Summer Math Packet

for

Rising Geometry Students

This packet is designed to help you review your Algebra Skills and help you prepare for your Geometry class. Your Geometry teacher will expect you to be proficient in these skills and will collect your work on the first day of class.

Directions:

1. Do ALL the exercises on separate paper. Please do not complete your work on the packet pages themselves. Label each topic clearly and compile your work in order. WORK SHOULD BE SHOWN ON THE SAME PAPER AS THE ANSWERS – ANSWERS WITHOUT WORK WILL **NOT** BE GIVEN CREDIT.
2. Each topic has an explanation and example problems at the top of the page. Some of the more difficult topics have embedded videos that you can watch. Be sure to read the example problems and watch the video before attempting the problems.
3. Check your work with the answers provided at the end of the packet. If your answer is incorrect, go back and rework the problems until it is correct.
4. If you need additional help, the following resources are available online:

Explanations and Example Problems:

http://www.classzone.com/books/algebra_1/index.cfm?state=MD

http://www.purplemath.com/

Videos and Tutorials:
http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=ate&wcsuffix=0775

https://www.khanacademy.org/math/algebra

Topic 1: Order of Operations

Evaluate Numerical Expressions Numerical expressions often contain more than one operation. To evaluate them, use the rules for order of operations shown below.

Order of Operations	**Step 1** Evaluate expressions inside grouping symbols. **Step 2** Evaluate all powers. **Step 3** Do all multiplication and/or division from left to right. **Step 4** Do all addition and/or subtraction from left to right.

Example 1 **Evaluate each expression.**

a. 3^4

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ Use 3 as a factor 4 times.

$= 81$ Multiply.

b. 6^3

$6^3 = 6 \cdot 6 \cdot 6$ Use 6 as a factor 3 times.

$= 216$ Multiply.

Example 2 **Evaluate each expression.**

a. $3[2 + (12 \div 3)^2]$

$3[2 + (12 \div 3)^2] = 3(2 + 4^2)$ Divide 12 by 3.

$= 3(2 + 16)$ Find 4 squared.

$= 3(18)$ Add 2 and 16.

$= 54$ Multiply 3 and 18.

b. $\frac{3 + 2^3}{4^2 \cdot 3}$

$\frac{3 + 2^3}{4^2 \cdot 3} = \frac{3 + 8}{4^2 \cdot 3}$ Evaluate power in numerator.

$= \frac{11}{4^2 \cdot 3}$ Add 3 and 8 in the numerator.

$= \frac{11}{16 \cdot 3}$ Evaluate power in denominator.

$= \frac{11}{48}$ Multiply.

Topic 1 Exercises:

Evaluate each expression

1. 5^2
2. 10^4
3. 8^3
4. $(8 - 4) \cdot 2$
5. $10 + 8 \cdot 3$
6. $12(20 - 17) - 3 \cdot 6$
7. $3^2 \div 3 + 2^2 \cdot 7 - 20 \div 5$
8. $250 \div [5(3 \cdot 7 + 4)]$
9. $\frac{4(5^2) - 4 \cdot 3}{4(4 \cdot 5 + 2)}$

Evaluate each expression when $x = 2, y = 3, z = 4, a = \frac{4}{5},$ and $b = \frac{3}{5}$

10. $x + 7$
11. $x + y^2$
12. $6a + 8b$
13. $\frac{y^2}{x^2}$
14. $x(2y + 3z)$
15. $\frac{3xy - 4}{7x}$
16. $\frac{z^2 - y^2}{x^2}$
17. $\frac{(z - y)^2}{x}$
18. $\frac{5a^2 b}{y}$
19. $\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2$
20. $\left(\frac{z \div x}{y}\right) + \left(\frac{y \div x}{z}\right)$

Topic 2: Distributive Property and Combining Like Terms

Evaluate Expressions The Distributive Property can be used to help evaluate expressions.

Distributive Property	For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Simplify Expressions A **term** is a number, a variable, or a product or quotient of numbers and variables. **Like terms** are terms that contain the same variables, with corresponding variables having the same powers. The Distributive Property and properties of equalities can be used to simplify expressions. An expression is in **simplest form** if it is replaced by an **equivalent** expression with no like terms or parentheses.

Example **Simplify $4(a^2 + 3ab) - ab$.**

$4(a^2 + 3ab) - ab$	$= 4(a^2 + 3ab) - 1ab$	Multiplicative Identity
	$= 4a^2 + 12ab - 1ab$	Distributive Property
	$= 4a^2 + (12 - 1)ab$	Distributive Property
	$= 4a^2 + 11ab$	Substitution

Topic 2 Exercises:

Simplify each expression. If not possible, write *simplified*.

1. $5(4x - 9)$
2. $12(6 - \frac{1}{2}x)$
3. $3(2x - y)$
4. $2(3x + 2y - z)$
5. $(x - 2)y$
6. $\frac{1}{4}(16x - 12y + 4x)$
7. $12a - a$
8. $3x - 1$
9. $3x^2 + 2x^2$
10. $2p + \frac{1}{2}p$
11. $21a + 18a + 31b - 3b$
12. $2 - 1 - 6x + x^2$
13. $10xy - 4(xy + xy)$
14. $4x + \frac{1}{4}(16x - 20y)$

Write an algebraic expression for each verbal expression, then simplify.

15. Six times the difference of $2a$ and b, increased by $4b$.

16. Two times the sum of x squared and y squared, increased by three times the sum of x squared and y squared.

Topic 3: Writing Equations

Write Equations Writing equations is one strategy for solving problems. You can use a variable to represent an unspecified number or measure referred to in a problem. Then you can write a verbal expression as an algebraic expression.

Example 1 **Translate each sentence into an equation or a formula.**

a. Ten times a number x is equal to 2.8 times the difference y minus z.

$10 \times x = 2.8 \times (y - z)$

The equation is $10x = 2.8(y - z)$.

b. A number m minus 8 is the same as a number n divided by 2.

$m - 8 = n \div 2$

The equation is $m - 8 = \frac{n}{2}$.

c. The area of a rectangle equals the length times the width. Translate this sentence into a formula.

Let A = area, ℓ = length, and w = width.

Formula: *Area equals length times width.*

$A = \ell \times w$

The formula for the area of a rectangle is $A = \ell w$.

Example 2 **Use the Four-Step Problem-Solving Plan.**

POPULATION **The population of the United States in July 2007 was about 301,000,000, and the land area of the United States is about 3,500,000 square miles. Find the average number of people per square mile in the United States.**

Step 1 *Explore* You know that there are 301,000,000 people. You want to know the number of people per square mile.

Step 2 *Plan* Write an equation to represent the situation. Let p represent the number of people per square mile.

$3,500,000 \times p = 301,000,000$

Step 3 *Solve* $3,500,000 \times p = 301,000,000$.

$3,500,000p = 301,000,000$ Divide each side by 3,500,000.

$p = 86$

There are 86 people per square mile.

Step 4 *Check* If there are 86 people per square mile and there are 3,500,000 square miles, $86 \times 3,500,000 = 301,000,000$. The answer makes sense.

Write Verbal Sentences You can translate equations into verbal sentences.

Example **Translate each equation into a sentence.**

a. $4n - 8 = 12$.

$4n \quad - \quad 8 \quad = \quad 12$

Four times n minus eight equals twelve.

Topic 3 Exercises:

Translate each sentence into an equation or formula.

1. Three times a number t minus twelve equals forty.
2. One-half of the difference of a and b is 54.
3. Three times the sum of d and 4 is 32.
4. The area A of a circle is the product of π and the radius, r, squared.
5. **WEIGHT LOSS** Lou wants to lose weight to audition for a part in a play. He weighs 160 pounds now. He wants to weigh 150 pounds.
   a. If p represents the number of pounds he wants to lose, write and equation to represent this situation.
   b. How many pounds does he need to lose to reach his goal?

Translate each equation into a sentence.

6. $4a - 6 = 23$
7. $x^2 + y^2 = 8$
8. $b = \frac{1}{3}(h - 1)$
9. $A = \frac{1}{2}bh$

Topic 4: Solving Equations, Ratios and Proportions

Variables on Each Side To solve an equation with the same variable on each side, first use the Addition or the Subtraction Property of Equality to write an equivalent equation that has the variable on just one side of the equation. Then solve the equation.

Example 1 **Solve $5y - 8 = 3y + 12$.**

$$5y - 8 = 3y + 12$$
$$5y - 8 - 3y = 3y + 12 - 3y$$
$$2y - 8 = 12$$
$$2y - 8 + 8 = 12 + 8$$
$$2y = 20$$
$$\frac{2y}{2} = \frac{20}{2}$$
$$y = 10$$

The solution is 10.

Example 2 **Solve $-11 - 3y = 8y + 1$.**

$$-11 - 3y = 8y + 1$$
$$-11 - 3y + 3y = 8y + 1 + 3y$$
$$-11 = 11y + 1$$
$$-11 - 1 = 11y + 1 - 1$$
$$-12 = 11y$$
$$\frac{-12}{11} = \frac{11y}{11}$$
$$-1\frac{1}{11} = y$$

The solution is $-1\frac{1}{11}$.

Grouping Symbols When solving equations that contain grouping symbols, first use the Distributive Property to eliminate grouping symbols. Then solve.

Example **Solve $4(2a - 1) = -10(a - 5)$.**

$4(2a - 1) = -10(a - 5)$	Original equation
$8a - 4 = -10a + 50$	Distributive Property
$8a - 4 + 10a = -10a + 50 + 10a$	Add 10a to each side.
$18a - 4 = 50$	Simplify.
$18a - 4 + 4 = 50 + 4$	Add 4 to each side.
$18a = 54$	Simplify.
$\frac{18a}{18} = \frac{54}{18}$	Divide each side by 18.
$a = 3$	Simplify.

The solution is 3.

Topic 4 Exercises:

Solve each Equation. It is possible for the equation to be an *identity* or have *no solution*.

1. $5x + 2 = 2x - 10$
2. $1.1x + 4.3 = 2.1$
3. $\frac{1}{2}b + 4 = \frac{1}{8}b + 88$
4. $8 - 5p = 4p - 1$
5. $-4 - 3x = 7x - 6$
6. $20 - a = 10a - 2$
7. $\frac{2}{5}y - 8 = 9 - \frac{3}{5}y$
8. $-4r + 5 = 5 - 4r$
9. $12 + 2y = 10y - 12$
10. $-3(x + 5) = 3(x - 1)$
11. $3(a + 1) - 5 = 3a - 2$
12. $5(f + 2) = 2(3 - f)$
13. $18 = 3(2t + 2)$
14. $3(d - 8) = 3d$
15. $1.2(x - 2) = 2 - x$
16. $2(w - 1) + 4 = 4(w + 1)$
17. $2[2 + 3(y - 1)] = 22$
18. $\frac{a-8}{12} = \frac{2a+5}{3}$
19. $-3(x - 8) = 24$
20. $6(2 - 2y) = 5(2y - 2)$

Solve for Variables Sometimes you may want to solve an equation such as $V = lwh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ. If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 **Solve $2x - 4y = 8$, for y.**

$$2x - 4y = 8$$
$$2x - 4y - 2x = 8 - 2x$$
$$-4y = 8 - 2x$$
$$\frac{-4y}{-4} = \frac{8 - 2x}{-4}$$
$$y = \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4}$$

The value of y is $\frac{2x - 8}{4}$.

Example 2 **Solve $3m - n = km - 8$, for m.**

$$3m - n = km - 8$$
$$3m - n - km = km - 8 - km$$
$$3m - n - km = -8$$
$$3m - n - km + n = -8 + n$$
$$3m - km = -8 + n$$
$$m(3 - k) = -8 + n$$
$$\frac{m(3 - k)}{3 - k} = \frac{-8 + n}{3 - k}$$
$$m = \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k}$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Topic 4 Exercises continued:

Solve each equation or formula for the variable indicated.

21. $ax - b = c$, for x
22. $xy + w = 9$, for y
23. $x(4 - k) = p$, for k
24. $4(r + 3) = t$, for r
25. $x(1 + y) = z$, for x
26. $16w + 4x = y$, for x
27. $A = \frac{h(a+b)}{2}$, for h
28. $C = \frac{5}{9}(F - 32)$, for F
29. $P = 2l + 2w$, for l

Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y, $x{:}y$ or $\frac{x}{y}$. Ratios are usually expressed in simplest form. An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 **Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.**

$\frac{24}{36} = \frac{2}{3}$ when expressed in simplest form.

$\frac{12}{18} = \frac{2}{3}$ when expressed in simplest form.

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 **Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.**

$\frac{10}{18} \stackrel{?}{=} \frac{25}{45}$ Write the proportion.

$10(45) \stackrel{?}{=} 18(25)$ Cross products

$450 = 450$ Simplify.

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

Topic 4 Exercises continued:

Evaluate each expression.

30. $\frac{-3}{x} = \frac{2}{8}$
31. $\frac{5}{8} = \frac{p}{24}$
32. $\frac{9}{y+1} = \frac{18}{54}$
33. $\frac{1.5}{x} = \frac{12}{x}$
34. $\frac{a-18}{12} = \frac{15}{3}$
35. $\frac{2+w}{6} = \frac{12}{9}$
36. Josh finished 24 math problems in one hour. At that rate, how many hours will it take him to complete 72 problems?

Topic 5: Graphing Linear Equations, Slope, and Direct Variation

Graph Linear Equations The graph of a linear equations represents all the solutions of the equation. An x-coordinate of the point at which a graph of an equation crosses the x-axis in an **x-intercept.** A y-coordinate of the point at which a graph crosses the y-axis is called a **y-intercept.**

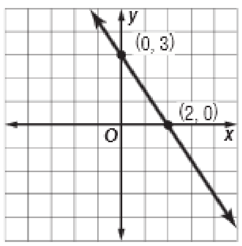
Example 1 **Graph the equation $3x + 2y = 6$ by using the x and y-intercepts.**

To find the x-intercept, let $y = 0$ and solve for x. The x-intercept is 2. The graph intersects the x-axis at (2, 0).

To find the y-intercept, let $x = 0$ and solve for y.

The y-intercept is 3. The graph intersects the y-axis at (0, 3).

Plot the points (2, 0) and (0, 3) and draw the line through them.



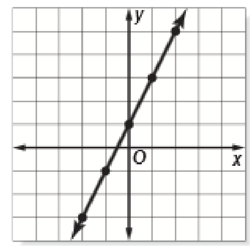
Example 2 **Graph the equation $y - 2x = 1$ by making a table.**

Solve the equation for y.

$y - 2x = 1$	Original equation.
$y - 2x + 2x = 1 + 2x$	Add $2x$ to each side.
$y = 2x + 1$	Simplify.

Select five values for the domain and make a table. Then graph the ordered pairs and draw a line through the points.

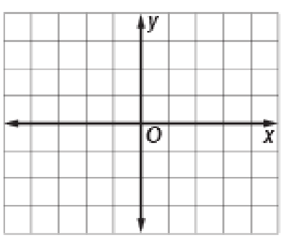
x	$2x + 1$	y	(x, y)
−2	2(−2) + 1	−3	(−2, −3)
−1	2(−1) + 1	−1	(−1, −1)
0	2(0) + 1	1	(0, 1)
1	2(1) + 1	3	(1, 3)
2	2(2) + 1	5	(2, 5)



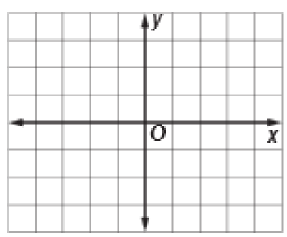
Topic 5 Exercises:

Graph each equation by using the x- and y-intercepts.

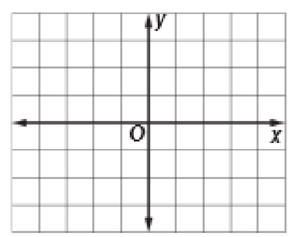
1. $2x + y = -2$



2. $3x - 6y = -3$

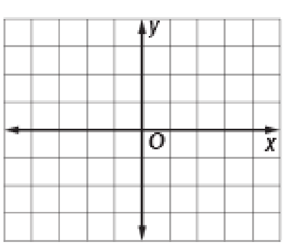


3. $-2x + y = -2$

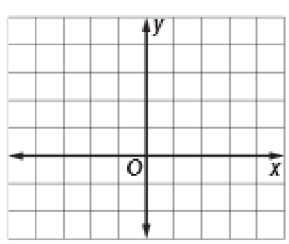


Graph each equation by making a table.

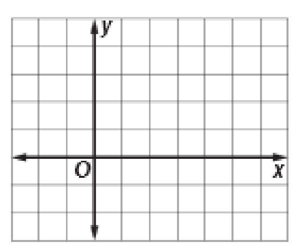
4. $y = 2x$



5. $x - y = -1$



6. $x + 2y = 4$



Find Slope The **slope** of a line is the ratio of change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move in the positive direction.

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line

Example 1 **Find the slope of the line that passes through (−3, 5) and (4, −2).**

Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-2 - 5}{-4 (-3)}$ $y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3$

$= \frac{-7}{7}$ Simplify.

$= -1$

Example 2 **Find the value of r so that the line through (10, r) and (3, 4) has a slope of $-\frac{2}{7}$.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$-\frac{2}{7} = \frac{4 - r}{3 - 10}$ $m = -\frac{2}{7}, y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10$

$-\frac{2}{7} = \frac{4 - r}{-7}$ Simplify.

$-2(-7) = 7(4 - r)$ Cross multiply.

$14 = 28 - 7r$ Distributive Property

$-14 = -7r$ Subtract 28 from each side.

$2 = r$ Divide each side by −7.

Topic 5 Exercises continued:

Find the slope that passes through each pair of points.

1. (4, 9), (1, 6) **2.** (−4, −1), (−2, −5) **3.** (−4, −1), (−4, −5)

4. (2, 1), (8, 9) **5.** (14, −8), (7, −6) **6.** (4, −3), (8, −3)

Find the value of *r* so that the line passing through the given points has the given slope.

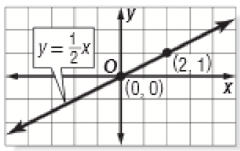
7. (6, 8), (r, −2), $m = 1$ **8.** (−1, −3), (7, r), $m = \frac{3}{4}$ **9.** (2, 8), (r, −4) $m = -3$

10. (7, −5), (6, r), $m = 0$ **11.** (r, 4), (7, 1), $m = \frac{3}{4}$ **12.** (7, 5), (r, 9), $m = 6$

Direct Variation Equations

A **direct variation** is described by an equation of the form $y = kx$, where $k \neq 0$. We say that y *varies directly as* x. In the equation $y = kx$, k is the **constant of variation**.

Example 1 **Name the constant of variation for the equation. Then find the slope of the line that passes through the pair of points.**



For $y = \frac{1}{2}x$, the constant of variation is $\frac{1}{2}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{1 - 0}{2 - 0}$ $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (2, 1)$

$= \frac{1}{2}$ Simplify.

The slope is $\frac{1}{2}$.

Example 2 **Suppose y varies directly as x, and $y = 30$ when $x = 5$.**

a. Write a direct variation equation that relates x and y.

Find the value of k.

$y = kx$ Direct variation equation

$30 = k(5)$ Replace y with 30 and x with 5.

$6 = k$ Divide each side by 5.

Therefore, the equation is $y = 6x$.

b. Use the direct variation equation to find x when $y = 18$.

$y = 6x$ Direct variation equation

$18 = 6x$ Replace y with 18.

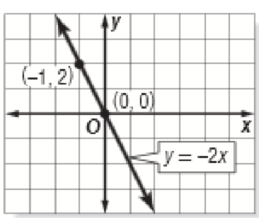
$3 = x$ Divide each side by 6.

Therefore, $x = 3$ when $y = 18$.

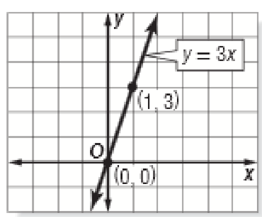
Topic 5 Exercises continued:

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

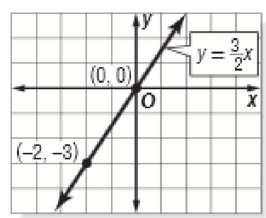
13.



14.



15.



Suppose y varies directly as x. Write a direct variation equation that relates x to y. Then solve.

16. If $y = 4$ when $x = 2$, find y when $x = 16$.
17. If $y = 9$ when $x = -3$, find y when $x = 6$.
18. If $y = -4.8$ when $x = -1.6$, find y when $x = -24$.
19. If $y = \frac{1}{4}$ when $x = \frac{1}{8}$, find y when $x = \frac{3}{16}$.

Topic 6: Writing Equations of Lines

Slope-Intercept Form

Slope-Intercept Form	$y = mx + b$, where m is the given slope and b is the y-intercept

Point-Slope Form

Point-Slope Form	$y - y_1 = m(x - x_1)$, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line

Example 1 **Write an equation in point-slope form for the line that passes through (6, 1) with a slope of $-\frac{5}{2}$.**

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 1 = -\frac{5}{2}(x - 6)$ $m = -\frac{5}{2}$; $(x_1, y_1) = (6, 1)$

Therefore, the equation is $y - 1 = -\frac{5}{2}(x - 6)$.

Example 2 **Write an equation in point-slope form for a horizontal line that passes through (4, −1).**

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-1) = 0(x - 4)$ $m = 0$; $(x_1, y_1) = (4, -1)$

$y + 1 = 0$ Simplify.

Therefore, the equation is $y + 1 = 0$.

Standard Form

Standard Form	$Ax + By = C$, where A and B are coefficients and C is a constant

** After finding point-slope form, manipulate the equation to put the equation in slope-intercept form and standard form

Watch this Video!

https://www.khanacademy.org/math/trigonometry/graphs/line_equation/v/point-slope-and-standard-form

Topic 6 Exercises:

Write the equation of the line passing through the given points in:

a.) Point-slope form

b.) Slope-intercept form

c.) Standard form using integers

(follow the example in the video. Each problem has 3 solutions)

1. $(-1,7)$ *and* $(0,4)$
2. $(6,-4)$ *and* $(-3,5)$
3. $(-3,-4)$ *and* $(3,-2)$
4. $(2,7)$ *and* $(1-4)$
5. $(0,-5)$ *and* $(3,2)$
6. $(4,-2)$ *and* $(9,-8)$

Parallel and Perpendicular Lines

Parallel Lines Two nonvertical lines are **parallel** if they have the same slope. All vertical lines are parallel.

Example **Write an equation in slope-intercept form for the line that passes through (–1, 6) and is parallel to the graph of $y = 2x + 12$.**

A line parallel to $y = 2x + 12$ has the same slope, 2. Replace m with 2 and (x_1, y_1) with (–1, 6) in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 6 = 2(x - (-1))$	$m = 2$; $(x_1, y_1) = (-1, 6)$
$y - 6 = 2(x + 1)$	Simplify.
$y - 6 = 2x + 2$	Distributive Property
$y = 2x + 8$	Slope-intercept form

Therefore, the equation is $y = 2x + 8$.

Perpendicular Lines Two non-vertical lines are **perpendicular** if their slopes are negative reciprocals of each other. Vertical and horizontal lines are perpendicular.

Example **Write an equation in slope-intercept form for the line that passes through (–4, 2) and is perpendicular to the graph of $2x - 3y = 9$.**

Find the slope of $2x - 3y = 9$.

$2x - 3y = 9$	Original equation
$-3y = -2x + 9$	Subtract $2x$ from each side.
$y = \frac{2}{3}x - 3$	Divide each side by –3.

The slope of $y = \frac{2}{3}x - 3$ is $\frac{2}{3}$. So, the slope of the line passing through (–4, 2) that is perpendicular to this line is the negative reciprocal of $\frac{2}{3}$, or $-\frac{3}{2}$.
Use the point-slope form to find the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = -\frac{3}{2}(x - (-4))$	$m = -\frac{3}{2}$; $(x_1, y_1) = (-4, 2)$
$y - 2 = -\frac{3}{2}(x + 4)$	Simplify.
$y - 2 = -\frac{3}{2}x - 6$	Distributive Property
$y = -\frac{3}{2}x - 4$	Slope-intercept form

Topic 6 Exercises continued:

Write the equation of the line in slope-intercept form that passes through the given point and is parallel to the given line.

7. $(-2,2), y = 4x - 2$

8. $(-1,6), 3x + y = 12$

9. $(-8,7), y = -\frac{1}{2}x - 4$

Write the equation of the line in slope-intercept form that passes through the given point and is perpendicular to the given line.

10. $(6,4), y = -3x + 1$

11. $(4,-2), y = -2x + 3$

12. $(0,4),\ y = 4$

Topic 7: Solving and Graphing Inequalities

Solve Inequalities Involving the Distributive Property When solving inequalities that contain grouping symbols, first use the Distributive Property to remove the grouping symbols. Then undo the operations in reverse of the order of operations, just as you would solve an equation with more than one operation.

Example **Solve $3a - 2(6a - 4) > 4 - (4a + 6)$.**

$3a - 2(6a - 4) > 4 - (4a + 6)$	Original inequality
$3a - 12a + 8 > 4 - 4a - 6$	Distributive Property
$-9a + 8 > -2 - 4a$	Combine like terms.
$-9a + 8 + 4a > -2 - 4a + 4a$	Add 4a to each side.
$-5a + 8 > -2$	Combine like terms.
$-5a + 8 - 8 > -2 - 8$	Subtract 8 from each side.
$-5a > -10$	Simplify.
$a < 2$	Divide each side by -5 and change $>$ to $<$.

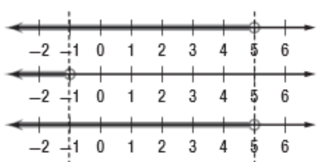
The solution in set-builder notation is $\{a \mid a < 2\}$.

Solving Compound Inequalities

Inequalities Containing *or* A compound inequality containing *or* is true if one or both of the inequalities are true. The graph of a compound inequality containing *or* is the **union** of the graphs of the two inequalities. The union can be found by graphing both inequalities on the same number line. A solution of the compound inequality is a solution of either inequality, not necessarily both.

Example **Solve $2a + 1 < 11$ or $a > 3a + 2$. Then graph the solution set.**

$2a + 1 < 11$	or	$a > 3a + 2$
$2a + 1 - 1 < 11 - 1$		$a - 3a > 3a - 3a + 2$
$2a < 10$		$-2a > 2$
$\frac{2a}{2} < \frac{10}{2}$		$\frac{-2a}{-2} < \frac{-2}{-2}$
$a < 5$		$a < -1$



Graph $a < 5$.

Graph $a < -1$.

Find the union.

The solution set is $\{a \mid a < 5\}$.

Topic 7 Exercises:

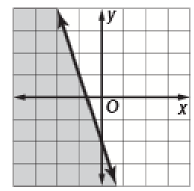
Solve each inequality and graph the solution on a number line.

1. $2(t + 3) \geq 16$
2. $3x + 10 > 8 - (x + 14)$
3. $2(y - 2) > -4 + 2y$
4. $m + 17 \leq -(3m - 13)$
5. $k - 6 \leq -(10 + k)$
6. $n - 4 \leq -3(2 + n)$
7. $3 < 3w$ *or* $3w \geq 9$
8. $2y + 2 < 12$ *or* $y - 3 \geq 2y$
9. $\frac{1}{2}n > -2$ *or* $2n - 2 < 6 + n$
10. $3a + 2 \geq 5$ *or* $7 + 3a < 2a + 6$

Graphing Inequalities in Two Variables

Graph Linear Inequalities The solution set of an inequality that involves two variables is graphed by graphing a related linear equation that forms a boundary of a **half-plane**. The graph of the ordered pairs that make up the solution set of the inequality fill a region of the coordinate plane on one side of the half-plane.

Example **Graph $y \le -3x - 2$.**



Graph $y = -3x - 2$.

Since $y \le -3x - 2$ is the same as $y < -3x - 2$ and $y = -3x - 2$, the boundary is included in the solution set and the graph should be drawn as a solid line.

Select a point in each half plane and test it. Choose (0, 0) and (−2, −2).

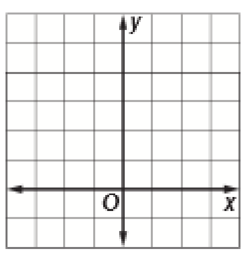
$y \le -3x - 2$	$y \le -3x - 2$
$0 \le -3(0) - 2$	$-2 \le -3(-2) - 2$
$0 \le -2$ is false.	$-2 \le 6 - 2$
	$-2 \le 4$ is true.

The half-plane that contains (−2, −2) contains the solution. Shade that half-plane.

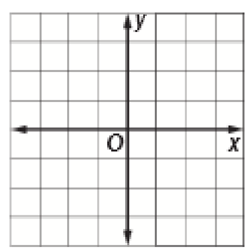
Topic 7 Exercises Continued:

Graph each inequality.

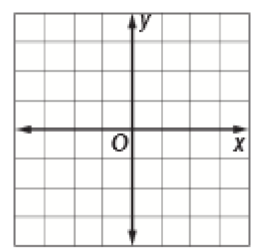
11. $y < 4$



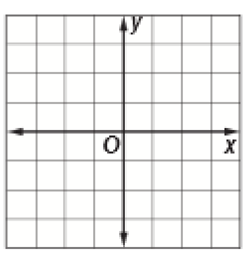
12. $x \ge 1$



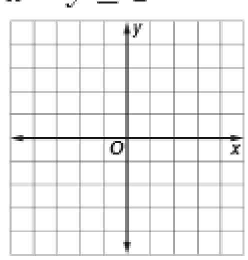
13. $3x \le y$



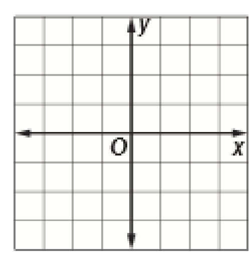
14. $-x > y$



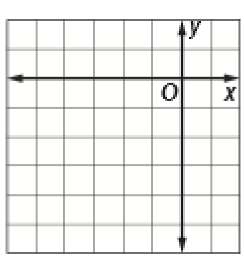
15. $x - y \ge 1$



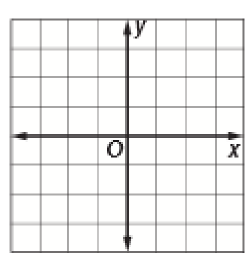
16. $2x - 3y \le 6$



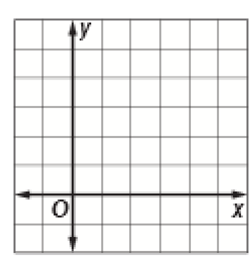
17. $y < -\frac{1}{2}x - 3$



18. $4x - 3y < 6$

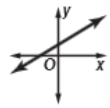
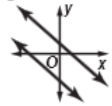


19. $3x + 6y \ge 12$



Topic 8: Solving Systems of Linear Equations

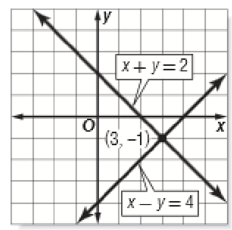
Possible Number of Solutions Two or more linear equations involving the same variables form a **system of equations**. A solution of the system of equations is an ordered pair of numbers that satisfies both equations. The table below summarizes information about systems of linear equations.

Graph of a System	intersecting lines 	same line 	parallel lines 
Number of Solutions	exactly one solution	infinitely many solutions	no solution
Terminology	consistent and independent	consistent and dependent	inconsistent

Solve by Graphing One method of solving a system of equations is to graph the equations on the same coordinate plane.

Example **Graph each system and determine the number of solutions that it has. If it has one solution, name it.**

$\boldsymbol{x + y = 2}$
$\boldsymbol{x - y = 4}$



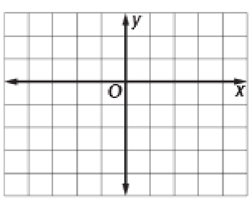
The graphs intersect. Therefore, there is one solution. The point $(3, -1)$ seems to lie on both lines. Check this estimate by replacing x with 3 and y with -1 in each equation.

$x + y = 2$
$3 + (-1) = 2$ 3
$x - y = 4$
$3 - (-1) = 3 + 1$ or 4 3
The solution is $(3, -1)$.

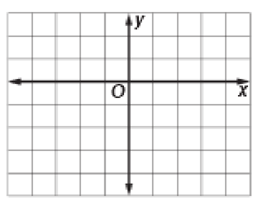
Topic 8 Exercises:

Graph each system and determine the number of solutions it has. If it has one solution, name it.

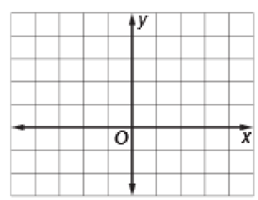
1. $y = -2$
$3x - y = -1$



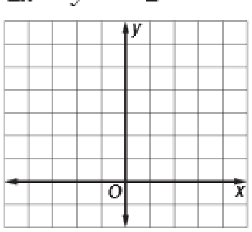
2. $x = 2$
$2x + y = 1$



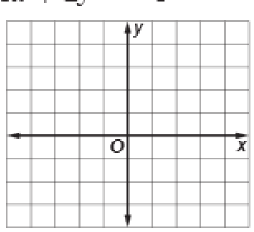
3. $y = \frac{1}{2}x$
$x + y = 3$



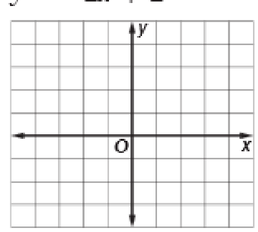
4. $2x + y = 6$
$2x - y = -2$



5. $3x + 2y = 6$
$3x + 2y = -4$



6. $2y = -4x + 4$
$y = -2x + 2$



Solve by Substitution

One method of solving systems of equations is **substitution.**

Example 1 **Use substitution to solve the system of equations.**

$y = 2x$

$4x - y = -4$

Substitute $2x$ for y in the second equation.

$4x - y = -4$	Second equation
$4x - 2x = -4$	$y = 2x$
$2x = -4$	Combine like terms.
$x = -2$	Divide each side by 2 and simplify.

Use $y = 2x$ to find the value of y.

$y = 2x$	First equation
$y = 2(-2)$	$x = -2$
$y = -4$	Simplify.

The solution is $(-2, -4)$.

Example 2 **Solve for one variable, then substitute.**

$x + 3y = 7$

$2x - 4y = -6$

Solve the first equation for x since the coefficient of x is 1.

$x + 3y = 7$	First equation
$x + 3y - 3y = 7 - 3y$	Subtract $3y$ from each side.
$x = 7 - 3y$	Simplify.

Find the value of y by substituting $7 - 3y$ for x in the second equation.

$2x - 4y = -6$	Second equation
$2(7 - 3y) - 4y = -6$	$x = 7 - 3y$
$14 - 6y - 4y = -6$	Distributive Property
$14 - 10y = -6$	Combine like terms.
$14 - 10y - 14 = -6 - 14$	Subtract 14 from each side.
$-10y = -20$	Simplify.
$y = 2$	Divide each side by -10 and simplify.

Use $y = 2$ to find the value of x.

$x = 7 - 3y$

$x = 7 - 3(2)$

$x = 1$

The solution is $(1, 2)$.

Topic 8 Exercises continued:

Use substitution to solve each system of equations.

7. $\begin{cases} y = 4x \\ 3x - y = 1 \end{cases}$

8. $\begin{cases} x = 2y - 3 \\ x = 2y + 4 \end{cases}$

9. $\begin{cases} c - 4d = 1 \\ 2c - 8d = 2 \end{cases}$

10. $\begin{cases} 2b = 6a - 14 \\ 3a - b = 7 \end{cases}$

11. $\begin{cases} y = -x + 3 \\ 2y + 2x = 4 \end{cases}$

12. $\begin{cases} x - 2y = -5 \\ x + 2y = -1 \end{cases}$

Elimination Using Multiplication Some systems of equations cannot be solved simply by adding or subtracting the equations. In such cases, one or both equations must first be multiplied by a number before the system can be solved by elimination.

Example 1 **Use elimination to solve the system of equations.**

$x + 10y = 3$
$4x + 5y = 5$

If you multiply the second equation by −2, you can eliminate the y terms.

$$\begin{aligned} x + 10y &= 3 \\ (+)\ -8x - 10y &= -10 \\ \hline -7x \qquad &= -7 \end{aligned}$$

$$\frac{7x}{-7} = \frac{-7}{-7}$$

$$x = 1$$

Substitute 1 for x in either equation.

$$\begin{aligned} 1 + 10y &= 3 \\ 1 + 10y - 1 &= 3 - 1 \\ 10y &= 2 \\ \frac{10y}{10} &= \frac{2}{10} \\ y &= \frac{1}{5} \end{aligned}$$

The solution is $\left(1, \frac{1}{5}\right)$.

Example 2 **Use elimination to solve the system of equations.**

$3x - 2y = -7$
$2x - 5y = 10$

If you multiply the first equation by 2 and the second equation by −3, you can eliminate the x terms.

$$\begin{aligned} 6x - 4y &= -14 \\ (+)\ -6x + 15y &= -30 \\ \hline 11y &= -44 \end{aligned}$$

$$\frac{11y}{11} = -\frac{44}{11}$$

$$y = -4$$

Substitute −4 for y in either equation.

$$\begin{aligned} 3x - 2(-4) &= -7 \\ 3x + 8 &= -7 \\ 3x + 8 - 8 &= -7 - 8 \\ 3x &= -15 \\ \frac{3x}{3} &= -\frac{15}{3} \\ x &= -5 \end{aligned}$$

The solution is (−5, −4).

Watch this Video!

https://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/solving-systems-addition-elimination/v/solving-systems-by-elimination-2

Topic 8 Exercises continued:

Use elimination to solve each system of equations.

13. $\begin{cases} 2x + 3y = 6 \\ x + 2y = 5 \end{cases}$

14. $\begin{cases} 3a - b = 2 \\ a + 2b = 3 \end{cases}$

15. $\begin{cases} 4c - 3d = 22 \\ 2c - d = 10 \end{cases}$

16. $\begin{cases} 4s - t = 9 \\ 5s + 2t = 8 \end{cases}$

17. $\begin{cases} 2x + 2y = 5 \\ 4x - 4y = 10 \end{cases}$

18. $\begin{cases} 4x + 2y = -5 \\ -2x - 4y = 1 \end{cases}$

19. The length of Sally's garden is 4 meters longer than 3 times the width. The perimeter of her garden is 72 meters. What are the dimensions of Sally's garden?

Topic 9: Polynomials

Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Group the coefficients and the variables

$= (3 \cdot 5)(x^{6+2})$ Product of Powers

$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= -12(a^{3+2})(b^{1+5})$

$= -12a^5b^6$

The product is $-12a^5b^6$.

Simplify Expressions An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n, $(ab)^m = a^mb^m$.

We can combine and use these properties to simplify expressions involving monomials.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power

$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product

$= (-2)^3(a^3)(a^8)(b^2)^3$ Group the coefficients and the variables

$= (-2)^3(a^{11})(b^2)^3$ Product of Powers

$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Topic 9 Exercises:

Simplify.

1. $y(y^5)$
2. $(2a^2)(8a)$
3. $(x^2y)(4xy^3)$
4. $(-4x^3)(-5x^7)$
5. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$
6. $(10x^3yz^2)(-2xy^5z)$
7. $(2a^3b^2)(b^3)^2$
8. $(-4xy)^3(-2x^2)^3$
9. $(-3j^2k^3)^2(2j^2k)^3$
10. $(25a^2b)\left(\frac{1}{5}abf\right)^2$
11. $(2xy)^2(-3x^2)(4y^4)$
12. $(2x^3y^2z^2)^3(x^2z)^4$

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.**

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5.

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.**

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.**

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Topic 9 Exercises Continued:

Simplify.

13. $\frac{x^5y^5}{x^5y^2}$

14. $\left(\frac{2a^2b}{a}\right)^3$

15. $\left(\frac{4p^4r^4}{3p^2r^2}\right)^2$

16. $\left(\frac{3r^6n^3}{2r^5n}\right)^4$

17. $\frac{(6a^{-1}b)^2}{(b^2)^4}$

18. $\left(\frac{4m^2n^2}{8m^{-1}n}\right)^0$

19. $\left(\frac{a^3b^{-1}}{a^{-2}b^2}\right)^{-2}$

20. $\left[\frac{2x^{-3}}{(2x)^3}\right]^{-1}$

21. $\left(\frac{2x^4}{-y^2}\right)^3$

Adding and Subtracting Polynomials

Add Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

Example 1 **Find $(2x^2 + x - 8) + (3x - 4x^2 + 2)$.**

Horizontal Method

Group like terms.

$(2x^2 + x - 8) + (3x - 4x^2 + 2)$

$= [(2x^2 + (-4x^2)] + (x + 3x) + [(-8) + 2)]$

$= -2x^2 + 4x - 6.$

The sum is $-2x^2 + 4x - 6$.

Example 2 **Find $(3x^2 + 5xy) + (xy + 2x^2)$.**

Vertical Method

Align like terms in columns and add.

$$\begin{array}{rl} & 3x^2 + 5xy \\ (+) & 2x^2 + \ \ xy \\ \hline & 5x^2 + 6xy \end{array}$$

Put the terms in descending order.

The sum is $5x^2 + 6xy$.

Subtract Polynomials You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Remember to distribute the negative!

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$

$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$

$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$

$= 2x^2 + (-9)$

$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rl} & 3x^2 + 2x - 6 \\ (-) & x^2 + 2x + 3 \\ \hline & 3x^2 + 2x - 6 \\ (+) & -x^2 - 2x - 3 \\ \hline & 2x^2 \qquad - 9 \end{array}$$

The difference is $2x^2 - 9$.

Topic 9 Exercises Continued:

Add or Subtract.

22. $(3a - 5) - (5a + 1)$

23. $(9xy + y - 2x) + (6xy - 2x)$

24. $(6p^2 + 4p + 5) - (2p^2 - 5p + 1)$

25. $(8p - 5q) + (6p^2 + 6q - 3)$

26. $(3x^2 - 2x) + (3x^2 + 5x - 1)$

27. $(2h - 6j - 2k) - (-7h - 5j - 4k)$

28. $(2a - 8b) - (-3a + 5b)$

29. $(6z^2 + 4z + 2) + (4z^2 + z)$

Polynomial Multiplied by Monomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 Find $-3x^2(4x^2 + 6x - 8)$.

Horizontal Method

$-3x^2(4x^2 + 6x - 8)$

$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$

$= -12x^4 + (-18x^3) - (-24x^2)$

$= -12x^4 - 18x^3 + 24x^2$

Vertical Method

$$\begin{array}{rr} & 4x^2 + 6x - 8 \\ (\times) & -3x^2 \\ \hline & -12x^4 - 18x^3 + 24x^2 \end{array}$$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.

$-2(4x^2 + 5x) - x(x^2 + 6x)$

$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$

$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$

$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$

$= -x^3 - 14x^2 - 10x$

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

FOIL – First - Outer - Inner - Last

Example:

$(x + 2)(x + 5)$

$= x^2 + 5x + 2x + 10$

$= x^2 + 7x + 10$

Topic 9 Exercises Continued:

Multiply.

30. $x(5x + x^2)$

31. $-2xy(2y + 4x^2)$

32. $(x - 8)(x + 3)$

33. $(-2x + 3)(x - 1)$

34. $-x(2x^2 - 4x) - 6x^2$

35. $(2x - 1)(x + 5)$

36. $(x + 2)(x + 2)$

37. $2x^2y^2(3xy + 2y + 5x)$

38. $(x + 3)(x - 3)$

39. $6a(2a - b) - 2a(-4a + 5b)$

40. $2(x + 9)(x - 2) - 3(x + 4)$

Topic 10: Factoring

Factoring Guide

Type of Factoring	When to Use It	Method	Example
1. Greatest Common Factor	Anytime, ALL THE TIME, if Possible. You should always look for the GCF before you do any other type of factoring.	Find the greatest common multiple of each term and factor it out.	$14x^2 + 21x$ Factored: $7x(2x+3)$
2. Difference of Two Squares	Two Terms. A perfect square minus another perfect square.	$a^2 - b^2$ Factored: $(a+b)(a-b)$	$x^2 - 4$ Factored: $(x+2)(x-2)$
3. Standard Form when $a = 1$	Three Terms. Trinomial in the form $x^2 + bx + c$	Look for factors of c that add up to b.	$x^2 - 7x + 12$ Factored: $(x-3)(x-4)$
4. Grouping	Four Terms. This is the only type of factoring you can try when there are 4 terms.	Group the first two and the second two terms. Find GCF of each group.	$3x^2 - 4x - 6x + 8$ Factored: $(3x^2 - 4x)(-6x + 8)$ $x(3x-4) - 2(3x-4)$ $(3x-4)(x-2)$
5. Perfect Square Trinomial	Three Terms. Trinomial in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$	$a^2 + 2ab + b^2$ Factored: $(a+b)^2$ $a^2 - 2ab + b^2$ Factored: $(a-b)^2$	$4x^2 - 12x + 9$ Factored: $(2x-3)^2$
6. Standard Form when $a \neq 1$	Three Terms. Trinomial in the form $ax^2 + bx + c$	1. Multiply $a \cdot c$ 2. Look for factors of $a \cdot c$ that add up to b. 3. Separate b term into those 2 factors. 4. Grouping	$2x^2 - 5x - 3$ Factored: $(2x^2 - 6x) + (x-3)$ $2x(x-3) + 1(x-3)$ $(2x+1)(x-3)$

Topic 10 Exercises:

Factor Completely.

1. $x^2 + 3x$
2. $x^2 - 4x + 3$
3. $x^2 - 6x + 9$
4. $x^2 - 9$
5. $4x^2 + 12x$
6. $x^2 - 3x + 2$
7. $x^2 + 10x + 9$
8. $-4x^2 + 19x - 21$
9. $5x^2 - 25x$
10. $6x^2y + 4xy^2$
11. $y^2 - 8y + 15$
12. $x^2 - y^2$
13. $-81 + a^4$
14. $25x^2 - 20x + 4$
15. $x^3 + 7x^2 + x$
16. $8y^2 - 200$
17. $3x^2 + 2x - 8$
18. $2x^2 - 4x + 2$
19. $2x^2 - 3x - 2$
20. $a^2 - 18a + 72$

Topic 11: Solving Quadratic Equations

Solve Equations by Factoring The following property, along with factoring, can be used to solve certain equations.

Zero Product Property	For any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0, *b* = 0, or both *a* and *b* equal 0.

Example 1 **Solve $x^2 + 6x = 7$. Check your solutions.**

$x^2 + 6x = 7$ Original equation

$x^2 + 6x - 7 = 0$ Rewrite equation so that one side equals 0.

$(x - 1)(x + 7) = 0$ Factor.

$x - 1 = 0$ or $x + 7 = 0$ Zero Product Property

$x = 1$ $\quad x = -7$ Solve each equation.

The solution set is $\{1, -7\}$. Since $1^2 + 6 = 7$ and $(-7)^2 + 6(-7) = 7$, the solutions check.

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation

$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.

$x^2 + 2x - 3 = 0$ Simplify.

Now let a = 1, b = 2, and c = −3 in the Quadratic Formula.

$$x = \frac{-b \pm \frac{b^2 - 4ac}{2a}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-2 + \sqrt{16}}{2}$$

$x = \frac{-2+4}{2}$ or $x = \frac{-2+4}{2}$

$= 1$ $\quad = -3$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{44}}{2}$$

$x = \frac{6 \pm \sqrt{44}}{2}$ or $x = \frac{6 - \sqrt{44}}{2}$

≈ 6.3 $\quad \approx -0.3$

The solution set is $\{-0.3, 6.3\}$.

Topic 11 Exercises:

Solve by Factoring and the Zero Product Property.

1. $x^2 + 7x - 8 = 0$
2. $4m^2 - 4m = 0$
3. $x^2 - 4x + 4 = 0$
4. $12x^2 = -6x$
5. $2k^2 - 5 = -3k$
6. $x^2 - 4x = 5$
7. $4x^2 = x + 3$
8. $-9 - 8x + x^2 = 0$
9. $x^2 = x$

Solve by using the Quadratic formula.

10. $x^2 - 3x + 2 = 0$
11. $-4c^2 + 19c = 21$
12. $2x^2 + 9x + 4 = 0$
13. $x^2 + 3x = 2$
14. $3x^2 + 5x - 2 = 0$
15. $3x^2 + 2x = 8$
16. $16x^2 - 8x = -1$
17. $48x^2 + 22x - 15 = 0$
18. $2p^2 + 5p = 8$

Topic 12: Radical Expressions

Product Property of Square Roots The **Product Property of Square Roots** and prime factorization can be used to simplify expressions involving irrational square roots. When you simplify radical expressions with variables, use absolute value to ensure nonnegative results.

Product Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1 Simplify $\sqrt{180}$.

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$	Prime factorization of 180
$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$	Product Property of Square Roots
$= 2 \cdot 3 \cdot \sqrt{5}$	Simplify.
$= 6\sqrt{5}$	Simplify.

Example 2 Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.

$\sqrt{120a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4}$

$= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2$

$= 2|a|b^2c^2\sqrt{30b}$

Quotient Property of Square Roots A fraction containing radicals is in simplest form if no radicals are left in the denominator. The **Quotient Property of Square Roots** and **rationalizing the denominator** can be used to simplify radical expressions that involve division. When you rationalize the denominator, you multiply the numerator and denominator by a radical expression that gives a rational number in the denominator.

Quotient Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example Simplify $\sqrt{\frac{56}{45}}$.

$\sqrt{\frac{56}{45}} = \sqrt{\frac{4 \cdot 14}{9 \cdot 5}}$	
$= \frac{2 \cdot \sqrt{14}}{3 \cdot \sqrt{15}}$	Simplify the numerator and denominator.
$= \frac{2\sqrt{14}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$	Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominator.
$= \frac{2\sqrt{70}}{15}$	Product Property of Square Roots

Topic 12 Exercises:

Simplify.

1. $\sqrt{28}$
2. $\sqrt{60}$
3. $\sqrt{2} \cdot \sqrt{5}$
4. $\sqrt{4x^2}$
5. $\sqrt{300a^4}$
6. $4\sqrt{10} \cdot 3\sqrt{6}$
7. $\frac{\sqrt{9}}{\sqrt{18}}$
8. $\frac{\sqrt{100}}{\sqrt{121}}$
9. $\frac{8\sqrt{2}}{2\sqrt{8}}$
10. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{2}}$
11. $\sqrt{\frac{3a^2}{10b^6}}$
12. $\sqrt{\frac{100x^4}{144b^8}}$

Topic 13: Rational Expressions

Multiply Rational Expressions To multiply rational expressions, you multiply the numerators and multiply the denominators. Then simplify.

Example 1 Find $\frac{2c^2f}{5ab^2} \cdot \frac{a^2b}{3cf}$.

$$\frac{2c^2f}{5ab^2} \cdot \frac{a^2b}{3cf} = \frac{2a^2bc^2f}{15ab^2cf} \quad \text{Multiply.}$$

$$= \frac{{}^{1}\cancel{(abcf)}(2ac)}{{}^{1}\cancel{(abcf)}(15b)} \quad \text{Simplify.}$$

$$= \frac{2ac}{15b} \quad \text{Simplify.}$$

Example 2 Find $\frac{x^2-16}{2x+8} \cdot \frac{x+4}{x^2+8x+16}$.

$$\frac{x^2-16}{2x+8} \cdot \frac{x+4}{x^2+8x+16} = \frac{(x-4)(x+4)}{2(x+4)} \cdot \frac{x+4}{(x+4)(x+4)} \quad \text{Factor.}$$

$$= \frac{(x-4)\cancel{(x+4)}^{1}}{2\cancel{(x+4)}_{1}} \cdot \frac{\cancel{x+4}^{1}}{(x+4)\cancel{(x+4)}_{1}} \quad \text{Simplify.}$$

$$= \frac{x-4}{2x+8} \quad \text{Multiply.}$$

Topic 13 Exercises:

Find each product. USE FACTORING!

1. $\frac{6ab}{a^2b^2} \cdot \frac{a^2}{b^2}$

2. $\frac{mp^2}{3} \cdot \frac{4}{mp}$

3. $\frac{x+2}{x-4} \cdot \frac{x-4}{x-1}$

4. $\frac{m-5}{8} \cdot \frac{16}{m-5}$

5. $\frac{2n-8}{n+2} \cdot \frac{2n+4}{n-4}$

6. $\frac{x^2-64}{2x+16} \cdot \frac{x+8}{x^2+16x+64}$

7. $\frac{8x+8}{x^2-2x+1} \cdot \frac{x-1}{2x+2}$

8. $\frac{a^2-25}{a+2} \cdot \frac{a^2-4}{a-5}$

9. $\frac{x^2+6x+8}{2x^2+9x+4} \cdot \frac{2x^2-x-1}{x^2-3x+2}$

10. $\frac{m^2-1}{2m^2-m-1} \cdot \frac{2m+1}{m^2-2m+1}$

11. $\frac{n^2-1}{n^2-7n+10} \cdot \frac{n^2-25}{n^2+6n+5}$

12. $\frac{3p-3r}{10pr} \cdot \frac{20p^2r^2}{p^2-r^2}$

13. $\frac{a^2+7a+12}{a^2+2a-8} \cdot \frac{a^2+3a-10}{a^2+2a-8}$

14. $\frac{v^2-4v-21}{3v^2+6v} \cdot \frac{v^2+8v}{v^2+11v+24}$

Topic 14: Relations and Functions

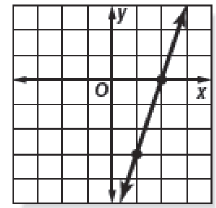
Identify Functions Relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

Example 1 **Determine whether the relation {(6, −3), (4, 1), (7, −2), (−3, 1)} is a function. Explain.**

Since each element of the domain is paired with exactly one element of the range, this relation is a function.

Example 2 **Determine whether $3x - y = 6$ is a function.**

Since the equation is in the form $Ax + By = C$, the graph of the equation will be a line, as shown at the right.

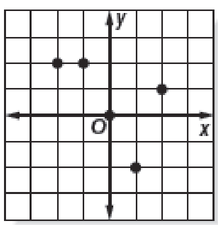


If you draw a vertical line through each value of x, the vertical line passes through just one point of the graph. Thus, the line represents a function.

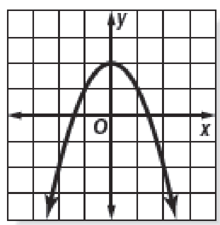
Topic 14 Exercises:

Determine whether each relation is a function. Write *function* or *relation*.

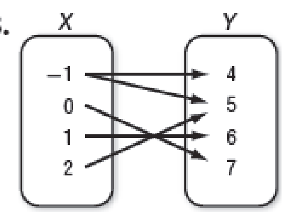
1.



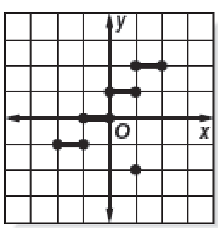
2.



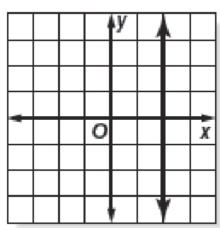
3.



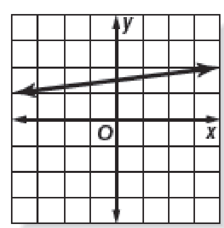
4.



5.



6.



7. {(4, 2), (2, 3), (6, 1)}

8. {(−3, −3), (−3, 4), (−2, 4)}

9. {(−1, 0), (1, 0)}

10. $-2x + 4y = 0$

11. $x^2 + y^2 = 8$

12. $x = -4$

Summer Math Packet <u>Answers</u>

For Rising Geometry Students

Topic 1 Answers:

1. 25
2. 1000
3. 512
4. 8
5. 34
6. 18
7. 27
8. 2
9. 1
10. 9
11. 11
12. $\frac{48}{5}$
13. $\frac{9}{4}$
14. 36
15. 1
16. $\frac{7}{4}$
17. $\frac{1}{2}$
18. $\frac{16}{5}$
19. $\frac{13}{16}$
20. $\frac{25}{24}$

Topic 2 Answers:

1. $20x-45$
2. $72-6x$
3. $6x-3y$
4. $6x+4y-2z$
5. $xy-2y$
6. $4x-3y+x$
7. $11a$
8. *Simplified*
9. $5x^2$
10. $\frac{5}{2}p$
11. $39a+34b$
12. x^2-6x+1
13. $2xy$
14. $8x-5y$
15. $12a-2b$
16. $5x^2+5y^2$

Topic 3 Answers:

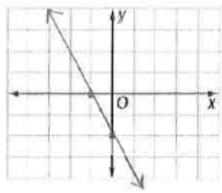
1. $3t-12=40$
2. $\frac{1}{2}(a-b)=54$
3. $3(d+4)=32$
4. $A=\pi r^2$
5. a.) $160-p=150$
   b.) $10\,pounds$
6. Four times a minus six equals twenty-three.
7. The sum of x squared and y y squared is eight.
8. One-third of the difference between the h and 1 equals b.
9. The area, A of a triangle is one-half the product of the height, h and the base, b.

Topic 4 Answers:

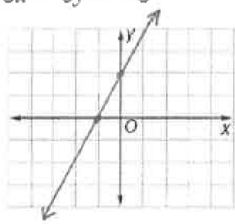
1. $x=-4$
2. $x=-2$
3. $b=224$
4. $p=1$
5. $x=\frac{1}{5}$
6. $a=2$
7. $y=17$
8. *identity*
9. $y=3$
10. $x=-2$
11. *identity*
12. $f=-\frac{4}{7}$
13. $t=2$
14. *no solution*
15. $x=2$
16. $w=-1$
17. $y=4$
18. $a=-4$
19. $x=0$
20. $y=1$
21. $x=\frac{c+b}{a}$
22. $y=\frac{a-w}{x}$
23. $k=\frac{4x-p}{x}$
24. $r=\frac{t-12}{4}$
25. $x=\frac{z}{1+y}$
26. $x=\frac{y-16w}{4}$
27. $h=\frac{2A}{a+b}$
28. $F=\frac{9}{5}C+32$
29. $l=\frac{P-2w}{2}$
30. $x=-12$
31. $p=15$
32. $y=26$
33. No solution
34. $a=78$
35. $w=6$
36. 3 hours

Topic 5 Answers:

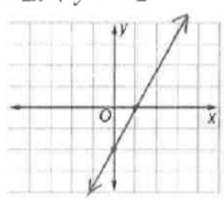
1. $2x + y = -2$



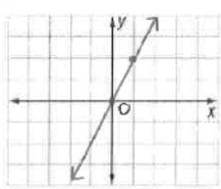
2. $3x - 6y = -3$



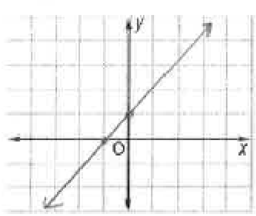
3. $-2x + y = -2$



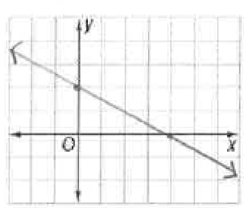
4. $y = 2x$



5. $x - y = -1$



6. $x + 2y = 4$



7. $m = 1$
8. $m = -2$
9. undefined
10. $m = \frac{4}{3}$
11. $m = -\frac{2}{7}$
12. $m = 0$
13. $r = -4$
14. $r = 3$
15. $r = 6$
16. $r = -5$
17. $r = 11$
18. $r = \frac{23}{3}$
19. $k = -2$
    $m = -2$
20. $k = 3$
    $m = 3$
21. $k = \frac{3}{2}$
    $m = \frac{3}{2}$
22. $y = 32$
23. $y = -18$
24. $y = -72$
25. $y = \frac{3}{8}$

Topic 6 Answers:

1. a.) $y - 4 = -3(x - 0)$
   or $y - 7 = -3(x + 1)$
   b.) $y = -3x + 4$
   c.) $3x + y = 4$

2. a.) $y + 4 = -1(x - 6)$
   or $y - 5 = -1(x + 3)$
   b.) $y = -x + 2$
   c.) $x + y = 2$

3. a.) $y + 2 = \frac{1}{3}(x - 3)$
   or $y + 4 = \frac{1}{3}(x + 3)$
   b.) $y = \frac{1}{3}x - 3$
   c.) $x - 3y = 9$

4. a.) $y + 4 = 11(x - 1)$
   or $y - 7 = 11(x - 2)$
   b.) $y = 11x - 15$
   c.) $11x - y = 15$

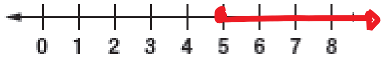
5. a.) $y - 2 = \frac{7}{3}(x - 3)$
   or $y + 5 = \frac{7}{3}(x - 0)$
   b.) $y = \frac{7}{3}x - 5$
   c.) $7x - 3y = 15$

6. a.) $y + 2 = -\frac{6}{5}(x - 4)$
   or $y + 8 = -\frac{6}{5}(x - 9)$
   b.) $y = -\frac{6}{5}x + \frac{14}{5}$
   c.) $6x + 5y = 14$

7. $y = 4x + 10$
8. $y = -3x + 3$
9. $y = -\frac{1}{2}x + 3$
10. $y = \frac{1}{3}x + 2$
11. $y = \frac{1}{2}x - 4$
12. $x = 0$

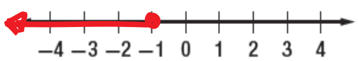
Topic 7 Answers:

1. $t \geq 5$

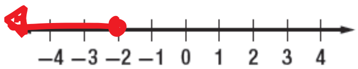


2. $x > -4$

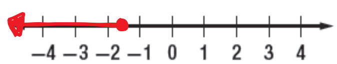
3. *No solution*
4. $m \leq -1$



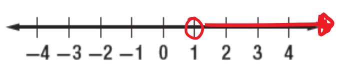
5. $k \leq -2$



6. $n \leq -\frac{1}{2}$



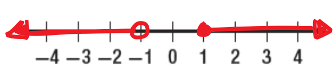
7. $w > 1$ *or* $w \geq 3$



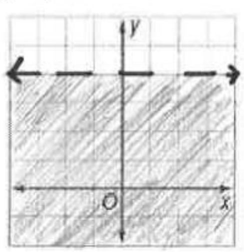
8. $y < 5$ *or* $y \leq -3$

9. $n > -4$ *or* $n < 8$
   All real numbers

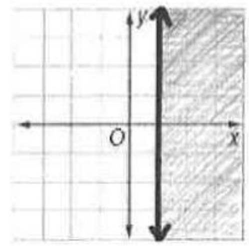
10. $a \geq 1$ *or* $a < -1$



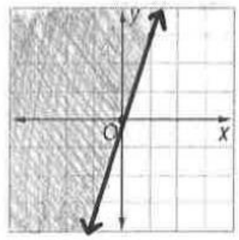
11. $y < 4$



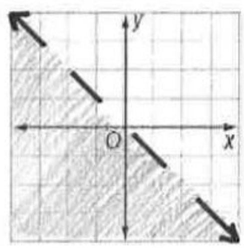
12. $x \geq 1$



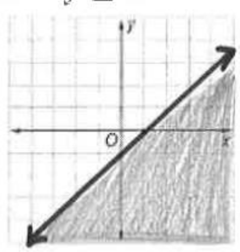
13. $3x \leq y$



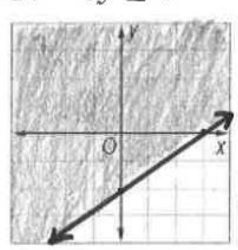
14. $-x > y$



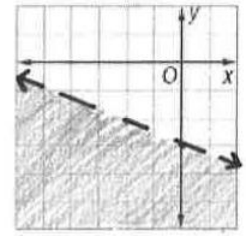
15. $x - y \geq 1$



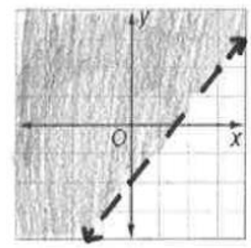
16. $2x - 3y \leq 6$



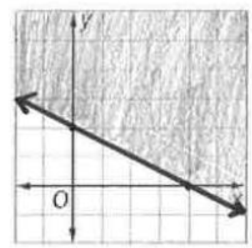
17. $y < -\frac{1}{2}x - 3$



18. $4x - 3y < 6$



19. $3x + 6y \geq 12$



Topic 8 Answers:

1. $(-1,-2)$
2. $(2,-3)$
3. $(2,1)$
4. $(1,4)$
5. *no solution*
6. *infinitely many solutions*
7. $(-1,-4)$
8. *no solution*
9. *infinitely many solutions*
10. *infinitely many solutions*
11. *no solution*
12. $(-3,1)$
13. $(-3,4)$

14. $(1,1)$
15. $(4,-2)$
16. $(2,-1)$
17. $\left(\frac{5}{2},0\right)$
18. $\left(-\frac{3}{2},\frac{1}{2}\right)$
19. $width-9m$
$length-28m$

Topic 9 Answers:

1. y^6
2. $16a^3$
3. $4x^3y^4$
4. $20x^{10}$
5. $a^3b^2c^7$
6. $-20x^4y^6z^3$
7. $2a^3b^8$
8. $512x^9y^3$
9. $72j^{10}k^9$
10. $a^4b^3f^2$
11. $-48x^4y^6$
12. $8x^{17}y^6z^{10}$
13. y^3
14. $8a^5b^3$
15. $\frac{4}{3}p^4r^4$
16. $\frac{81}{16}r^4n^8$
17. $\frac{36}{a^2b^6}$
18. 1
19. $\frac{b^6}{a^{10}}$
20. $4x^6$
21. $-\frac{8x^{12}}{y^6}$
22. $-2a-6$
23. $15xy+y-4x$
24. $4p^2+9p+4$
25. $6p^2+8p+q-3$
26. $6x^2+3x-1$
27. $9h-j+2k$
28. $5a-13b$
29. $10z^2+5z+2$
30. x^3+5x^2
31. $-2xy^2-4x^3y$
32. x^2-x-24
33. $-2x^2+5x-3$
34. $-2x^3-2x^2$
35. $2x^2+9x-5$
36. x^2+4x+4
37. $6x^3y^3+4x^2y^3+10x^3y^2$
38. x^2-9
39. $20a^2-16ab$
40. $2x^2+11x-48$

Topic 10 Answers:

1. $x(x+3)$
2. $(x-3)(x-1)$
3. $(x-3)^2$
4. $(x-3)(x+3)$
5. $4x(x+3)$
6. $(x-2)(x-1)$
7. $(x+9)(x+1)$
8. $-(2x-7)(2x-3)$
9. $5x(x-5)$
10. $2xy(3x+2y)$
11. $(y-3)(y-5)$
12. $(x+y)(x-y)$
13. $(a^2+9)(a+3)(a-3)$
14. $(5x-2)^2$
15. $x(x^2+7x+1)$
16. $8(y+5)(y-5)$
17. $(3x-4)(x+2)$
18. $2(x-1)^2$
19. $(2x+1)(x-2)$
20. $(a-12)(a-6)$

Topic 11 Answers:

1. $x=-8,1$
2. $m=0,1$
3. $x=2$
4. $x=0,-\frac{1}{2}$
5. $k=-\frac{5}{2},1$
6. $x=5,-1$
7. $x=-\frac{3}{4},1$
8. $x=9,-1$
9. $x=0,1$
10. $x=2,1$
11. $c=3,\frac{7}{4}$
12. $x=-4,-\frac{1}{2}$
13. $x=\frac{-3\pm\sqrt{17}}{2}$
14. $x=-2,\frac{1}{3}$
15. $x=-2,\frac{4}{3}$
16. $x=\frac{1}{4}$
17. $x=\frac{3}{8},-\frac{5}{6}$
18. $p=\frac{-5\pm\sqrt{89}}{4}$

Topic 12 Answers:

1. $2\sqrt{7}$

2. $2\sqrt{15}$
3. $\sqrt{10}$
4. $2|x|$
5. $10a^2\sqrt{3}$
6. $24\sqrt{15}$
7. $\frac{\sqrt{2}}{2}$
8. $\frac{10}{11}$
9. 2
10. $\frac{\sqrt{30}}{4}$
11. $\frac{a\sqrt{30}}{10b^3}$
12. $\frac{10x^2}{12b^4}$

Topic 13 Answers:

1. $\frac{6a}{b^3}$
2. $\frac{4p}{3}$
3. $\frac{x+2}{x-1}$
4. 2
5. 4
6. $\frac{x-8}{2x+16}$
7. $\frac{4}{x-1}$
8. $a^2+3a-10$
9. $\frac{x+2}{x-2}$
10. $\frac{m+1}{m^2-2m+1}$
11. $\frac{n-1}{n-2}$
12. $\frac{6pr}{p+r}$
13. $\frac{a^2+8a+15}{a^2+2a-8}$
14. $\frac{v^2-7v}{3v^2+6v}$

Topic 14 Answers:

1. *yes*
2. *yes*
3. *no*
4. *no*
5. *no*
6. *yes*
7. *yes*
8. *no*
9. *yes*
10. *yes*
11. *no*
12. *no*