3$

57. $g(x) = (x - 1)^3 + 2$

58. $g(x) = -(x + 3)^3 - 10$

59. $g(x) = |x + 4| + 8$

60. $g(x) = |x + 3| + 9$

61. $g(x) = -2|x - 1| - 4$

62. $g(x) = \frac{1}{2}|x - 2| - 3$

63. $g(x) = -\frac{1}{2}\sqrt{x + 3} - 1$

64. $g(x) = -\sqrt{x + 1} - 6$

65. ***Profit*** The profit P per week on a case of soda pop is given by the model

$$P(x) = 80 + 20x - 0.5x^2, \qquad 0 \le x \le 20$$

where x is the amount spent on advertising. In this model, x and P are both measured in hundreds of dollars.

(a) Use a graphing utility to graph the profit function.

(b) The business estimates that taxes and operating costs will increase by an average of \$2500 per week during the next year. Rewrite the profit function to reflect this expected decrease in profits. Describe the transformation applied to the graph of the function.

(c) Rewrite the profit function so that x measures advertising expenditures in dollars. [Find $P(\frac{x}{100})$.] Describe the transformation applied to the graph of the profit function.

66. ***Automobile Aerodynamics*** The number of horsepower H required to overcome wind drag on an automobile is approximated by the model

$$H(x) = 0.002x^2 + 0.005x - 0.029, \quad 10 \le x \le 100$$

where x is the speed of the car in miles per hour.

(a) Use a graphing utility to graph the function.

(b) Rewrite the function so that x represents the speed in kilometers per hour. [Find $H(x/1.6)$.] Describe the transformation applied to the graph of the function.

67. ***Fuel Use*** The amount of fuel F (in billions of gallons) used by trucks from 1980 through 2000 can be approximated by the function $F(t) = 0.036t^2 + 20.1$, where $t = 0$ represents 1980. (Source: U.S. Federal Highway Administration)

(a) Describe the transformation of the common function $f(t) = t^2$. Then sketch the graph over the interval $0 \le t \le 20$.

(b) Rewrite the function so that $t = 0$ represents 1990. Explain how you got your answer.

68. ***Finance*** The amount M (in billions of mortgage debt outstanding in the United S 1990 through 2001 can be approximat function $M(t) = 29.9t^2 + 3892$, where t sents 1990. (Source: Board of Govern Federal Reserve System)

(a) Describe the transformation of the function $f(t) = t^2$. Then sketch the the interval $0 \le t \le 11$.

(b) Rewrite the function so that $t = 0$ 2000. Explain how you got your answ

Synthesis

True or False? **In Exercises 69 and 70, whether the statement is true or false. Ju answer.**

69. The graphs of $f(x) = |x| - 5$ and $g(x) =$ are identical.

70. Relative to the graph of $f(x) = \sqrt{x}$, the g function $h(x) = -\sqrt{x + 9} - 13$ is shifte the left and 13 units downward, then refle x-axis.

71. ***Exploration*** Use a graphing utility to g function. Describe any similarities and you observe among the graphs.

(a) $y = x$ (b) $y = x^2$ (c)

(d) $y = x^4$ (e) $y = x^5$ (f)

72. ***Conjecture*** Use the results of Exercise 7

(a) Make a conjecture about the shapes of of $y = x^7$ and $y = x^8$. Use a graphin verify your conjecture.

(b) Sketch the graphs of $y = (x -$ $y = (x + 1)^2$ by hand. Use a graphin verify your graphs.

Review

In Exercises 73 and 74, determine whether t and L_2 passing through the pairs of p parallel, perpendicular, or neither.

73. L_1: $(-2, -2), (2, 10)$
L_2: $(-1, 3), (3, 9)$

74. L_1: $(-1,$
L_2: $(1, 5),$

In Exercises 75–78, find the domain of the f

75. $f(x) = \dfrac{4}{9 - x}$

76. $f(x) = \dfrac{\sqrt{}}{x}$

77. $f(x) = \sqrt{100 - x^2}$

78. $f(x) = \sqrt[3]{}$

Combinations of Functions

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the sum, difference, product, and quotient of f and g are defined as follows.

1. **Sum:** $(f + g)(x) = f(x) + g(x)$
2. **Difference:** $(f - g)(x) = f(x) - g(x)$
3. **Product:** $(fg)(x) = f(x) \cdot g(x)$
4. **Quotient:** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 1.59.)

1.5 Exercises

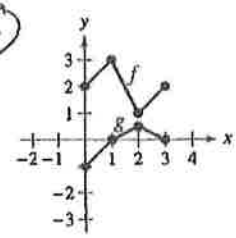
Vocabulary Check

Fill in the blanks.

1. Two functions f and g can be combined by the arithmetic operations of ______, ______, ______, and ______ to create new functions.
2. The ______ of the function f with g is $(f \circ g)(x) = f(g(x))$.
3. The domain of $f \circ g$ is the set of all x in the domain of g such that ______ is in the domain of f.
4. To decompose a composite function, look for an ______ and ______ function.

In Exercises 1–4, use the graphs of f and g to graph $h(x) = (f + g)(x)$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

1.



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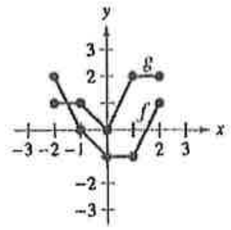

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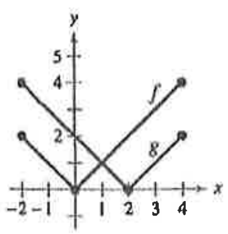

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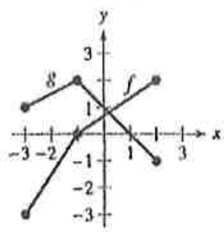


In Exercises 5–12, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. What is the domain of f/g?

5. $f(x) = x + 3, \quad g(x) = x - 3$
6. $f(x) = 2x - 5, \quad g(x) = 1 - x$
7. $f(x) = x^2, \quad g(x) = 1 - x$
8. $f(x) = 2x - 5, \quad g(x) = 4$
9. $f(x) = x^2 + 5, \quad g(x) = \sqrt{1 - x}$
10. $f(x) = \sqrt{x^2 - 4}, \quad g(x) = \dfrac{x^2}{x^2 + 1}$
11. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x^2}$
12. $f(x) = \dfrac{x}{x + 1}, \quad g(x) = x^3$

In Exercises 13–26, evaluate the indicated function for $f(x) = x^2 + 1$ and $g(x) = x - 4$ algebraically. If possible, use a graphing utility to verify your answer.

13. $(f + g)(3)$
14. $(f - g)(-2)$
15. $(f - g)(0)$
16. $(f + g)(1)$
17. $(fg)(4)$
18. $(fg)(-6)$
19. $\left(\dfrac{f}{g}\right)(-5)$
20. $\left(\dfrac{f}{g}\right)(0)$
21. $(f - g)(2t)$
22. $(f + g)(t - 4)$
23. $(fg)(-5t)$
24. $(fg)(3t^2)$
25. $\left(\dfrac{f}{g}\right)(-t)$
26. $\left(\dfrac{f}{g}\right)(t + 2)$

In Exercises 27–30, use a graphing utility to graph the functions f, g, and $f + g$ in the same viewing window.

27. $f(x) = \frac{1}{2}x, \quad g(x) = x - 1$
28. $f(x) = \frac{1}{3}x, \quad g(x) = -x + 4$
29. $f(x) = x^2, \quad g(x) = -2x$
30. $f(x) = 4 - x^2, \quad g(x) = x$

In Exercises 31–34, use a graphing utility to graph f, g, and $f + g$ in the same viewing window. Which function contributes most to the magnitude of the sum when $0 \le x \le 2$? Which function contributes most to the magnitude of the sum when $x > 6$?

31. $f(x) = 3x, \quad g(x) = -\dfrac{x^3}{10}$
32. $f(x) = \dfrac{x}{2}, \quad g(x) = \sqrt{x}$
33. $f(x) = 3x + 2, \quad g(x) = -\sqrt{x + 5}$
34. $f(x) = x^2 - \frac{1}{2}, \quad g(x) = -3x^2 - 1$

In Exercises 35–38, find (a) $f \circ g$, (b) $g \circ f$, and, if possible, (c) $(f \circ g)(0)$.

35. $f(x) = x^2, \quad g(x) = x - 1$
36. $f(x) = \sqrt[3]{x - 1}, \quad g(x) = x^3 + 1$
37. $f(x) = 3x + 5, \quad g(x) = 5 - x$
38. $f(x) = x^3, \quad g(x) = \dfrac{1}{x}$

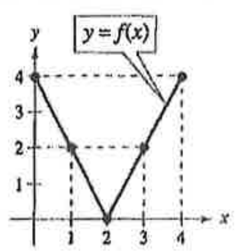
In Exercises 39–44, (a) find $f \circ g$, $g \circ f$, and the domain of $f \circ g$. (b) Use a graphing utility to graph $f \circ g$ and $g \circ f$. Determine whether $f \circ g = g \circ f$.

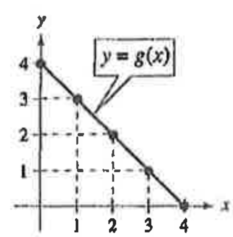
39. $f(x) = \sqrt{x + 4}, \quad g(x) = x^2$
40. $f(x) = \sqrt[3]{x + 1}, \quad g(x) = x^3 - 1$
41. $f(x) = \frac{1}{3}x - 3, \quad g(x) = 3x + 1$
42. $f(x) = \sqrt{x}, \quad g(x) = \sqrt{x}$
43. $f(x) = x^{2/3}, \quad g(x) = x^6$
44. $f(x) = |x|, \quad g(x) = x + 6$

In Exercises 45–50, (a) find $(f \circ g)(x)$ and $(g \circ f)(x)$, (b) determine algebraically whether $(f \circ g)(x) = (g \circ f)(x)$, and (c) verify your answer to part (b) by comparing a table of values for each composition.

45. $f(x) = 5x + 4, \quad g(x) = 4 - x$
46. $f(x) = \frac{1}{4}(x - 1), \quad g(x) = 4x + 1$
47. $f(x) = \sqrt{x + 6}, \quad g(x) = x^2 - 5$
48. $f(x) = x^3 - 4, \quad g(x) = \sqrt[3]{x + 10}$
49. $f(x) = |x + 3|, \quad g(x) = 2x - 1$
50. $f(x) = \dfrac{6}{3x - 5}, \quad g(x) = -x$

In Exercises 51–54, use the graphs of f and g to evaluate the functions.





51. (a) $(f + g)(3)$ (b) $(f/g)(2)$
52. (a) $(f - g)(1)$ (b) $(fg)(4)$
53. (a) $(f \circ g)(2)$ (b) $(g \circ f)(2)$
54. (a) $(f \circ g)(1)$ (b) $(g \circ f)(3)$

In Exercises 55–62, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

55. $h(x) = (2x + 1)^2$
56. $h(x) = (1 - x)^3$
57. $h(x) = \sqrt[3]{x^2 - 4}$
58. $h(x) = \sqrt{9 - x}$
59. $h(x) = \dfrac{1}{x + 2}$
60. $h(x) = \dfrac{4}{(5x + 2)^2}$
61. $h(x) = (x + 4)^2 + 2(x + 4)$
62. $h(x) = (x + 3)^{3/2} + 4(x + 3)^{1/2}$

In Exercises 63–72, determine the domains of (a) f, (b) g, and (c) $f \circ g$. Use a graphing utility to verify your results.

63. $f(x) = \sqrt{x + 4}, \quad g(x) = x^2$
64. $f(x) = \sqrt{x + 3}, \quad g(x) = \dfrac{x}{2}$
65. $f(x) = x^2 + 1, \quad g(x) = \sqrt{x}$
66. $f(x) = x^{1/4}, \quad g(x) = x^4$
67. $f(x) = \dfrac{1}{x}, \quad g(x) = x + 3$
68. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{2x}$
69. $f(x) = |x - 4|, \quad g(x) = 3 - x$
70. $f(x) = \dfrac{2}{|x|}, \quad g(x) = x - 1$
71. $f(x) = x + 2, \quad g(x) = \dfrac{1}{x^2 - 4}$
72. $f(x) = \dfrac{3}{x^2 - 1}, \quad g(x) = x + 1$

73. ***Stopping Distance*** The research and development department of an automobile manufacturer has determined that when required to stop quickly to avoid an accident, the distance (in feet) a car travels during the driver's reaction time is given by $R(x) = \frac{3}{4}x$, where x is the speed of the car in miles per hour. The distance (in feet) traveled while the driver is braking is given by $B(x) = \frac{1}{15}x^2$.
    (a) Find the function that represents the total stopping distance T.
    (b) Use a graphing utility to graph the functions R, B, and T in the same viewing window for $0 \le x \le 60$.
    (c) Which function contributes most to the magnitude of the sum at higher speeds? Explain.

Inverse Functions

Definition of Inverse Function

Let f and g be two functions such that

$$f(g(x)) = x \qquad \text{for every } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \qquad \text{for every } x \text{ in the domain of } f.$$

Under these conditions, the function g is the **inverse function** of the function f. The function g is denoted by f^{-1} (read "f-inverse"). So,

$$f(f^{-1}(x)) = x \qquad \text{and} \qquad f^{-1}(f(x)) = x.$$

The domain of f must be equal to the range of f^{-1}, and the range of f must be equal to the domain of f^{-1}.

The Graph of an Inverse Function

The graphs of a function f and its inverse function f^{-1} are related to each other in the following way. If the point (a, b) lies on the graph of f, then the point (b, a) must lie on the graph of f^{-1}, and vice versa. This means that the graph of f^{-1} is a *reflection* of the graph of f in the line $y = x$, as shown in Figure 1.68.

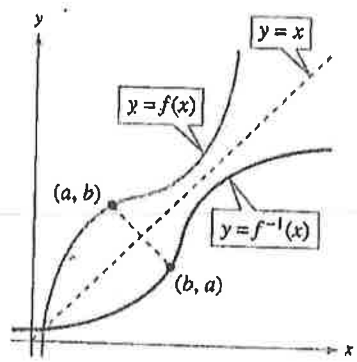


Figure 1.68

Definition of a One-to-One Function

A function f is **one-to-one** if, for a and b in its domain, $f(a) = f(b)$ implies that $a = b$.

Existence of an Inverse Function

A function f has an inverse function f^{-1} if and only if f is one-to-one.

Finding an Inverse Function

1. **Use the Horizontal Line Test to decide whether f has an inverse function.**
2. In the equation for $f(x)$, replace $f(x)$ by y.
3. Interchange the roles of x and y, and solve for y.
4. Replace y by $f^{-1}(x)$ in the new equation.
5. Verify that f and f^{-1} are inverse functions of each other by showing that the domain of f is equal to the range of f^{-1}, the range of f is equal to the domain of f^{-1}, and $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

1.6 Exercises

Vocabulary Check

Fill in the blanks.

1. If the composite functions $f(g(x)) = x$ and $g(f(x)) = x$, then the function g is the _______ function of f, and is denoted by _______ .
2. The domain of f is the _______ of f^{-1}, and the _______ of f^{-1} is the range of f.
3. The graphs of f and f^{-1} are reflections of each other in the line _______ .
4. To have an inverse function, a function f must be _______ ; that is, $f(a) = f(b)$ implies $a = b$.
5. A graphical test for the existence of an inverse function is called the _______ Line Test.

In Exercises 1–8, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

1. $f(x) = 6x$
2. $f(x) = \frac{1}{3}x$
3. $f(x) = x + 7$
4. $f(x) = x - 3$
5. $f(x) = 2x + 1$
6. $f(x) = \dfrac{x-1}{4}$
7. $f(x) = \sqrt[3]{x}$
8. $f(x) = x^5$

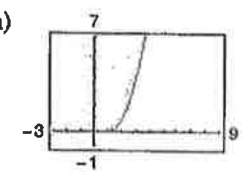
In Exercises 9–14, (a) show that f and g are inverse functions algebraically and (b) verify that f and g are inverse functions numerically by creating a table of values for each function.

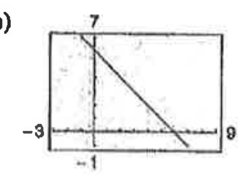
9. $f(x) = -\dfrac{7}{2}x - 3, \quad g(x) = -\dfrac{2x+6}{7}$
10. $f(x) = \dfrac{x-9}{4}, \quad g(x) = 4x + 9$
11. $f(x) = x^3 + 5, \quad g(x) = \sqrt[3]{x-5}$
12. $f(x) = \dfrac{x^3}{2}, \quad g(x) = \sqrt[3]{2x}$
13. $f(x) = -\sqrt{x-8}; \quad g(x) = 8 + x^2, \quad x \le 0$
14. $f(x) = \sqrt[3]{3x-10}, \quad g(x) = \dfrac{x^3+10}{3}$

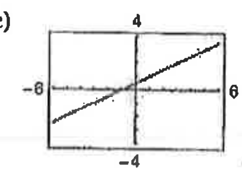
In Exercises 15–20, show that f and g are inverse functions algebraically. Use a graphing utility to graph f and g in the same viewing window. Describe the relationship between the graphs.

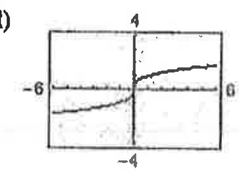
15. $f(x) = x^3, \quad g(x) = \sqrt[3]{x}$
16. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x}$
17. $f(x) = \sqrt{x-4}; \quad g(x) = x^2 + 4, \; x \ge 0$
18. $f(x) = 9 - x^2, \quad x \ge 0; \quad g(x) = \sqrt{9-x}$
19. $f(x) = 1 - x^3, \quad g(x) = \sqrt[3]{1-x}$
20. $f(x) = \dfrac{1}{1+x}, \; x \ge 0; \quad g(x) = \dfrac{1-x}{x}, \; 0 < x \le 1$

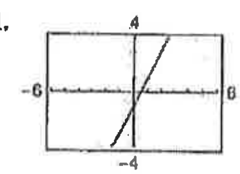
In Exercises 21–24, match the graph of the function with the graph of its inverse function. [The graphs of the inverse functions are labeled (a), (b), (c), and (d).]

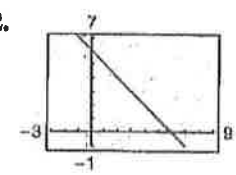
(a)


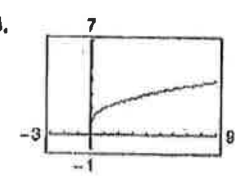
(b)


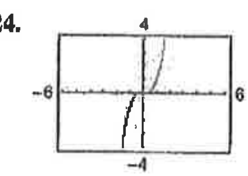
(c)


(d)


21.


22.


23.


24.


In Exercises 25–28, show that f and g are inverse functions (a) graphically and (b) numerically.

25. $f(x) = 2x, \quad g(x) = \frac{x}{2}$

26. $f(x) = x - 5, \quad g(x) = x + 5$

27. $f(x) = \frac{x - 1}{x + 5}, \quad g(x) = -\frac{5x + 1}{x - 1}$

28. $f(x) = \frac{x + 3}{x - 2}, \quad g(x) = \frac{2x + 3}{x - 1}$

In Exercises 29–40, use a graphing utility to graph the function and use the Horizontal Line Test to determine whether the function is one-to-one and so has an inverse function.

29. $f(x) = 3 - \frac{1}{2}x$

30. $f(x) = \frac{1}{4}(x + 2)^2 - 1$

31. $h(x) = \frac{x^2}{x^2 + 1}$

32. $g(x) = \frac{4 - x}{6x^2}$

33. $h(x) = \sqrt{16 - x^2}$

34. $f(x) = -2x\sqrt{16 - x^2}$

35. $f(x) = 10$

36. $f(x) = -0.65$

37. $g(x) = (x + 5)^3$

38. $f(x) = x^5 - 7$

39. $h(x) = |x + 4| - |x - 4|$

40. $f(x) = -\frac{|x - 6|}{|x + 6|}$

In Exercises 41–52, determine algebraically whether the function is one-to-one. If it is, find its inverse function. Verify your answer graphically.

41. $f(x) = x^4$

42. $g(x) = x^2 - x^4$

43. $f(x) = \frac{3x + 4}{5}$

44. $f(x) = 3x + 5$

45. $f(x) = \frac{1}{x^2}$

46. $h(x) = \frac{4}{x^2}$

47. $f(x) = (x + 3)^2, \quad x \geq -3$

48. $q(x) = (x - 5)^2, \quad x \leq 5$

49. $f(x) = \sqrt{2x + 3}$

50. $f(x) = \sqrt{x - 2}$

51. $f(x) = |x - 2|, \quad x \leq 2$

52. $f(x) = \frac{x^2}{x^2 + 1}$

In Exercises 53–62, find the inverse function of f. Use a graphing utility to graph both f and f^{-1} in the same viewing window. Describe the relationship between the graphs.

53. $f(x) = 2x - 3$

54. $f(x) = 3x$

55. $f(x) = x^5$

56. $f(x) = x^3 + 1$

57. $f(x) = x^{3/5}$

58. $f(x) = x^2, \quad x \geq 0$

59. $f(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$

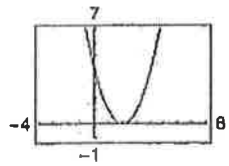
60. $f(x) = \sqrt{16 - x^2}, \quad -4 \leq x \leq 0$

61. $f(x) = \frac{4}{x}$

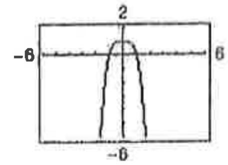
62. $f(x) = \frac{6}{\sqrt{x}}$

Think About It **In Exercises 63–66, delete part of the graph of the function so that the part that remains is one-to-one. Find the inverse function of the remaining part and give the domain of the inverse function. (There are many correct answers.)**

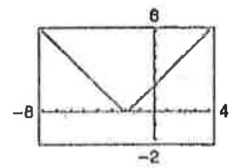
63. $f(x) = (x - 2)^2$



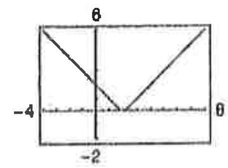
64. $f(x) = 1 - x^4$



65. $f(x) = |x + 2|$

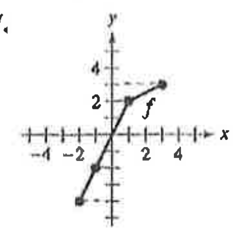


66. $f(x) = |x - 2|$



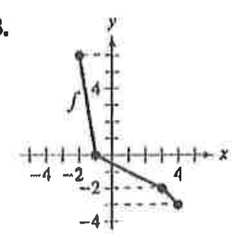
In Exercises 67 and 68, use the graph of the function f to complete the table and sketch the graph of f^{-1}.

67.



x	$f^{-1}(x)$
-4	
-2	
2	
3	

68.



x	$f^{-1}(x)$
-3	
-2	
0	
6	

Graphical Reasoning **In Exercises 69–72, (a) use a graphing utility to graph the function, (b) use the *draw inverse* feature of the graphing utility to draw the inverse of the function, and (c) determine whether the graph of the inverse relation is an inverse function, explaining your reasoning.**

69. $f(x) = x^3 + x + 1$

70. $h(x) = x\sqrt{4 - x^2}$

71. $g(x) = \dfrac{3x^2}{x^2 + 1}$

72. $f(x) = \dfrac{4x}{\sqrt{x^2 + 15}}$

In Exercises 73–78, use the functions $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$ to find the indicated value or function.

73. $(f^{-1} \circ g^{-1})(1)$

74. $(g^{-1} \circ f^{-1})(-3)$

75. $(f^{-1} \circ f^{-1})(6)$

76. $(g^{-1} \circ g^{-1})(-4)$

77. $(f \circ g)^{-1}$

78. $g^{-1} \circ f^{-1}$

In Exercises 79–82, use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the specified function.

79. $g^{-1} \circ f^{-1}$

80. $f^{-1} \circ g^{-1}$

81. $(f \circ g)^{-1}$

82. $(g \circ f)^{-1}$

83. ***Transportation*** The total value of new car sales f (in billions of dollars) in the United States from 1995 through 2001 is shown in the table. The time (in years) is given by t, with $t = 5$ corresponding to 1995. (Source: National Automobile Dealers Association)

Year, t	Sales, $f(t)$
5	456.2
6	490.0
7	507.5
8	546.3
9	606.5
10	650.3
11	690.4

(a) Does f^{-1} exist?

(b) If f^{-1} exists, what does it mean in the context of the problem?

(c) If f^{-1} exists, find $f^{-1}(650.3)$.

(d) If the table above were extended to 2002 and if the total value of new car sales for that year were \$546.3 billion, would f^{-1} exist? Explain.

84. ***Hourly Wage*** Your wage is \$8.00 per hour plus \$0.75 for each unit produced per hour. So, your hourly wage y in terms of the number of units produced is $y = 8 + 0.75x$.

(a) Find the inverse function. What does each variable in the inverse function represent?

(b) Use a graphing utility to graph the function and its inverse function.

(c) Use the *trace* feature of a graphing utility to find the hourly wage when 10 units are produced per hour.

(d) Use the *trace* feature of a graphing utility to find the number of units produced when your hourly wage is \$22.25.

Synthesis

True or False? **In Exercises 85 and 86, determine whether the statement is true or false. Justify your answer.**

85. If f is an even function, f^{-1} exists.

86. If the inverse function of f exists, and the graph of f has a y-intercept, the y-intercept of f is an x-intercept of f^{-1}.

87. ***Proof*** Prove that if f and g are one-to-one functions, $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

88. ***Proof*** Prove that if f is a one-to-one odd function, f^{-1} is an odd function.

Review

In Exercises 89–92, write the rational expression in simplest form.

89. $\dfrac{27x^3}{3x^2}$

90. $\dfrac{5x^2y}{xy + 5x}$

91. $\dfrac{x^2 - 36}{6 - x}$

92. $\dfrac{x^2 + 3x - 40}{x^2 - 3x - 10}$

In Exercises 93–98, determine whether the equation represents y as a function of x.

93. $4x - y = 3$

94. $x = 5$

95. $x^2 + y^2 = 9$

96. $x^2 + y = 8$

97. $y = \sqrt{x + 2}$

98. $x - y^2 = 0$

Selected Solutions

Lines

1) a) L_2

b) L_3

c) L_1

11) $y-1=0(x-2) \rightarrow y=1$

$(3,1), (-5,1), (7,1)$

(any real x value as long as y value is 1)

15) $y+9=-2(x-0) \rightarrow y=-2x-7$

$(1,-11)\ (-2,-5)\ (-4,-1)$

19) $5x-y+3=0$

$y=5x+3$

a) $m=5 \quad b=3$

b) see graph paper

c) on calculator

25) $(0,-2)\ m=3$

$y+2=3(x-0)$

$y=3x-2$

$-3x+y=-2$

$\boxed{3x-y=2}$

see graphpaper for graph

31) $(-\frac{1}{2},\frac{3}{2})\ m=0$

$y-\frac{3}{2}=0(x+\frac{1}{2})$

$\boxed{y=\frac{3}{2}}$

see graph paper for graph

37) $(2,\frac{1}{2})(\frac{1}{2},\frac{5}{4})$

$m=\dfrac{\frac{5}{4}-\frac{1}{2}}{\frac{1}{2}-2}=\dfrac{\frac{3}{4}}{-\frac{3}{2}}=\frac{3}{4}\cdot\frac{-2}{3}=-\frac{1}{2}$

$y-\frac{1}{2}=-\frac{1}{2}(x-2)$

$y-\frac{1}{2}=-\frac{1}{2}x+1$

$\boxed{y=-\frac{x}{2}+\frac{3}{2}}$

43) $(0,28500)\ (2,32900)$

$m=\dfrac{32900-28500}{2-0}=\dfrac{4400}{2}=2200$

$y-28500=2200(x-0)$

$y=2200x+28500$

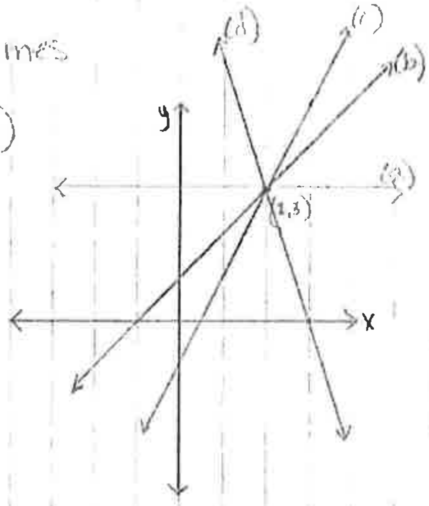
in 2006 $\rightarrow y=2200(6)+28500$

$y=13200+28500$

$\boxed{y=\$41{,}700}$

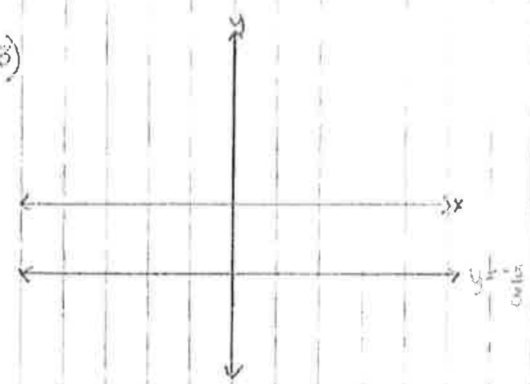
Lines

3)



19)

23)



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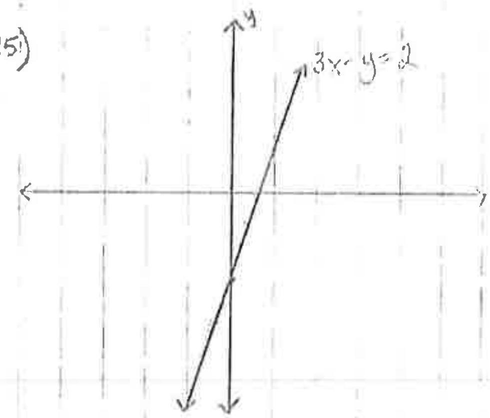

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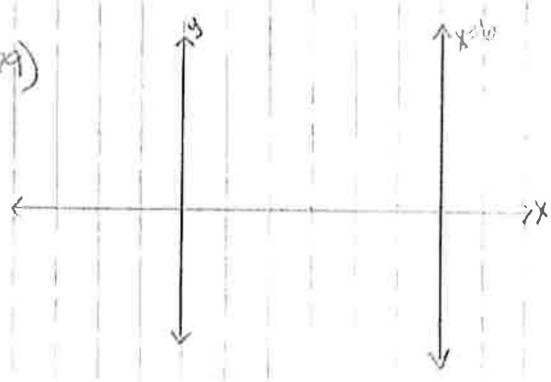

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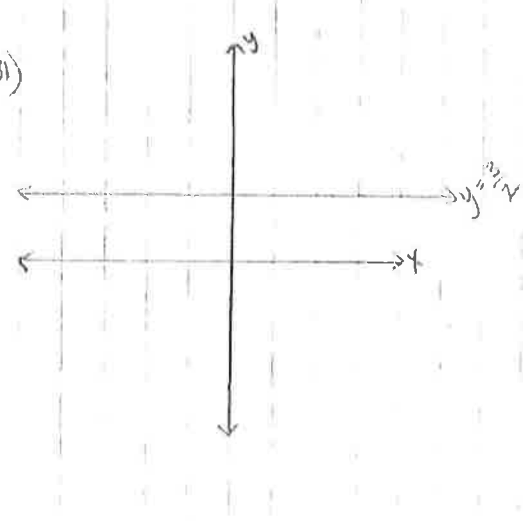


59) (3, -2) $x-4=0$
$x=4$ (vertical line)

a) $x=3$
b) $y=-2$

69) $y=\frac{3}{4}x$
since the house is 32 ft wide so high pt will be 16 ft in
$y=\frac{3}{4}(16)=12$

top height is 12 ft.

Functions

1) yes each number in the domain maps to only one number in the range

9) A, C, D

17) $2x+3y=4$

$y=-\frac{2}{3}x+\frac{4}{3}$

yes it is a function

25) $f(x)=\frac{1}{x+1}$

a) $f(4)=\frac{1}{4+1}=\frac{1}{5}$

b) $f(0)=\frac{1}{0+1}=1$

c) $f(4t)=\frac{1}{4t+1}$

d) $f(x+c)=\frac{1}{x+c+1}$

33) $g(x)=\frac{1}{x^2-9}$

a) $g(0)=\frac{1}{0-9}=-1/9$

b) $g(3)=\frac{1}{9-9}=$ undefined

c) $g(y+3)=\frac{1}{y^2+6y+9-9}=\frac{1}{y^2+6y}$

39) $h(t)=\frac{1}{2}|t+3|$

t	-5	-4	-3	-2	-1
h(t)	1	1/2	0	1/2	1

47) $f(x)=x^2$ $g(x)=x+2$

$x^2=x+2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$ $x=-1$

73) $4x+y=108$

$y=108-4x$

a) $V=x^2(108-4x)$

b) $0<108-4x$

$4x<108$

$x<27$

$(0,27)$

c) on calculator

d) found on calculator max volume when $x=18$

85) $f(x)=x^2-x+1$ $\frac{f(2+h)-f(2)}{h}$

$\frac{4+4h+h^2-2-h+1-(4-2+1)}{h}$

$=\frac{3h+h^2}{h}=3+h$

Graphs of Functions

3) domain: [-4,4]
range: [0,4]
f(0) = 4

9) $f(x) = |x+3|$
from calculator:
domain: $(-\infty,\infty)$
range: $[0,\infty)$
Algebraically:
domain $(-\infty,\infty)$ → no fractions, no radicals → no restrictions
range: Absolute value so "v" that opens up so range: $[0,\infty)$

13) $x - y^2 = 1$
fails vertical line test so not a function
to graph on calculator plug in $y_1 = \sqrt{x-1}$ and $y_2 = -\sqrt{x-1}$

25) graphed on calculator
$f(x) = x\sqrt{x+3}$
increasing: $(-2,\infty)$
decreasing: $(-3,-2)$

31) $y = 2x^3 + 3x^2 - 12x$
done on calculator
maximum: 20 at $x=-2$
minimum: -7 at $x=1$

41) see graph paper

49) $f(t) = t^2 + 2t - 3$
$f(-t) = (-t)^2 + 2(-t) - 3$
$= t^2 - 2t - 3$
neither

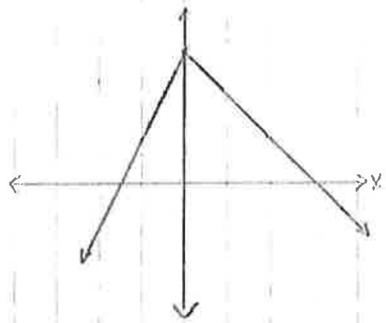
59) (4,9)
a) (-4,9)
b) (-4,-9)

67) from calculator: even
$h(x) = x^2 - 4$
$h(-x) = (-x)^2 - 4$
$= x^2 - 4$ even

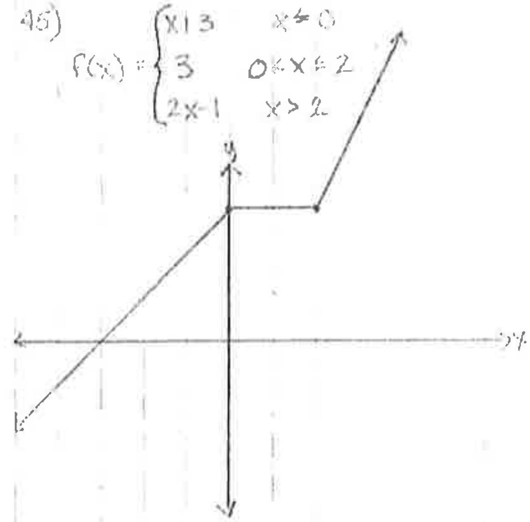
75) see graph paper

Graphs of functions

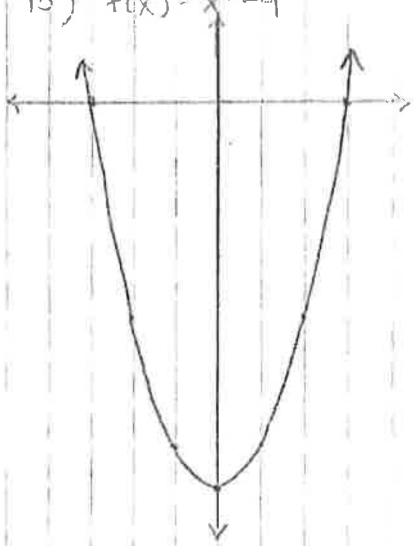
41) $f(x) = \begin{cases} 2x+3 & x<0 \\ 3-x & x\geq 0 \end{cases}$



45) $f(x) = \begin{cases} x+3 & x\leq 0 \\ 3 & 0<x\leq 2 \\ 2x-1 & x>2 \end{cases}$



75) $f(x) = x^2-9$



$f(x) \geq 0$ for $(-\infty,-3] \cup [3,\infty)$

Translations

numbers 5, 9, 11, 13 see graph

27) $y = -\sqrt{x} - 1$
the graph is $\sqrt{x}$
flipped over the x-axis
and moved down 1

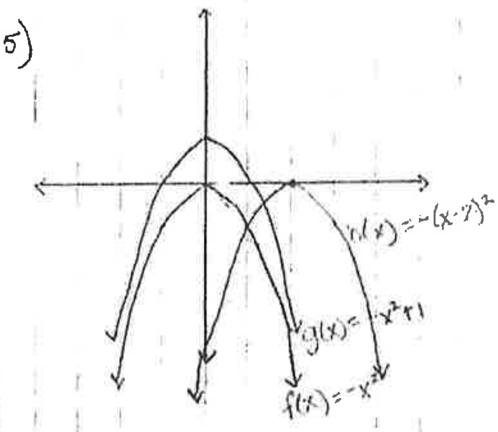
43) $p(x) = (\frac{1}{3}x)^3 + 2$
the graph is x^3
stretched horizontally
by a factor of 3 then
moved up 2

51) $g(x) = 2 - (x+5)^2$
a) x^2
b) x^2 is shifted left 5
flipped over the x-axis
and shifted up 2
c) see graph paper
d) $g(x) = 2 - f(x+5)$

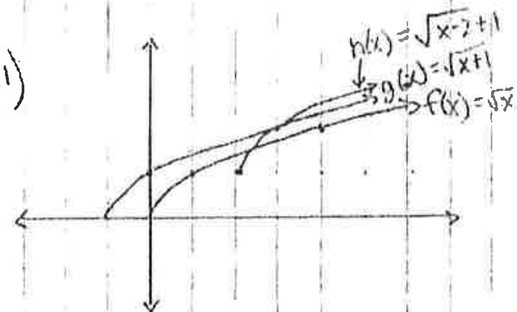
61) $g(x) = -2|x-1| - 4$
a) $|x|$
b) $|x|$ is shifted right 1,
stretched vertically by
a factor of 2, flipped over
x-axis, moved down 4
c) see graph paper
d) $g(x) = -2f(x-1) - 4$

Translations

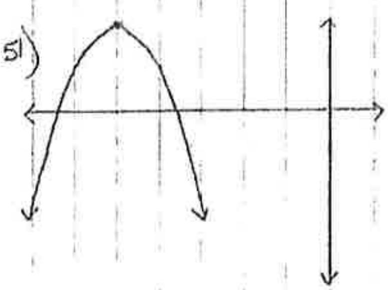
5)



11)



51)



Combinations of Functions

1) see graph paper

$f(x)=x^2+1$ $g(x)=x-4$

13) $(f+g)(3)=3^2+1+3-4$
$=9$

17) $(fg)(4)=(4^2+1)(4-4)$
$=0$

29) see graph paper

37) $f(x)=3x+5$ $g(x)=5-x$

a) $(f\circ g)(x)=f(g(x))=f(5-x)$
$=3(5-x)+5=15-3x+5$
$=20-3x$

b) $(g\circ f)(x)=g(f(x))=g(3x+5)$
$=5-(3x+5)=-3x$

43) $f(x)=x^{2/3}$ $g(x)=x^6$

a) $f(g(x))=f(x^6)=(x^6)^{2/3}$
$=x^4$

domain: $(-\infty,\infty)$

$g(f(x))=g(x^{2/3})=(x^{2/3})^6$
$=x^4$

b) $f\circ g=g\circ f$ for $x\in(-\infty,\infty)$

51) a) $(f+g)(3)=f(3)+g(3)$
$=2+1=3$

b) $(f/g)(2)=f(2)/g(2)$
$=0/2=0$

57) $h(x)=\sqrt[3]{x^2-4}$
$f(x)=\sqrt[3]{x}$
$g(x)=x^2-4$

63) $f(x)=\sqrt{x+4}$ $g(x)=x^2$

a) $x+4\geq 0$
$x\geq -4$ domain $[-4,\infty)$

b) domain: $(-\infty,\infty)$

c) $f(g(x))=f(x^2)=\sqrt{x^2+4}$
domain $(-\infty,\infty)$

71) $f(x)=x+2$ $g(x)=\frac{1}{x^2-4}$

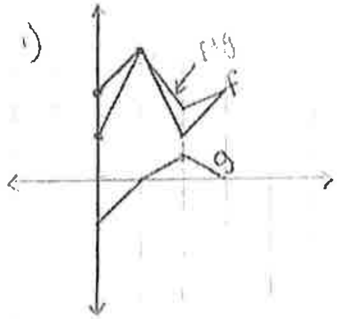
a) domain $(-\infty,\infty)$

b) domain $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

c) $f(g(x))=f\left(\frac{1}{x^2-4}\right)=\frac{1}{x^2-4}+2$
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

Combinations of functions

1)



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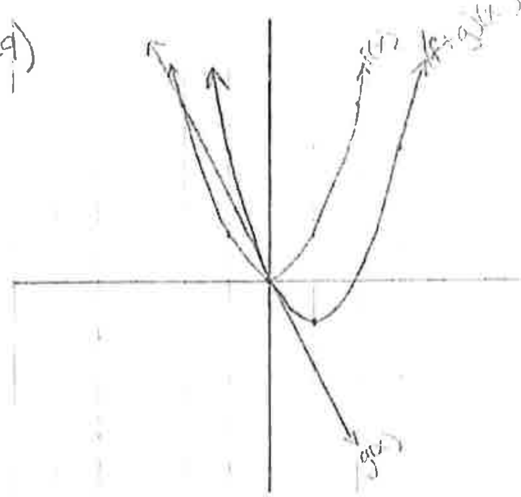


Inverse Functions

3) $f(x) = 2x+1$

$x = 2y+1$

$\frac{x-1}{2} = f^{-1}(x)$

$f(f^{-1}(x)) = f\left(\frac{x-1}{2}\right)$

$= 2\left(\frac{x-1}{2}\right)+1$

$= x-1+1$

$= x$

$f^{-1}(f(x)) = f^{-1}(2x+1)$

$= \frac{2x+1-1}{2}$

$= \frac{2x}{2} = x$

13) $f(x) = -\sqrt{x-8}$ $g(x) = 8+x^2$ $x \le 0$

a) $f(g(x)) = f(8+x^2) = -\sqrt{8+x^2-8} = -x$

$g(f(x)) = g(-\sqrt{x-8}) = 8+(-\sqrt{x-8})^2$

$= 8+x-8 = x$

b)

x	f(x)	g(x)
9	-1	89
89	-9	7929
12	-2	152
152	-12	23112

17) $f(x) = 1-x^3$ $g(x)$ $\sqrt[3]{1-x}$

$f(g(x)) = f(\sqrt[3]{1-x}) = 1-(\sqrt[3]{1-x})^3$

$= 1-(1-x)$

$= x$

$g(f(x)) = g(1-x^3) = \sqrt[3]{1-(1-x^3)} = x$

see graph paper

they are reflections over $y=x$

27) a) see graph paper

b)

x	f(x)	g(x)
2	$1/7$	-11
$1/7$	$-1/10$	2
-11	2	$-9/2$

33) $h(x) = \sqrt{16-x^2}$ graphed on calc.

fails horizontal line test so not one to one

43) $f(x) = \frac{3x+4}{5}$

$\frac{3a+4}{5} = \frac{3b+4}{5}$

$3a+4 = 3b+4$

$3a = 3b$

$a = b$

yes one to one

$x = \frac{3y+4}{5}$

$5x = 3y+4$

$5x-4 = 3y$

$f^{-1}(x) = \frac{5x-4}{3}$

75) $f(x) = \frac{1}{8}x-3$

$x = \frac{1}{8}y-3$

$x+3 = \frac{1}{8}y$

$f^{-1}(x) = 8x+24$

$f^{-1}(f^{-1}(6)) = f^{-1}(8(6)+24)$

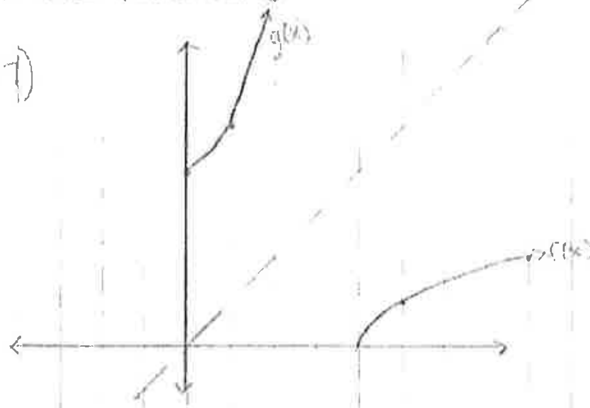
$= f^{-1}(72)$

$= 8(72)+24$

$= 600$

Inverse functions

17)



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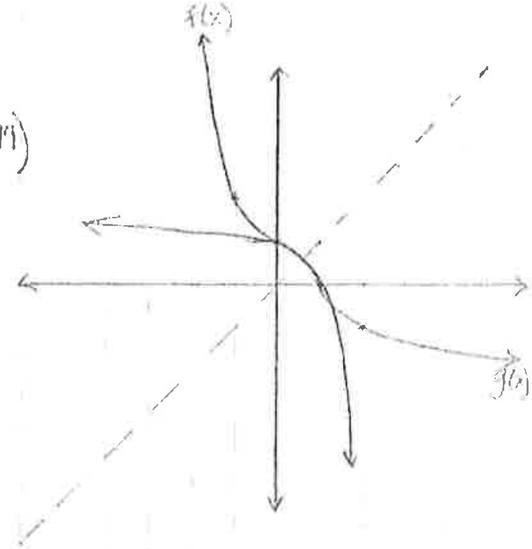

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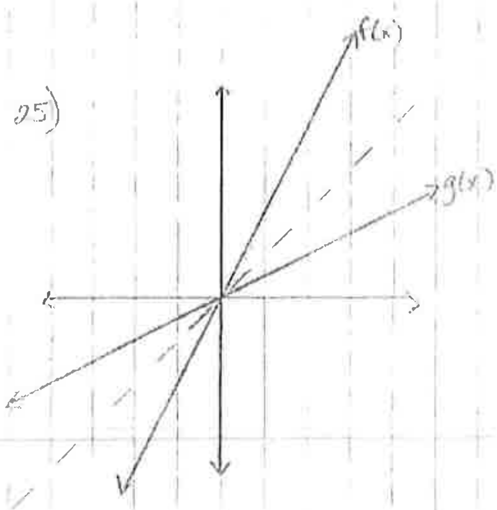

27)