Algebra II: Unit 4 – Rational and Radical Functions

Investigation 1

Activity 4.1.2 Evaluating and Graphing $f(x) = kx^{-n}$ for n a natural number
Activity 4.1.3 Direct Variation and Inverse Variation
Activity 4.1.4 Other Applications of an Inverse Square Law
Activity 4.1.6 A Review of Rational Exponents and Radical Notation

Investigation 2

Activity Sheet 4.2.2 Applications Problems Needing Direct and Indirect Variation
Activity Sheet 4.2.3 and 4.2.3a Table for Pace vs Speed Data
Activity Sheet 4.2.5 Applications Problems that Include Joint Variation

Investigation 3

Activity 4.3.3 Graphing Rational Functions III
Activity 4.3.4 Applications of Rational Functions
Activity 4.3.5 Graphing Rational Functions IV
Activity 4.3.6 (+)Graphing Rational Functions V

Investigation 4

Activity sheet 4.4.1 Rational Expressions I
Activity sheet 4.4.2 Rational Expressions II (Adding and Subtracting)
Activity sheet 4.4.3 Rational Expressions III (Adding, Subtracting, and Multiplying) Pg. 1-4

Investigation 5

Activity Sheet 4.5.1 More Equations and Problems Pages 1-2
Activity Sheet 4.5.2 Solving Equations with Fractions and Rational Expressions
Activity Sheet 4.5.3 More Equations and Problems

Activity 4.1.2 – Evaluating and Graphing $f(x) = kx^{-n}$ for n a Natural Number

Indirect Variation: If the product of two variables equals a constant the variables are *indirectly related*. Variables x and y are indirectly related if they satisfy the equation $xy = k$ for some constant k. When we solve for y, we get $y = \frac{k}{x}$ or $y = kx^{-1}$.

To investigate the effect k has on the graph of $y = kx^{-1}$ we will use the graphing calculator and the *Transform* application.

Follow the steps below:

- Press $\boxed{\text{APPS}}$ and choose Transfrm.
- In Y_1 enter AX^{-1}.
- Set the window to the ZStandard by pressing $\boxed{\text{ZOOM}}$ 6
- Press $\boxed{\text{WINDOW}}$ and key cursor over to SETTINGS: Set A = 3 and Step = 1.
- Press $\boxed{\text{Graph}}$
- Use the left and right arrow keys to investigate how the change in A affects the graph.

1. When A is positive what quadrants will the graph appear?

2. When A is negative what quadrants will the graph appear?

3. As $|x|$ increases what happens to y?

4. As $|x|$ decreases what happens to y?

5. What happens when $k = 0$?

6. Does this graph model an inverse (indirect) variation graph? Why or why not?

7. Fill in the tables using the table feature.

A = 3

x	y
-1	
-.5	
0	
.5	
1	

A = -20

x	y
-1	
-.5	
0	
.5	
1	

A = 30

x	y
-1	
-.5	
0	
.5	
1	

Let's continue to investigate how increasing the absolute value of A – increasing $|A|$ – affects the graph of $Y = AX^{-1}$.

8. Sketch the graph of $Y = AX^{-1}$.

   a) When A is positive:

   b) When A is negative:

Next, we will investigate how the value of n affects the graph of $y = kx^{-n}$.

- In Y_1 enter AX^{-B}
- Set the window to the ZStandard by pressing ZOOM 6
- Press WINDOW and cursor over to SETTINGS: Set A = 3, B = 1 Step = 1.
- Press Graph
- Use the right arrow key to investigate how the change in B affects the graph (B is even and odd.)

9. What do you notice?

10. Next Set B to 4 and use the left and right arrow keys to see what happens when A is positive and when A is negative. What do you notice?

11. Set B to 5 and use the left and right arrow keys to see what happens when A is positive and when A is negative. What do you notice?

12. Let's summarize using the equation of the form $y = kx^{-n}$. In the graphing calculator we substituted A for *k* and B for *n*.

    a. If *n* is even and *k* is positive what quadrants will the graph appear?

    b. If *n* is odd and *k* is positive what quadrants will the graph appear?

    c. If *n* is even and *k* is negative what quadrants will the graph appear?

    d. If *n* is odd and *k* is negative what quadrants will the graph appear?

    e. As $|x|$ increases what happens to *y*?

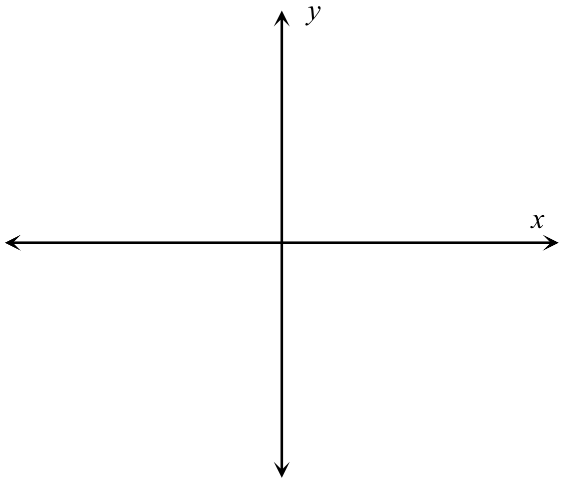
    f. As $|x|$ decreases what happens to *y*?

    g. What happens if *n* = 0?

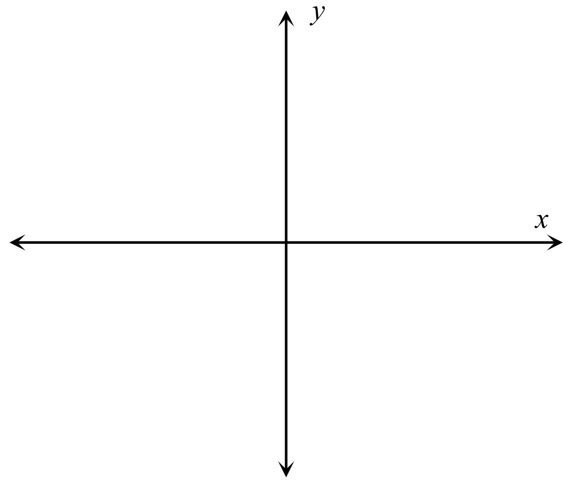
    h. What type of graph do you have?

13. Sketch the graph of $y = kx^{-n}$ when:

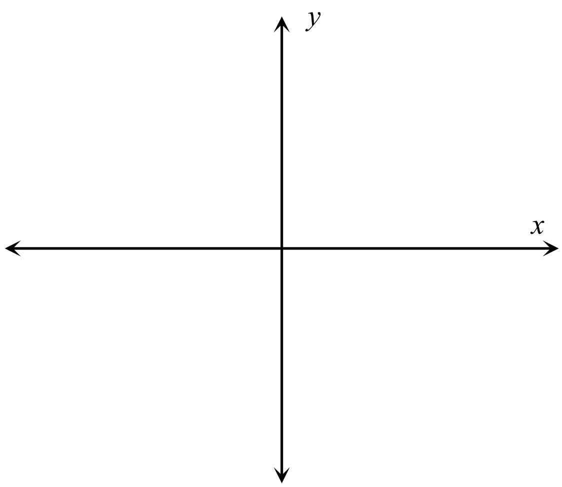
a) *k* is positive and *n* is odd:



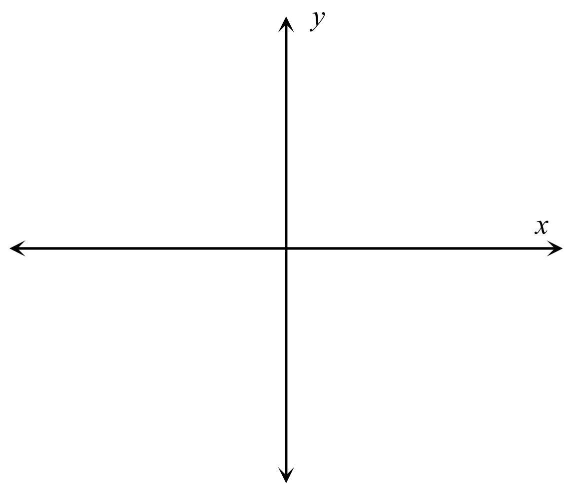
b) *k* is negative and *n* is odd:



c) *k* is positive and *n* is even:

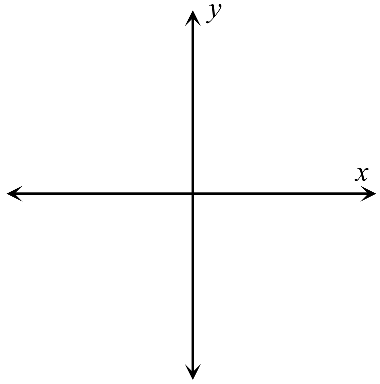


b) *k* is negative and *n* is even



14. Use your calculator to sketch the following graphs. Make sure you uninstall the Transfrm app prior to graphing the functions.

a) $y = 4x$

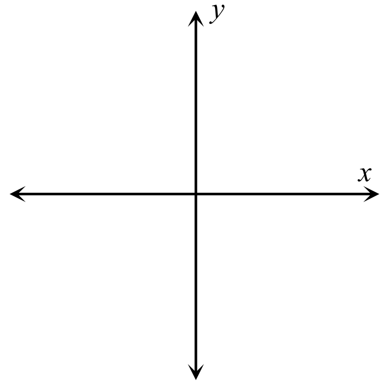


Domain:

Range:

End behavior:

b) $y = 4x^2$

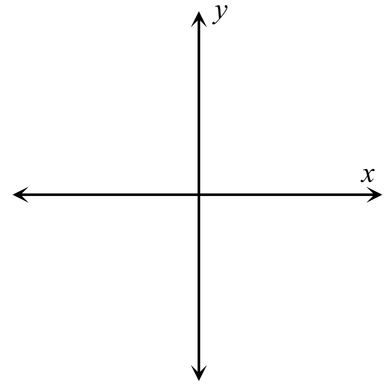


Domain:

Range:

End behavior:

c) $y = \frac{4}{x}$

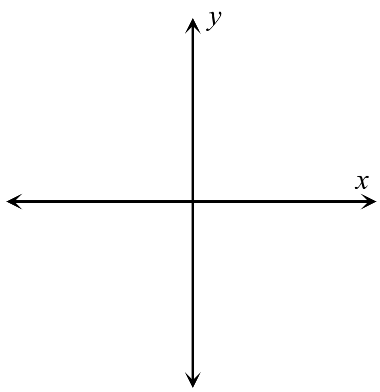


Domain:

Range:

End behavior:

d) $y = 4x^{-2}$

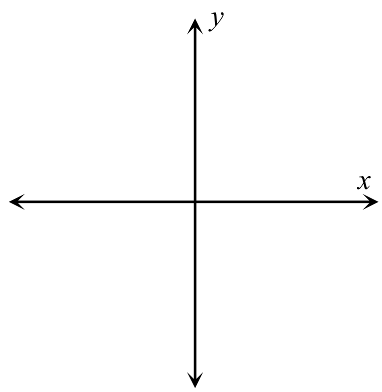


Domain:

Range:

End behavior:

e) $y = \frac{1}{4}x^2$

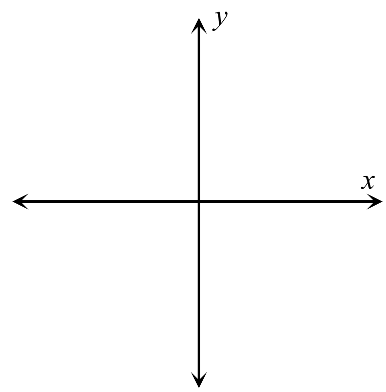


Domain:

Range:

End behavior:

f) $y = x^3 + 5x$



Domain:

Range:

End behavior:

Review Problems

Simplify the following expressions containing negative exponents. Express the final result using only positive exponents.

1. $x^2 \cdot x^{-7}$

2. $\dfrac{5x^6y^{-4}}{30x^{-4}y^5}$

3. $(5x^{-2}y^3)(2x^5y^8)$

4. $\dfrac{x^{-5}}{x^{-7}}$

5. $x^{-3} \cdot x^5 \cdot x$

6. $\dfrac{-4x^4y^{-2}}{5x^{-1}y^4}$

7. $(3a^{-5}b^{-8})(2a^2b^{-3})$

8. $(5x^{-2}y^{-3})(3x^{-4}y)$

9. $\dfrac{-6x^{-3}y^5z^8}{24x^{-5}y^6z^{-3}}$

Activity 4.1.3 Direct and Inverse Variation

1. How can you determine if two variables are related directly?

2. How can you determine if two variables are related by an inverse variation relationship?

3. Find the constant of variation *k* if y varies directly as x, and y = 14 when x = 3. Then write and equation relating x and y.

Which relationships have variables that are "directly proportional", "inversely proportional," or is the relationship a "neither."

4. The amount of salary you make to the number of hours worked.

5. The faster your speed the less time the trip takes.

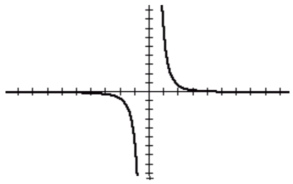
6. The volume of the water in a swimming pool as it is filled at a rate of 150 gallons per minute.

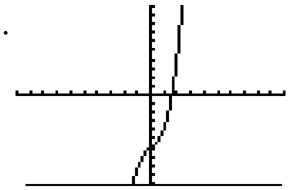
7. The temperature in measured in degrees Celcius and also in degree Fahrenheit.

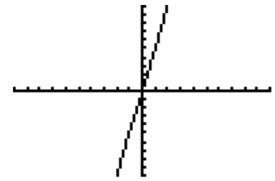
8. The area of a rectangle remains constant and the relationship between of the width to the length.

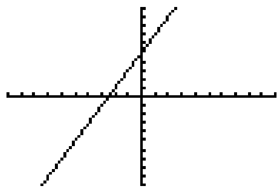
9. Violet purchased a new car, as the car's age increases, the resale price decreases.

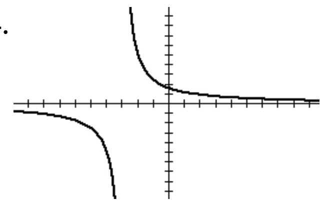
Identify which graph displays two variables which are "directly proportional", "inversely proportional", or "neither."

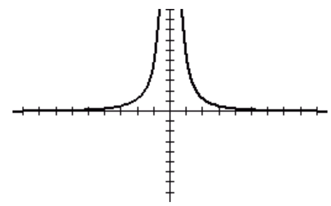
10. 

11. 

12. 

13. 

14. 

15. 

Given a table identify which relationship is "directly proportional", "inversely proportional", or "neither":

16.

x	y
2	18
4	9
8	4.5
9	4
12	3
16	2.25
18	2

17.

x	y
3	18
4	32
5	50
6	72
7	98
8	128
9	0.625

18.

x	y
1	7
2	13
3	23
4	37
5	55
6	77
7	103

Solve the following:

19. The Law of Lever states that to balance a given person seated on a seesaw, the distance the other person is from the pivot (or fulcrum) is inversely proportional to that other persons' weight.
    a. Write an equation for this situation. Let d = distance and w = weight.

    b. If Casandra, who weighs 40 lbs., is sitting 5 feet from the fulcrum how far away from the fulcrum will her friend Carlos, who weighs 100 lbs. need to sit to balance Casandra?

20. The weight of a body above the surface of the Earth varies inversely as the square of its distance from the center of the Earth.
    a. Write an equation to describe this situation. Define your variables.

    b. Assume the Earth's radius is 4000 miles. If a man weights 165 lbs. at the Earth's surface, how much would he weigh if he is 12,000 miles above the earth's surface?

21. The amount of money raised at a basketball game is directly proportional to the number of attendees. The amount of money raised for 300 attendees was $2100. How much money will be raised for 750 attendees?

22. The volume of gas in a container varies inversely as the pressure. If the volume is 340 ft^3 with pressure of 48 lbs/in^2, what pressure has to be applied to have a volume of 376 ft^3?

23. Distance in feet it takes a car to stop after the brakes are applied varies directly with the square of the car's speed. The formula d = $\frac{1}{12}s^2$ describes this relation for a certain car.
   a. If the car is traveling at a rate of 75 mph what is the stopping distance?

   b. The average size car is about 13.5 feet. If you are going 75mph, how many cars away would you have to start breaking if a stopped car is in front of you and you want to avoid an accident?

Activity 4.1.4 Other Applications of an Inverse Square Law

1. Radiation spreads out as it travels away from the gamma or X-ray source. Like light therefore, the intensity of the radiation depends on how far you are from the source. Suppose that for a particular source, the intensity is 90 milliroentgen/hour at 2 meters and 10 milliroentgen/hour at 6 meters.

   a. How does the intensity vary with distance?

   b. Write the equation for this situation.

   c. What would be the intensity if your distance from the source were:

   15 meters? 10 meters? 3 meters? 10 centimeters?

2. Radiation machines produce an intensity of radiation that varies inversely as the square of the distance from the machine. Suppose that for a particular source, the radiation intensity is 80 milliroentgens per hour at 3 meters.

   a. Write the general equation that represents this situation.

   b. What is the intensity at a distance of 2 meters?

3. Sound Intensity I is inversely proportional to the square of the distance d from the sound source.

   a. Write an equation that represents this situation.

   b. If a person moves 6 times as far from the source, how will the intensity of sound be affected?

4. The Inverse square law applies to many situations such as Newton's Law of Universal Gravitation as do the effects of electric, magnetic, light, sound, radiation phenomena and photography. Below are suggested websites to research how the inverse square law works in these fields. You are not limited to our suggestions. You can choose a different website but you must bring in the addresses to class. Be prepared to give a brief talk in class on what you have learned.

The Inverse Square Law– as it applies to photography
http://digital-photography-school.com/an-introduction-to-the-inverse-square-law/
http://photography.tutsplus.com/articles/rules-for-perfect-lighting-understanding-the-inverse-square-law--photo-3483

The Inverse Square Law – as it applies to gravity
http://www.tutorvista.com/physics/inverse-square-law-gravitation
http://www.astronomynotes.com/gravappl/s5.htm

The Inverse Square Law – as it applies to sound
http://www.sengpielaudio.com/calculator-squarelaw.htm
http://www.acousticalsurfaces.com/acoustic_IOI/101_5.htm

Activity 4.1.6 Review

Simplify: Express with positive exponents.

1. $x^{25} \cdot x^{-7}$

2. $\dfrac{35x^0y^{-4}}{30x^{-4}y^{-5}}$

3. $(-5x^{-12}y^{13})(2x^2y^4)$

4. $\dfrac{x^{-5.3}}{x^{-7.4}}$

5. $x^{-3/5} \cdot x^{5/3} \cdot x$

6. $\dfrac{-40x^4y^{-2}}{5x^{-1}y^{4/3}}$

7. $\left(3a^{-\frac{5}{2}}b^{-\frac{8}{5}}\right)\left(-2a^2b^{-\frac{3}{5}}\right)$

8. $\left(5x^{-\frac{2}{5}}y^{-6}\right)\left(3x^{-\frac{4}{5}}y\right)$

9. $\dfrac{-60x^{-3}y^5z^{-8}}{-24x^5y^{-6}z^{-3}}$

10. Express: the 3rd power of the 7th root of x. Use radical notation for final answer.

11. Rewrite with rational exponents and simplify. Assume that the variables represent nonnegative real numbers.

a) $\sqrt[5]{x^{15}}$ b) $\sqrt[6]{y^8}$ c) $\sqrt[9]{m^{18}}$ d) $\sqrt[3]{p^{12}}$ e) $\sqrt[8]{9^8x^{32}}$

f) $\sqrt[6]{x^4}$ g) $\sqrt[6]{a^2}$ h) $(\sqrt[5]{x^2y^4})^{15}$ i) $\sqrt[6]{(xy)^{18}}$

12. Write the expression using a single radical sign. Assume that the variables represent nonnegative real numbers.

a) $(x^{\frac{2}{3}})(x^{\frac{3}{4}})$ b) $(x^{\frac{3}{4}})(x^{\frac{1}{8}})$ c) $(y^{\frac{1}{6}})(y^{\frac{3}{4}})$

13. Use rational exponents to simplify. Assume that the variables represent nonnegative real numbers. Write the final answer as a radical.

a) $\sqrt[12]{a^8}$ b) $\sqrt[10]{a^6}$ c) $\sqrt[14]{a^6}$ d) $\sqrt[10]{(3a)^5}$

14. Show that if x ≥ 0, $\sqrt[12]{a^2} = \sqrt[18]{a^3}$

15. Show that $(x^8 + 25)^{.5} \neq x^4 + 5$

Activity 4.2.2 Application Problems for Direct and Inverse Variation

1. The **area** of a circle varies directly with the square of the radius. Given the formula of the area of a circle: A = πr^2. What happens to A when r is doubled, when r is tripled?

2. The **surface area** of a cube varies directly with the square of the length of its side. Given the formula of the surface area of a cube: SA = $6x^2$, what happens to the surface area when the side is doubled, when it is tripled, when it is 100 times more?

3. The **volume** of a cube is directly proportional to the cube of the length of its edge. What is the effect on the volume when the side is doubled, when it is tripled, when it is 100 times more?

4. Compare the surface area to the volume as the side was doubled, tripled, 100 times more?

5. **Can Giants exist**? Two people of different size who have the same proportions will have the same build.
   a. A person’s mass is directly proportional to the cube of his/her height.
   How much more would a giant's mass be, who has similar build, and height of 2256 cm tall (about 74 feet) to a person who is 188 cm tall, (about 6’ 2”)?

   b. The mass a person’s legs will support is directly proportional to the square of his/her height. How much more would a giant's surface area be, who has similar build, and height of 2256 cm tall (about 74 feet) to a person who is 188 cm tall, (about 6’ 2”)?

   c. Explain why Giants can’t exist.

6. Boyle's Law states that the volume *v* of a fixed amount of gas (at a constant temperature) is inversely proportional to the pressure *p* of the gas.
   a. If a pressure of 58 pounds per square inch (psi) compresses the gas to a volume of 460 cubic feet, write the equation expressing volume in terms of pressure.

   b. What would be the volume if the pressure is 96 psi?

   c. If you want to compress the gas to 375 cubic feet, what pressure would you need? (Round to the nearest whole number)

7. When you swim in deep water, there can be a lot of water pressure. The pressure *p* in your ears varies directly with the depth *d* at which you swim. Each foot of water creates water pressure of 0.43 per square inch (psi).
   a. Write an equation to express this situation.

   b. Predict the pressure at:
   9ft 25ft 50ft 150ft

8. Suppose y varies inversely as the fourth power of x. If y = 60 when x = 2
   a. Write an equation expressing y in terms of x.

   b. Find y when x = 5.

9. The intensity *I* of light varies inversely as the square of the observer's distance *d* from the light source. Suppose the light intensity is 40 lumens when the observer is 5.8 yards from the light.
   a. Write the equation for the intensity in terms of its distance for this situation.

   b. Find the light intensity when the distance between the observer and the light source is 12 yards away.

Activity 4.2.3 – Pace vs. Speed on a Treadmill

Pace and *speed* both describe how fast an object is moving. Runners generally use pace to describe the rate of their movement by the number of minutes it takes to cover a mile. Cyclists generally use speed; speed is the measure of how fast an object is moving. Speed applies to all moving objects such as bicycles, cars, buses, trains, and airplanes. One unit of speed is miles per hour. For example, if you are running and take 15 minutes to complete a mile, your pace is 15 minutes per mile, while your speed is 4 miles per hour.

The table to the right shows the relationship between a person's pace (number of minutes per mile) and their speed (in miles per hour) on a treadmill.

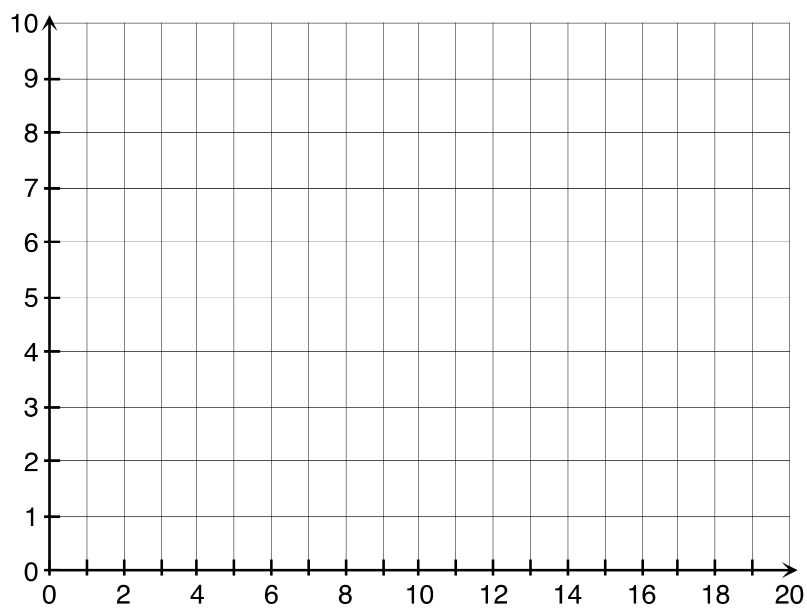
Pace (minutes per mile)	Speed (miles per hour)
20:00	3.0
17:39	3.4
15:47	3.8
13:38	4.4
12:00	5
10:43	5.6
9:41	6.2
8:49	6.8
8:20	7.2
7:54	7.6
7:30	8.0
6:59	8.6
6:15	9.6

We will use statistical regression to create a power function to describe this relationship. Follow the steps below.

- Press the STAT key, select 1:edit, and enter the data into lists L1 and L2
- Press STAT PLOT to create a scatterplot of the data. Be sure to set an appropriate window.
- Press the STAT key, go to the CALC menu, and select A:PwrReg for Power Regression.

1. Write the power function that models these data?

2. Sketch a graph of the function. Label the axes.



Activity 4.2.3A Pace vs Speed on a Treadmill

Pace is a method to describe one's speed. Runners use pace to describe the rate of their movement by the number of minutes it takes to cover a mile. Pace is typically given in minutes and seconds.

Speed is the measure of how fast an object is moving. It applies to all moving objects such as a cycle, car, bus, train, or an airplane. The units of speed are miles per hour.

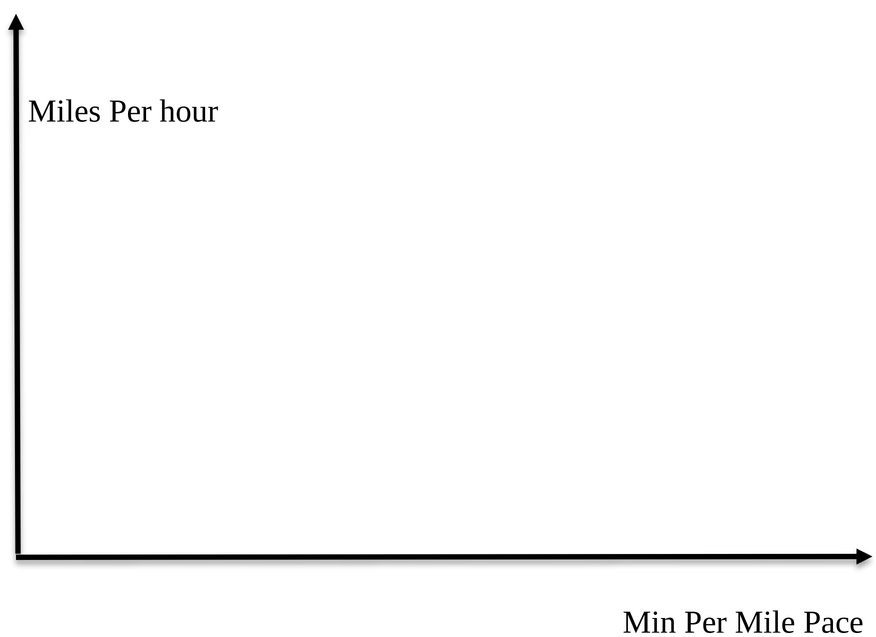
Example: If you are running and take 15 minutes to complete a mile, your pace is 15 (minutes per mile) while your speed is 4 miles per hour.

Pace and speed both describe how fast you are moving. Runners generally use pace, while bicyclists use speed.

1. You will gather data on Pace vs Speed using a treadmill. Fill in the table below.
2. Enter your data into the lists L1 and L2 using the STATS key, 1:edit
3. Make a stats plot. Be sure to set an appropriate window.
4. Use the STATS key, and select A:PwrReg for Power Regression. Write the equation below.
5. Sketch the function.

Power function:____________________

Min/Mile Pace	MPH



Activity 4.2.5 Variation Application Problems – Direct, Indirect, and Joint

1. The volume of a cone varies jointly as the height and the square of the radius of the base. The constant of variation is $\frac{\pi}{3}$. The equation is $V = \frac{1}{3}\pi r^2 h$. Find the volume of a cone if the radius is 5 cm and height is 6 cm.

2. The temperature, T (in degrees Kelvin), of an enclosed gas varies jointly with the product of the volume, V (in cubic centimeters), and the pressure, P (in kilograms per square centimeter). The temperature of a gas is 266^{o}K when the volume is 7000 cubic centimeters and the pressure is 0.76 kilogram per square centimeter. What is the temperature when the volume is 8000 cubic centimeters and the pressure is 0.87 kilogram per square centimeter?

3. Supposed ***x*** varies jointly with ***y*** and the square of ***z***. When $\boldsymbol{x} = -81$ and $\boldsymbol{y} = 3$, then $\boldsymbol{z} = 3$**. Find *y* when *x* = 15 and *z* = 5.**

4. When a boat is going at a high speed, most of the boat's motor goes into generating the wake (the track of waves left in the water). The power *p* used to generate the wake is directly proportional to the seventh power of the boat's speed *s*.
   a. If a boat's speed is 10 knots (nautical miles per hour) and uses 0.2 horsepower, write the equation for this situation.

   b. If a boat's speed is 30 knots, how much power will it need to generate a wake?

5. The maximum weight M (in pounds) that a board could hold varies directly as its width w (in inches) and the square of its thickness t (in inches) and inversely from the distance d (in feet) of its supports. Find the maximum weight that a board can support if it the supports are 18 feet apart, the board is 11 in wide and 2 in. thick and the constant of variation is
$70\,\frac{ft-lb}{in^3}$.

6. The electrical resistance r of a wire varies directly as its length l and inversely as the square of its diameter d. A wire with a length of 300 inches and a diameter of one-half of an inch has a resistance of 20 ohms. If a 600 inch wire has the same diameter find its electrical resistance.

7. The kinetic energy of an object (measured in Joules) varies jointly with the mass of the object and the square of its velocity. If the kinetic energy of an object with mass 12 kg moving at 8 meters per second is 384 Joules, what is the mass of an object moving 12 meters per second that generates 576 Joules?

8. It is known that gravitational force, F varies directly as the masses of two objects and inversely as the square of the distance between them. Which of the following equations correctly represents the phenomenon of gravitational force (F) if m_1 and m_2 are the masses of the bodies between which the force of gravity exists and d is the distance between them.
   a. $F = \dfrac{kd^2}{m_1 m_2}$
   b. $F = km_1m_2d^2$
   c. $F = \dfrac{km_1m_2}{\sqrt{d}}$
   d. $F = \dfrac{km_1m_2}{d^2}$

9. Animals usually defend their territory vigorously against intruders, especially of the same species, by attacking intruders that come in to the region. According to the statistical studies reported in J.M. Emlen's Ecology: An Evolutionary Approach, the defended region's area a varies directly with the 1.31 power of the animal's body mass $m$in km.
   a. Suppose that a normal 20 kilogram beaver will defend a region of area 200 square meters. Write an equation for this function.

   b. Skeletons show that thousands of years ago North American beavers were up to 3.5 meters long and had a mass of 200 kg. How many square meters should a beaver have defended?

10. The maximum weight M (in pounds) that a board could hold varies directly as its width w (in inches) and the square of its thickness t (in inches) and inversely from the distance d (in feet) of its supports.
   a. Write the general equation of this function.

   b. A beam 2 inches by 12 inches by 10 feet long is turned on “edge” so that the width is two inches, its thickness is 12 inches and it can support a load of 1,584 pounds. Write an equation for this particular function.

   c. If it is laid flat, what load can it support?

   d. By what factor is the maximum weight changed if the thickness is doubled.

Activity 4.3.3 – Graphing Rational Functions III

In **Activity 4.3.2** you worked with vertical and horizontal asymptotes where the degree of the numerator and the degree of the denominator were equal. You found that when the degrees are equal the equation of the horizontal asymptote is defined by the quotient of the leading coefficients of the numerator and denominator. You also found that, if the degree of the numerator is less than the degree of the denominator, the equation of the horizontal asymptote is $y = 0$.

1. Sketch the graph of $g(x) = \frac{2(x+3)(x-2)}{(3x-2)(x+4)}$ and identify the important features.

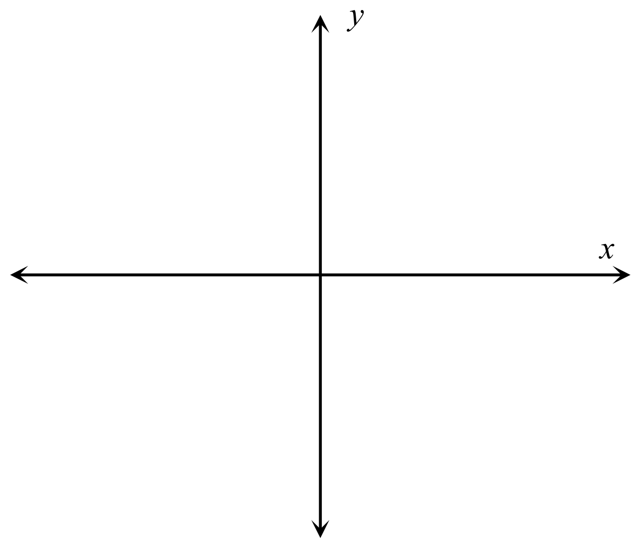
   VA:

   HA:

   Zeros:

   y-intercept:

   Domain:



2. Sketch the graph of $f(x) = \frac{3x(x+4)}{(2x-1)(x+3)}$ and identify the important features.

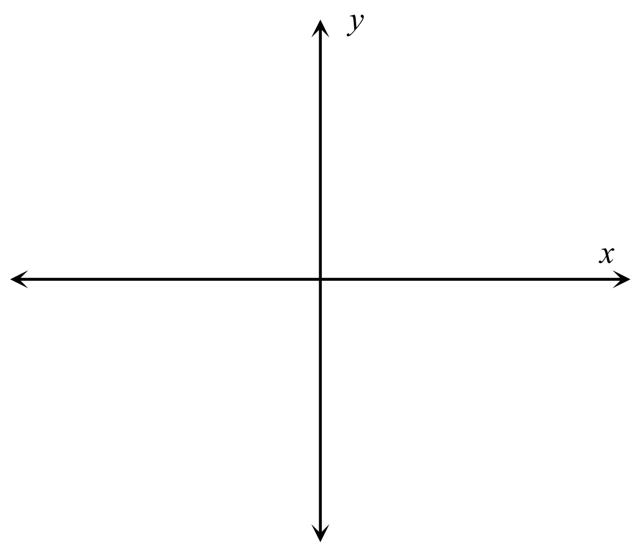
   VA:

   HA:

   Zeros:

   y-intercept:

   Domain:



By now your class definition of a vertical asymptote might be something like: a **vertical asymptote** is a vertical line that a graph approaches but **never** crosses or touches. But as you saw in Questions 1 and 2 of this activity, perhaps to your surprise, the graph of a function can (and often does) touch, and even cross, a horizontal asymptote. You may need to adjust your class definition of a horizontal asymptote now.

3. Sketch the graph of $g(x) = \frac{x}{(x-3)(x+2)}$ and identify the important features.

VA:

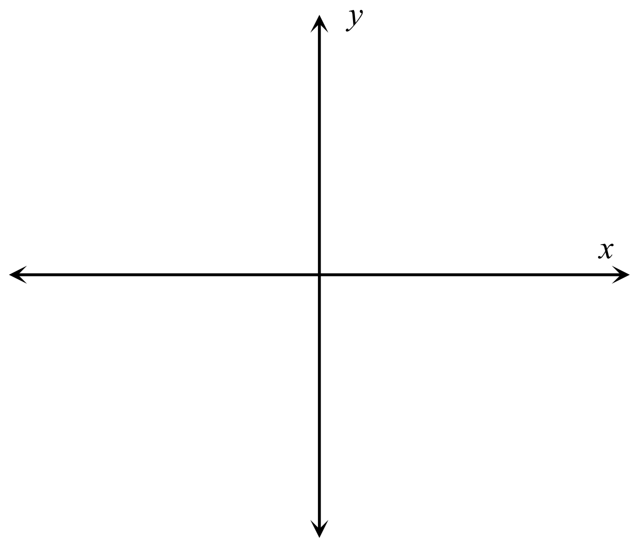
HA:

Zeros:

x-intercepts:

y-intercept:

Domain:



To determine if a function's graph crosses its horizontal asymptote, we need to see if there is an input *a* that has the output *b*, that is, *g*(*a*) = *b*, where *y* = *b* is the equation of the horizontal asymptote.

In above example the HA has the equation $y = 0$.

Let $\frac{x}{(x-3)(x+2)} = 0$ and solve.

The numerator equals 0 when $x = 0$.

The graph crosses the horizontal asymptote at (0, 0).

We will study techniques for solving rational equations in Investigation 5 of this unit. Being able to determine if a graph will cross its horizontal asymptote provides one reason to study rational equations and their solutions. For now we will informally state that we multiplied both sides of the above equation by the product $(x-3)(x+2)$. We know it is okay to multiply both sides of an equation by a nonzero number. In Investigation 5 we will explore how it is possible to multiply both sides of an equation by an expression that is not a number.

4. Find the point, if one exists, where the graph of the function crosses its horizontal asymptote.

$$r(x) = \frac{3(x+3)(x-2)}{x^2}$$

5. Sketch the graph of $k(x) = \dfrac{x^2 + 3x}{x + 5}$ and identify the important features.

VA:

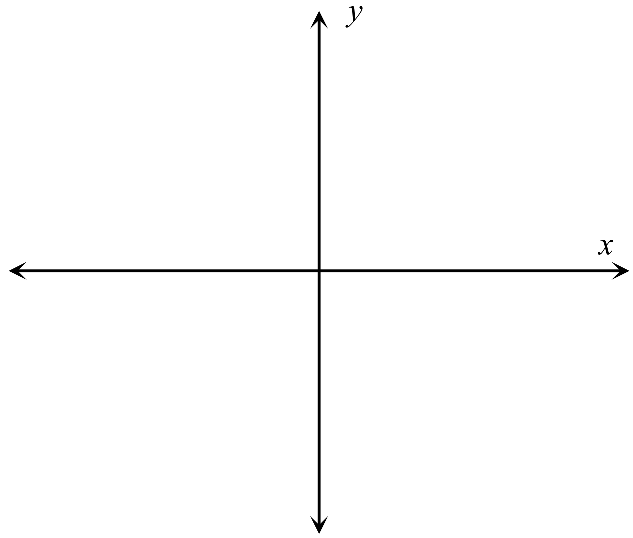
HA:

x-intercepts:

Zeros:

y-intercept:

Domain:



6. Sketch the graph of $r(x) = \frac{x^2-1}{x}$ and identify the important features.

VA:

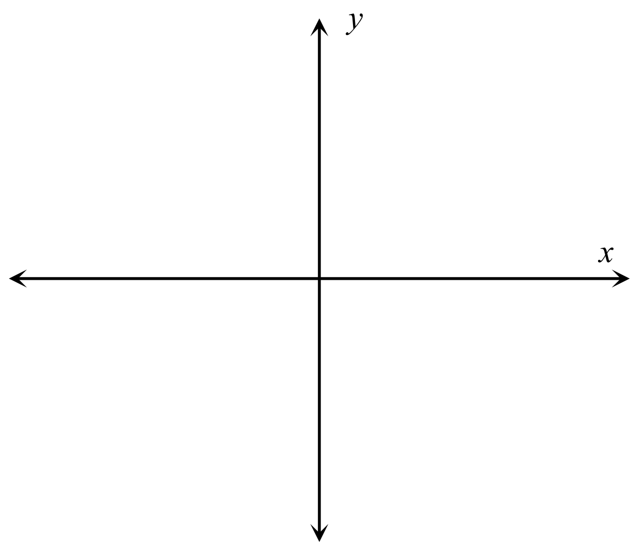
HA:

x-intercepts:

Zeros:

y-intercept:

Domain:



7. Does every rational function have a horizontal asymptote?

Let's summarize what we discovered so far about the HA and VA for rational functions.

8. Fill in the blanks below:

- When the degree of the numerator and denominator are the same, we find the value of the HA by ______________________
- When the degree of the denominator is larger than the degree of the numerator, the HA is always the line y = _______
- When the degree of the denominator is smaller than the degree of the numerator, a HA ____
- The VA is related to the restrictions of the domain and can be found by setting the denominator of the function equal to _________ and solving for the variable. (This will be refined later in future activities.)
- If the denominator is a quadratic or higher degree, then you will need to ______________ in order to find the vertical asymptote(s).

9. Sketch the graph of $r(x) = \frac{1}{1+x^2}$ and identify the important features.

VA:

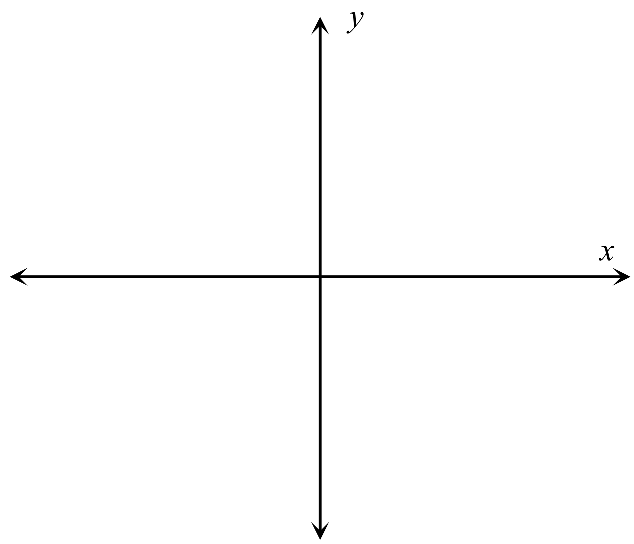
HA:

x-intercepts:

Zeros:

y-intercept:

Domain:



10. Do all rational functions have graphs with a vertical asymptote? Explain.

11. Sketch the graph of $f(x) = \frac{3}{(x+3)^2}$ and identify the important features.

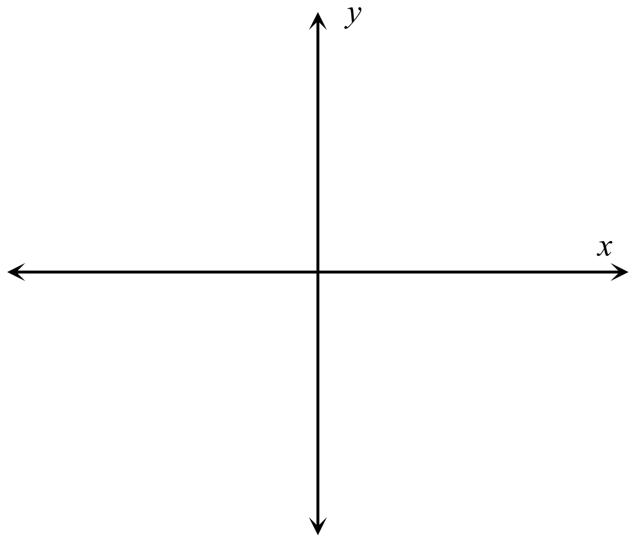
VA:

HA:

Zeros:

y-intercept:

Domain:



Consider the function $f(x) = \frac{2x-8}{x^2-16} = \frac{2(x-4)}{(x-4)(x+4)}$. Notice that when $x = 4$ both the numerator and denominator equal 0.

- Put the above equation in Y1 and graph. You see no holes in the graph.
- Evaluate the function at $x = 4$ using the value command. Select 2nd TRACE, select 1:value, enter 4, press ENTER.
- You will see y =
- This means that when $x = 4$, there is no value for y.

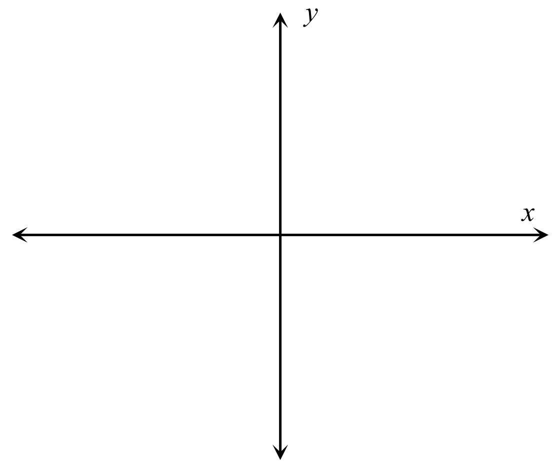
We mark this on the graph with a hole, *an open circle*.

When a value of *x* sets both the denominator and the numerator of a rational function equal to 0, there is a hole* in the graph; this is a point at which the function has no output value.

To find the coordinates of the hole, cancel the common factors in the numerator and denominator. This will create a "new" function since reducing the expression changes the domain of the function. Evaluate the new function at the excluded value to obtain the output for the new function. ***Note*: The functions are no longer equal if they have different domains.**

*If you evaluate the new function and the denominator is still zero but the numerator is not zero you have a vertical asymptote again and not a hole! We will not in this course work with functions that are this bizarre.

12. Graph $f(x) = \frac{2x-8}{x^2-16} = \frac{2(x-4)}{(x-4)(x+4)}$ and identify important features.



VA:

HA:

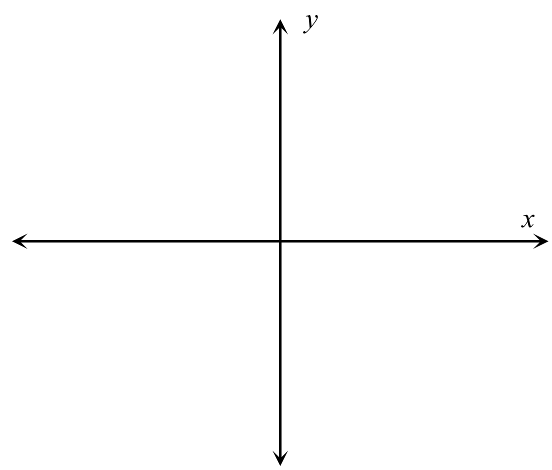
Zeros:

y-intercept:

Domain:

Hole: $(4, \frac{1}{4})$ – Remember to put an open circle at the point $(4, \frac{1}{4})$.

13. Graph $f(x) = \frac{4(x-2)}{x^2+x-6}$ and identify important features.



VA:

HA:

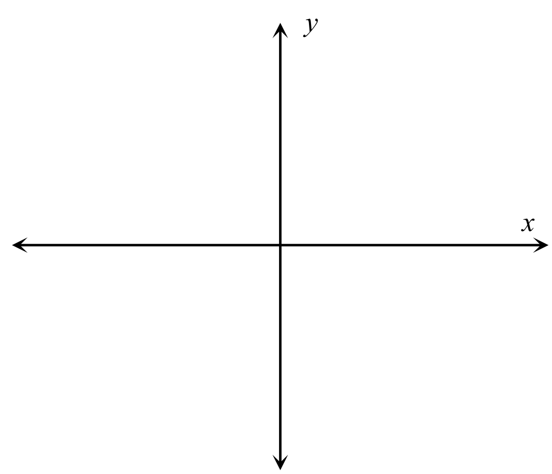
Hole:

Zeros:

y-intercept:

Domain:

14. Graph $f(x) = \frac{(x+2)(x-4)^2}{(x+2)(x+1)}$ and identify important features.



VA:

HA:

Hole:

Zeros:

y-intercept:

Domain:

Activity 4.3.4 – Applications of Rational Functions

1. Below is a list of the average yearly cost for electricity for common household appliances.

Appliance	Average Cost/Year in Electricity
Home Computer	$9
Television	$13
Microwave	$13
Dishwasher	$51
Clothes Dryer	$75
Washing Machine	$79
Refrigerator	$92

a. Assume that a new washing machine costs $715; determine the total annual average cost for a washing machine that lasts for 15 years. The only costs associated with the washing machine are its purchase price and electricity.

b. Write a function $C(x)$ that gives the annual average cost of a washing machine as a function of the number of years you own the washing machine.

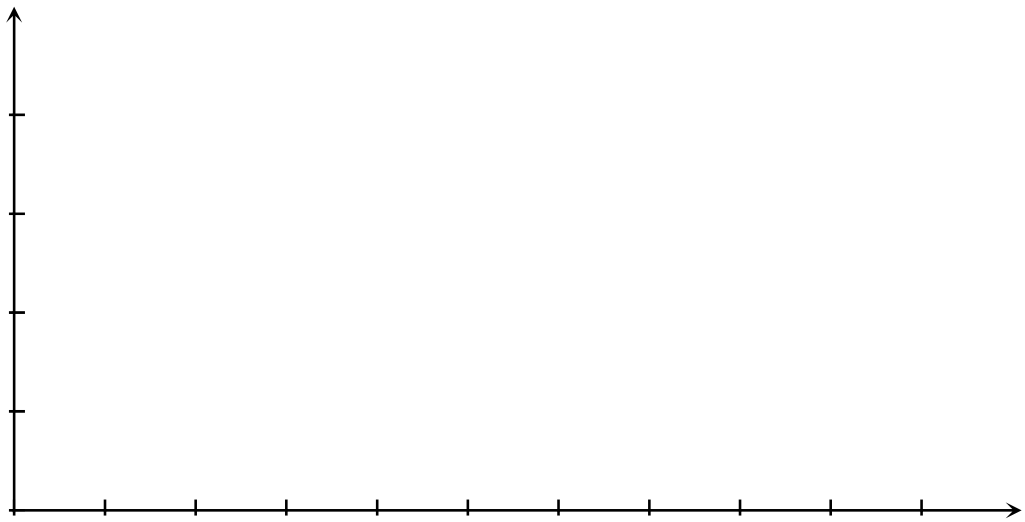
c. Determine the asymptotes of this function?

d. Explain the meaning of the horizontal asymptote in terms of the washing machine.

e. If a company offers a washing machine that costs $1100, but says that it will last at least 25 years, determine the total cost of the washing machine for 25 years. Assume no repairs were needed. Is the washing machine worth the difference in cost? Explain.

2. The function $C(t) = \frac{5t}{t^2+1}$ describes the concentration of a drug in the blood stream over time. $C(t)$ is measured in micrograms per liter and t is measured in hours.

   a. Use a graphing calculator to sketch the graph of the function over the first 10 hours after the dose is given. Label and scale the axes.



   b. When will the highest concentration of drug be reached and what is the amount in the patient's bloodstream at that time?

   c. How long does it take for the concentration to drop below 0.2 mg/L?

   d. What are the asymptotes of the rational function $C(t) = \frac{5t}{t^2+1}$?

   e. What is the meaning of the horizontal asymptote within the context of the problem?

3. The rabbit population on Bishop's Farm can be found by the function $p(t) = \frac{4000t}{t+1}$, where $t \geq 0$ is the time in months since the beginning of the year.

   a. Determine the asymptotes of this function?

   b. Explain the meaning of the horizontal asymptote in terms of the rabbit population.

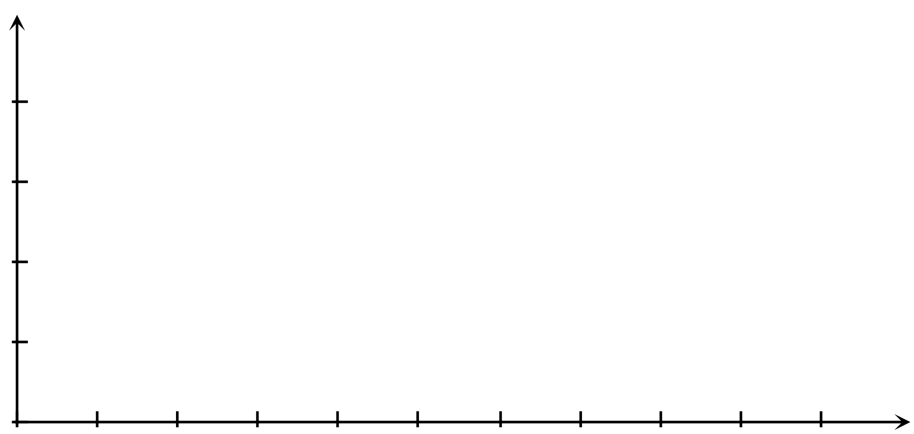
   c. Explain the meaning of the vertical asymptote in terms of the rabbit population.

4. You are traveling in your car for 200 miles.

   a. How long will the trip take if you average 30 miles per hour, 55 miles per hour, or 65 miles per hour?

   b. Write a function *f*(*t*) that describes the time it takes to travel 200 miles as a function of your speed *s*.

   c. Sketch the graph of this function. Label and scale the axes.



   d. What does the graph tell you about the time it takes you to travel depending on the speed of the car?

5. A rocket fired upward from the surface of the earth with an initial velocity *v* (in meters/second) will attain a maximum height h (in meters) according to the formula

$$h(v) = \frac{rv^2}{2gr - v^2}$$

where *r* is the radius of the earth, about $6.4 * 10^6$ meters, and g is 9.8m/s^2.

   a. Find *h*(1000). Interpret the result in terms of the problem and convert it from meters to miles.

   b. Find *h*(5000). Interpret the result in terms of the problem and convert it from meters to miles.

   c. Use your graphing calculator to graph this function in the first quadrant since h and *v* will be positive. Let the interval for *v* be [0, 12000]. You should see a vertical asymptote. Identify the vertical asymptote from the graph.

   d. Do all rockets we launch come back to earth or do some escape the earth's gravitational pull?

   e. What might be the significance of the velocity where you have the vertical asymptote?

6. An aluminum can is to be constructed to have a volume of 298 cm^3. Using a graphing calculator, find the dimensions of the can that will minimize the amount of material?

   Surface Area = $2\pi r^2 + 2\pi rh$ Volume = $\pi r^2 h$

   where *r* is the radius of the circle and *h* is height of can.

Activity 4.3.5 – Graphing Rational Functions IV

Revisiting Rational Functions and Comparing their Behavior to other Function Families

Sketch the graphs of the following functions and its parent function. For each graph, identify the domain, range, *y*-intercept, *x*-intercept(s), zero(s) of the function, end behavior, horizontal asymptote(s), and vertical asymptote(s). If something does not exist, state so.

1. $f(x) = 2(x+3)^2 - 5$ Parent: $p(x) = x^2$

VA:

HA:

Zero(s):

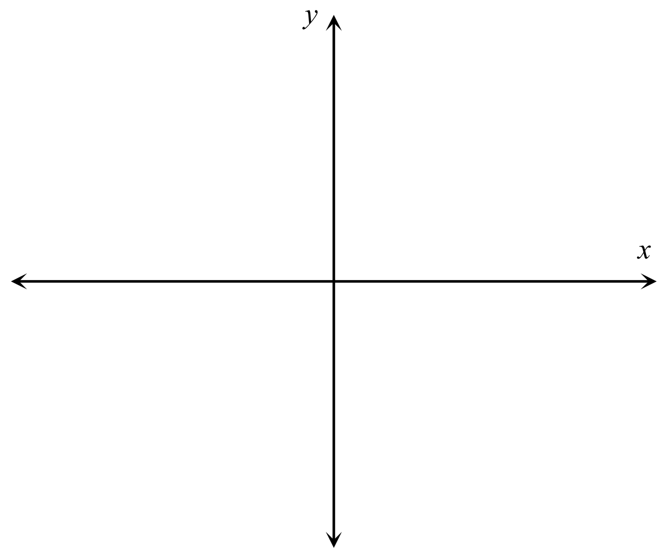
x-intercept(s):

y-intercept:

Domain:

Range:

End Behavior:



2. $f(x) = 2(x+3)^3 - 5$ Parent: $p(x) = x^3$

VA:

HA:

Zero(s):

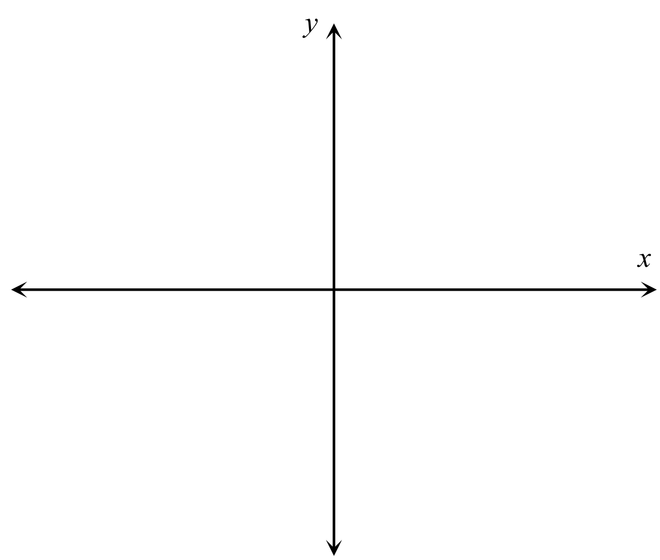
x-intercept(s):

y-intercept:

Domain:

Range:

End Behavior:



3. $r(x) = 2(4^{x+3}) - 5$ Parent: $p(x) = 4^x$

VA:

HA:

Zero(s):

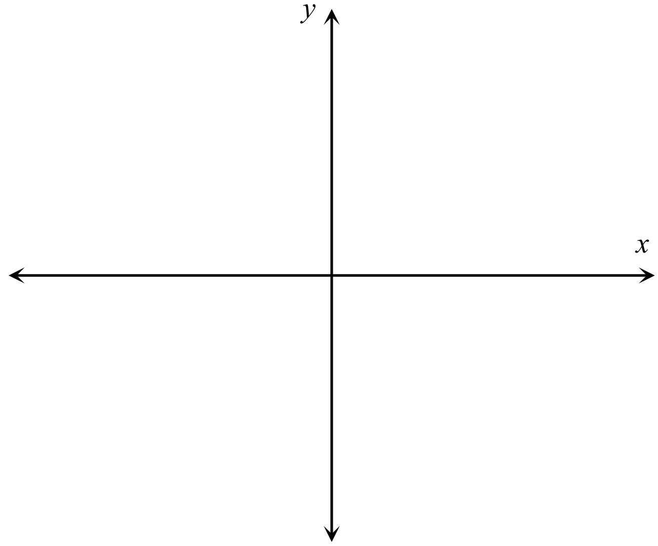
x-intercept(s):

y-intercept:

Domain:

Range:

End Behavior:



4. $k(x) = \frac{2}{x+3} - 5$ Parent: $p(x) = \frac{1}{x}$

VA:

HA:

Zero(s):

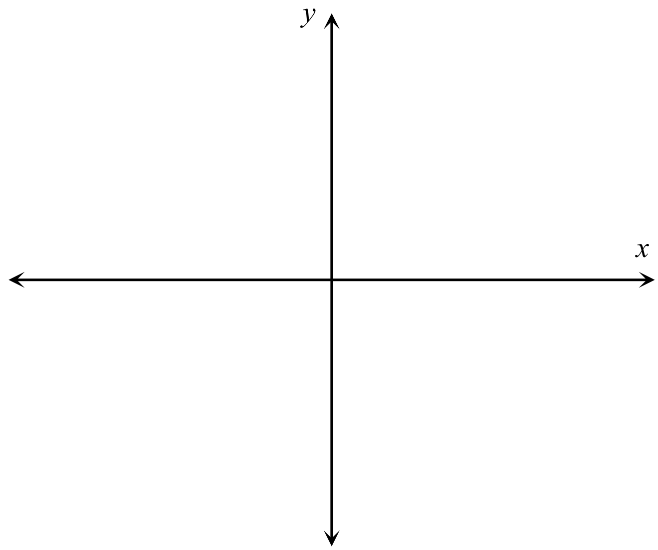
x-intercept(s):

y-intercept:

Domain:

Range:

End Behavior:



5. Did the number "3" play the same role for all the graphs? Explain.

6. Did the number "2" play the same role for all the graphs? Explain.

7. Did the number “5” play the same role for all the graphs? Explain.

8. How were the graphs similar? How were they different? Explain.

Graphing Rational Functions

Sketch the graphs of the following functions. For each graph, identify the domain, range, *y*-intercept, *x*-intercept(s), zero(s), horizontal asymptote(s), and vertical asymptote(s). If something does not exist, state so.

9. $k(x) = \frac{x^2 + 4}{x + 9}$

VA:
HA:

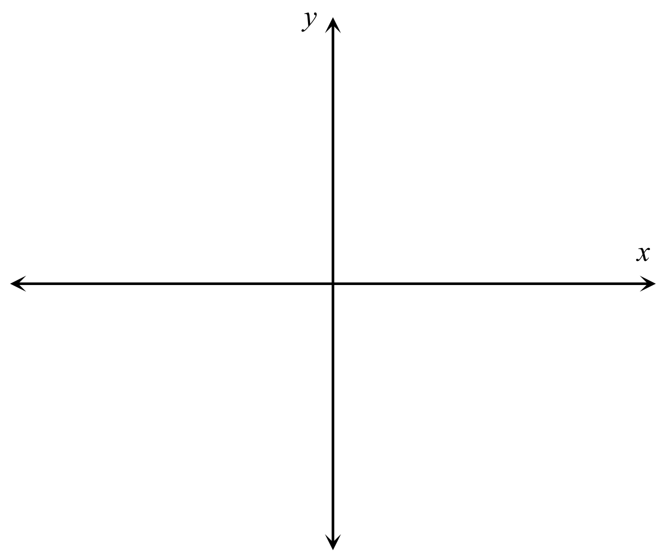
Zero(s):

x-intercept(s):

y-intercept:

Domain:

Range:



10. $k(x) = \frac{x + 4}{x^2 - 9}$

VA:
HA:

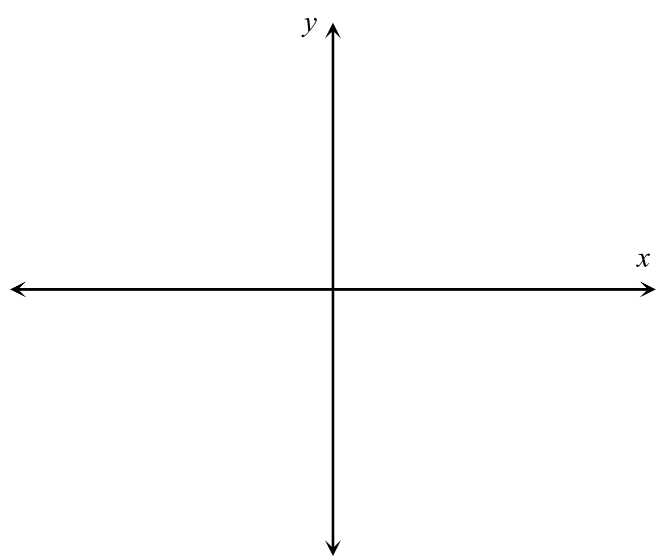
Zero(s):

x-intercept(s):

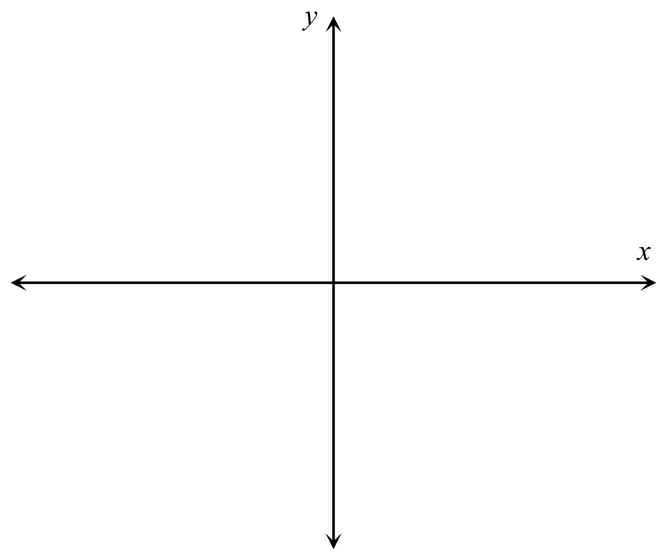
y-intercept:

Domain:

Range:



11. $k(x) = \frac{5x+4}{3x+9}$



VA:

HA:

Zero(s):

x-intercept(s):

y-intercept:

Domain:

Range:

What are the Rules for Finding a Horizontal Asymptote?

12. Fill in the blanks below:

- When the degree of the numerator and denominator are the same, we find the value of the HA by ____.
- When the degree of the denominator is larger than the degree of the numerator, the HA is always the line, $y =$ __________.
- When the degree of the denominator is smaller than the degree of the numerator, the HA ____.

(+) Activity 4.3.6 – Graphing Rational Functions V

Section I (+)

Sketch the graphs of the following functions. For each graph, identify the vertical and horizontal asymptotes, holes, zeros, *x*-intercept(s), *y*-intercept, and domain. If something does not exist, state so.

1. $f(x) = \frac{x-1}{x^2+2x-3}$

V.A.:

H.A.:

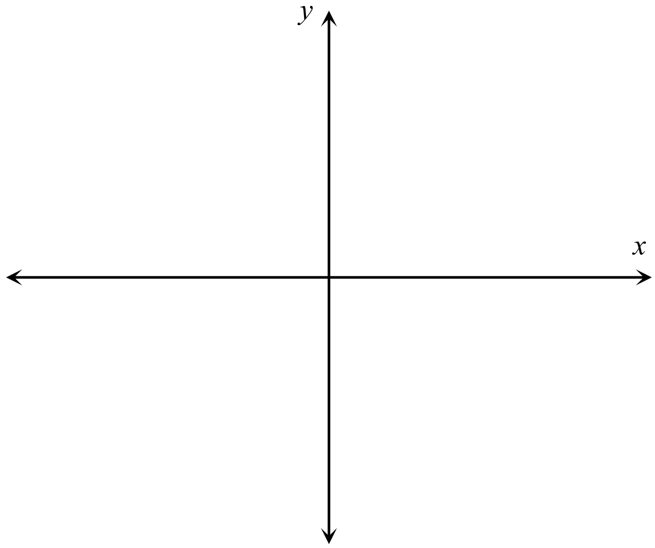
Hole:

Zeros:

x-intercept(s):

y-intercept:

Domain:



2. $g(x) = \frac{x-1}{x^2+5x-6}$

V.A.:

H.A.:

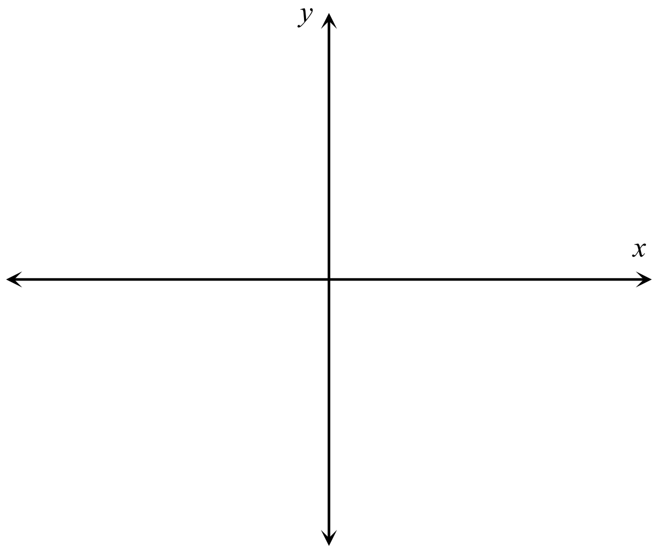
Hole:

Zeros:

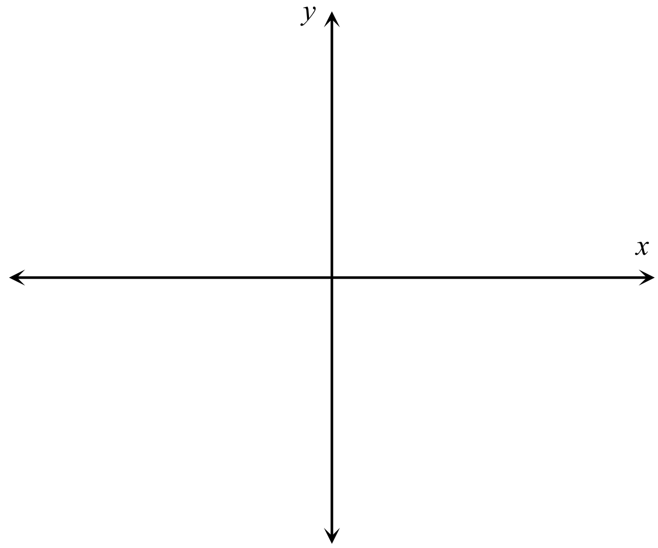
x-intercept(s):

y-intercept:

Domain



3. $k(x) = \frac{x^2 + 3x - 10}{x + 5}$



V.A.:

H.A.:

Hole:

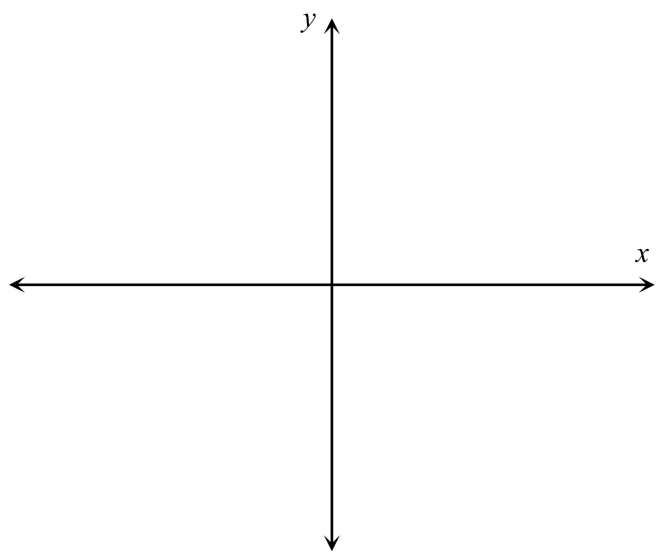
Zeros:

x-intercept(s):

y-intercept:

Domain:

4. $h(x) = \frac{x^4+x^2+1}{x^2-1}$



V.A.:

H.A.:

Hole:

Zeros:

x-intercept(s):

y-intercept:

Domain:

Asymptote or Hole?

To determine whether the graph of a rational function has a vertical asymptote or a hole at a restriction, proceed as follows:

1. Factor the numerator and denominator of the original rational function *f*. Identify any restrictions on *f*.
2. Reduce the rational function to lowest terms, naming the new function g. Identify any restrictions on the function g.
3. Those restrictions of *f* that remain restrictions of the function g will be used to define the vertical asymptotes of the graph of *f*.
4. Those restrictions of *f* that are no longer restrictions of the function g will be the x-coordinates of the "holes" of the graph of *f*.
5. To determine the coordinates of the "holes", substitute each restriction of *f* that is not a restriction of g into the function g to determine the *y*-value of the hole.

5. Write the equation of a rational function $g(x)$ who has a zero at 0, vertical asymptotes at $x = 2$ and $x = -1$, and a horizontal asymptote at $y = 3$.

6. Write the equation of a rational function $f(x)$ who has zeroes at 3 and -4, a hole at $x = -2$, a vertical asymptote at $x = 5$, and a horizontal asymptote at $y = 4$.

Name the vertical asymptotes, horizontal asymptotes and holes in the graphs of the following equations. **Do not use a calculator**.

7. $f(x) = \frac{3(x-2)(x+4)}{(x+3)(x+4)}$ V.A. H.A.

   Hole

8. $f(x) = \frac{(x-4)^2}{(x+4)(2x-3)(\frac{1}{4}x+8)}$ V.A.

   H.A. Hole

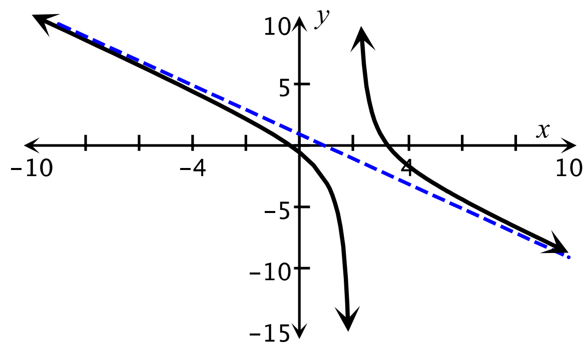
Section II (+)

If the numerator's degree is one degree greater than the denominator's degree, you have a *slant asymptote* of the form $y = mx + b$. You will need to use long division to find the equation of the slant asymptote.

Example: Find the slant asymptote of the rational function $f(x) = \frac{-x^2+3x+1}{x-2}$.

Solution: Perform long division.

$$\begin{array}{r|l} & \underline{-x \;+\; 1} \\ x-2 & -x^2+3x+1 \\ & \underline{-x^2+2x} \\ & \quad x+1 \\ & \quad \underline{x-2} \\ & \qquad 3 \end{array}$$



Ignore the remainder and use only the polynomial part. So, $y = -x + 1$ is the slant asymptote (S.A.) as shown above with the dotted line.

Graph the following functions and identify the features listed below.

9. $f(x) = \frac{x^2+1}{x}$

V.A.:

S.A.:

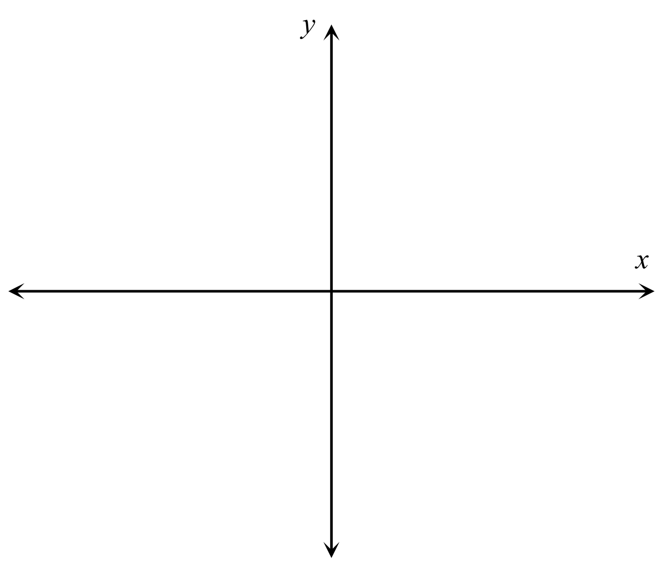
Hole:

Zero(s):

x-intercept(s):

y-intercept:

Domain:



10. $f(x) = \frac{x^2 - x - 12}{x - 2}$

V.A.:

S.A.:

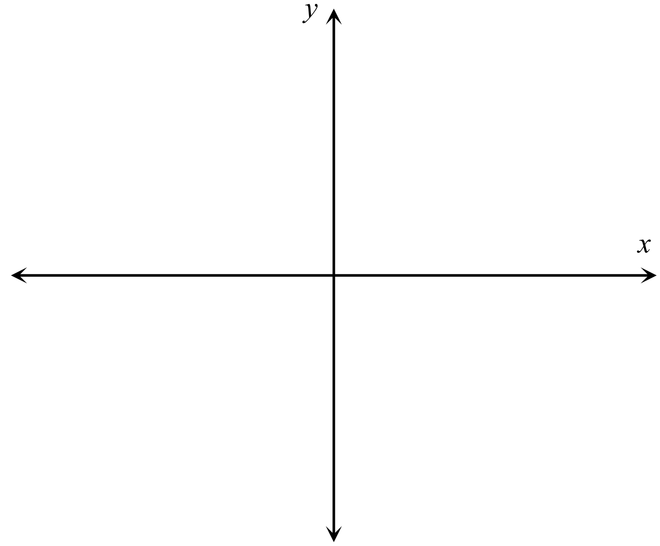
Hole:

Zero(s):

x-intercept(s):

y-intercept:

Domain:



11. $f(x) = \frac{3x^2 + 2}{x - 1}$

V.A.:

S.A.:

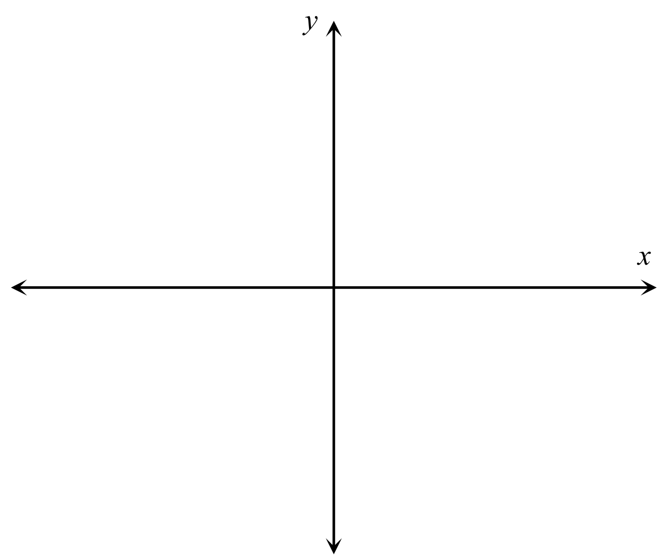
Hole:

Zero(s):

x-intercept(s):

y-intercept:

Domain:



Activity 4.4.1 Rational Expressions I

1. The concentration of a given substance in a mixture is determined by the ratio of the amount of the substance to the total quantity. For example, if 6 cups of lemonade contains 1 cup of pure lemon juice, the concentration of lemon juice in the lemonade is 1/6. Suppose we have 12 cups of lemonade that contains only 1 cup of pure lemon juice and we decide it is too weak. So we will add x cups of pure lemon juice.
   a. Write an expression that contains x for the amount of lemon juice present after x cups have been added.

   b. Write an expression that contains x for the total amount of lemonade present after the x cups have been added.

   c. Now write an expression for the concentration of lemon juice in the lemonade after x cups have been added.

   d. Ignore the context and give the domain of the expression in part c.

   e. Now consider the context and give the domain of the expression in part c.

   f. If we keep adding lemon juice, what is the end behavior of the function defined by your expression in part c?

   g. Interpret the end behavior in terms of this context.

2. If you own a retail store you purchase merchandise at a wholesale cost, W and then sell it at a retail price also called the selling price, S. The retailer must sell the merchandise for more than he paid for it. The retailer's markup is the difference between the selling price S and the wholesale price W.
   a. If Sam owns a store and purchases a pair of men's shorts for $30 and sells them for $55, what is the amount of markup?

   b. In the retail business the term percent markup is often used. This percentage, P, known as percent markup requires that we find what percent the markup is of the selling price. For the shorts Sam purchased in part a, what is the percent markup?

c. The rational expression W/(1 – P) defines the selling price, S where W is the wholesale cost, and P is the percent markup but expressed as a decimal. Consider for our shorts S = 30/(1 - .45) . Make a table For S

P	0	0.01	0.05	0.10	0.25	0.5	0.75	0.95	0.99
S									

d. Can the percent markup ever be 100%, that is can P = 1?

e. Can the percent markup P be greater than 1?

f. Does this function have a vertical asymptote? ____ If so what is the equation that defines it?

g. As P gets closer to 1 from the left, written $P \rightarrow 1^-$, say P increases by .01 near 1, say from .94 to .95 to .96, what happens to the Selling Price?

h. What is a practical domain for this problem?

3. A container holds b balls numbered 1, 2, 3, …, b and only one ball has the winning number.

a. Write an expression for drawing the winning ball.

b. Write an expression for the complement, that is of not drawing a winning ball.

c. Suppose a second container holds 2 fewer balls than the one above and three of its balls are winning balls. Write an expression for drawing a winning ball from the first container and a winning ball from the second container.

Activity 4.4.2 Rational Expressions II

In this activity, you will simplify rational expressions that have both like and unlike denominators.

The process is similar to simplifying expressions with fractions. When we add or subtract fractions, we need to have a ________________ denominator.

Remember that fractions in the form of $\frac{a}{b}$ can be written in the form of $a\left(\frac{1}{b}\right)$. For example, $\frac{3}{x}$ can be written as $3\left(\frac{1}{x}\right)$. These are equivalent expressions. Hence, using the distributive property $3\left(\frac{1}{x}\right) + 4\left(\frac{1}{x}\right)$ would be ___________.

Without a common denominator, we cannot use the distributive property. Sometimes it is useful to be able to convert between these two forms in order to simplify rational expressions. Some rational expressions will require you to factor first in order to simplify them. Let's simplify the following rational expression.

Ex. 1: $\frac{3x}{x+2} + \frac{5x+1}{x+2}$

We already have a CD so we need to keep the denominator so we can use the distributive property and combine the numerators.

$$\frac{3x+5x+1}{x+2}$$

Now, we can combine like terms in the numerator.

$$\frac{8x+1}{x+2}$$

… One last thing...

In this example, there is a **restriction**. The domain cannot contain the value -2 because the denominator of any fraction cannot be equal to zero. (Set x + 2 = 0 and solve for x to get the restriction. Therefore, x cannot equal -2 in the expression.) The domain of the expression is $(-\infty,-2)U(-2,\infty)$.

Simplify the following rational expressions and list the domain and any restrictions.

1. $\dfrac{5y}{2y-1}+\dfrac{4-2y}{2y-1}$

2. $\dfrac{5a-6}{3a+2}-\dfrac{10a+7}{3a+2}$

3. $\dfrac{3}{6x+1}+\dfrac{4}{6x+1}$

4. A square has a side with measure $\dfrac{x}{x-3}$. What is an expression for the perimeter of the square? What is the domain of this expression? (Hint: Be sensitive to the fact that we are talking about the measure of the side of a square.) What is the restriction?

5. An equilateral triangle has a side length of $\dfrac{2x-1}{3x+2}$. Find an expression for the perimeter of the triangle. What is the domain of the expression? What is the restriction?

Let's simplify the following rational expression.

Ex. 2: $\dfrac{3}{x}+\dfrac{5}{4x}$

First, we should **find the least common denominator** (CD). The CD in this example is 4x. We now need to change the fractions so they have a CD.

$\dfrac{3}{x}\cdot\left(\dfrac{4}{4}\right)+\dfrac{5}{4x}\cdot\left(\dfrac{1}{1}\right)$ Multiply the fraction on the left by $\dfrac{4}{4}$.

$\dfrac{12}{4x}+\dfrac{5}{4x}$

Simplify the fraction on the left and then add the fractions using the distributive property.

$\dfrac{17}{4x}$

The restriction is that x cannot equal zero because the denominator of any fraction cannot be zero. The domain of the expression is $(-\infty,0)U(0,\infty)$.

Ex. 3: $\dfrac{3}{x(x+2)}-\dfrac{4x}{x}$

The CD is $x(x+2)$, so we need to change the fraction on the right so that we have a common denominator.

$$\frac{3}{x(x+2)}-\frac{4x}{x}\cdot\left(\frac{x+2}{x+2}\right)$$

$\dfrac{3}{x(x+2)}-\dfrac{4x(x+2)}{x(x+2)}$ Now, distribute in the numerator.

$\dfrac{3}{x(x+2)}-\dfrac{4x^2+8x}{x(x+2)}$ Combine the numerators. Be careful with the subtraction!

$\dfrac{3-(4x^2+8x)}{x(x+2)}$ Distribute the subtraction.

$$\frac{3-4x^2-8x}{x(x+2)}$$

The restrictions are that x cannot be equal to 0 or -2. (Set x(x + 2) = 0 and solve for x to get the restrictions.) The domain of the expression is $(-\infty,-2)U(-2,0)U(0,\infty)$.

Simplify the following rational expressions and list the domain and any restrictions.

6. $\dfrac{4}{3x}+\dfrac{8}{x}$

7. $\dfrac{4}{2x^3}-\dfrac{6}{x}$

8. $\dfrac{2x}{x(x-4)}+\dfrac{4}{x}$

9. A rectangle has a length of $\frac{4}{x+3}$ and a width of $\frac{2}{x}$. Find an expression for the perimeter. What is the domain? Are there any restrictions? If so, list them.

10. A parallelogram has side lengths of $\frac{3}{x}$ and $\frac{2x+1}{x(x+1)}$. Find an expression for the perimeter. What is the domain? List the restrictions.

Sometimes you may need to factor first in order to simplify rational expressions.

Ex. 4: $\frac{1}{x-2}-\frac{2}{x^2-6x+8}$

First, factor the denominator in the fraction on the right.

$\frac{1}{x-2}-\frac{2}{(x-2)(x-4)}$ Now find the CD. The CD = (x – 2)(x – 4).

$\frac{1}{x-2}\cdot\left(\frac{x-4}{x-4}\right)-\frac{2}{(x-2)(x-4)}$ Multiply the fraction on the left by $\left(\frac{x-4}{x-4}\right)$.

$\frac{x-4}{(x-2)(x-4)}-\frac{2}{(x-2)(x-4)}$ Combine the numerators.

$\frac{x-4-2}{(x-2)(x-4)}$ Simplify.

$\frac{x-6}{(x-2)(x-4)}$

The restrictions are that x cannot be equal to 2 or 4. The domain of the expression is $(-\infty,2)U(2,4)U(4,\infty)$.

Simplify the following rational expressions and list the domain and any restrictions.

11. $\frac{2x-3}{x-2}+\frac{4x}{3x-6}$

12. $\frac{3}{x^2-x-20}-\frac{4x}{x^2+7x+12}$

13. $\frac{x+7}{x^2+13x+42}+\frac{2x}{x^2+8x+7}$

14. An isosceles triangle has a base of $\frac{4}{x+3}$ and sides of equal measure that are each $\frac{7}{x^2+7x+12}$. Find an expression for the perimeter. List the domain. List the restrictions.

15. Consider two positive numbers whose product is 9. Find an expression for the sum of the two numbers. Use that expression to define a function. When will that sum be a minimum? (Hint: Graph your function and try to find the lowest point on the curve.)

16. Describe in your own words how to simplify a rational expression. Create an example to illustrate your procedure.

17. English schools' Track and Field puts forth benchmarks for sprinting times. After 52 weeks of training, 12 and 13 year olds should aim for a time of 13.2 seconds for a 100 meter sprint. Fourteen and 15 year olds should aim for a time of 11.6 seconds for a 100 meter sprint. Sixteen and 17 year olds should aim for a time of 12.5 seconds for a 100 meter sprint. Find an expression to determine the speed for each of the three age groups in meters per second. Then convert each speed to miles per hour.

Simplify the following. List the domain and any restrictions.

18. $\dfrac{3y+1}{2y+1}+\dfrac{7y+4}{2y+1}$

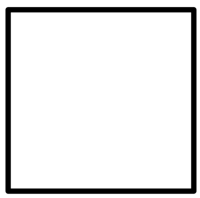
19. $\dfrac{3a+3}{a^2+2a+1}+\dfrac{a-1}{a^2-1}$

20. $\dfrac{x-5}{x^2-3x-10}+\dfrac{x}{x^2+x-2}$

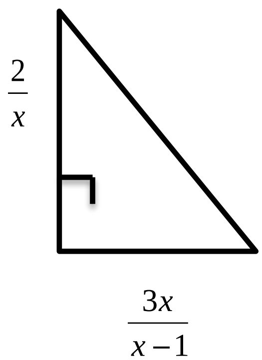
Activity 4.4.3 Rational Expressions III

In this activity, you will use what you know about the properties of geometric figures and simplifying rational expressions to answer the following questions.

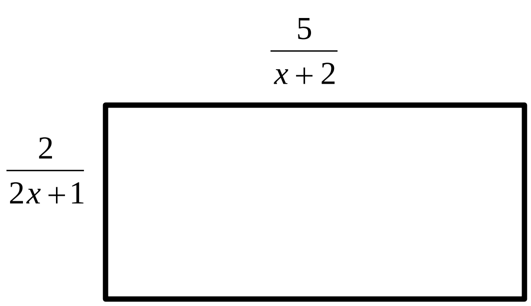
1. Find an expression for the area of the square having a side length of $\frac{4}{x+2}$.



2. Find an expression for the area of the triangle below.



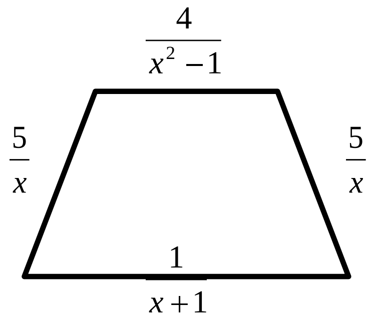
3. Find expressions for the area and perimeter for the rectangle below.



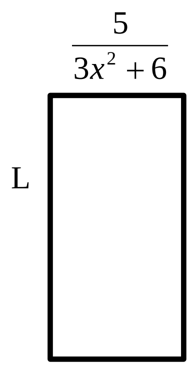
Area:____________________

Perimeter:________________

4. Find an expression for the perimeter of the quadrilateral.

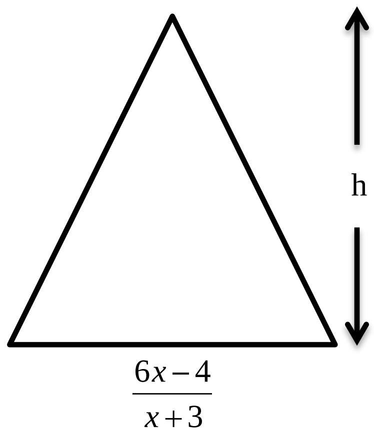


5. The area for the rectangle below is $\frac{15x+5}{2x^3+4x}$. Find the length, L of the rectangle if the width is $\frac{5}{3x^2+6}$.



6. Find an expression for the perimeter for the rectangle in problem #5.

7. Find an expression for the height for the triangle below if the area is $\frac{3x^2-11x+6}{4x}$.



8. A rectangular prism has a height of x, a width of $\frac{x}{x+1}$ and a length of $\frac{x+1}{x-1}$.
   a. Find an expression for the volume of the prism.

   b. Find an expression for the surface area of the prism.

9. A cube has a side length of $\frac{x}{x-1}$ units.

   a. Find an expression for the volume of the cube.

   b. Find an expression for the surface area of the cube.

   c. Find the volume and surface area if x = 4 inches.

10. A paint can has a diameter of $\frac{16}{x}$ inches and a height of $\frac{18}{2x-2}$ inches.

   a. Find an expression for the volume of the can.

   b. Find an expression for the surface area of the can.

11. Suppose b balls, numbered 1 to b, are placed in a container. Two of the balls that are in the container are winning balls. The container is shaken.

    a. What is the probability of drawing a winning ball ?

    b. Calculate the probability of drawing a winning ball if there are 100 balls in the container (b = 100). Then calculate for b = 10,000 and for b = 100,000.

    c. As the number of balls increases, what happens to the probability of drawing a winning ball?

12. Suppose b balls, numbered 1 to b, are placed in a container. Two of the balls that are in the container are winning balls. The container is shaken. You now draw a first ball, replace it and draw again. What is the probability of drawing two winning balls?

13. Suppose b balls, numbered 1 to b, are placed in a container. Two of the balls that are in the container are winning balls. The container is shaken. You now draw a ball, do not replace it and draw a second ball. What is the probability of drawing two winning balls?

14. A. Ms Agnesi, a math teacher at a local high school is planning a field trip to the Connecticut Science Museum in Hartford. Bus transportation, parking and entrance fees will $1000 even with a museum discount. The total cost will be shared equally by all who go on the trip. She needs to at least have 30 students sign up but only has busses to transport 156 students and chaperones. She needs to have 5 chaperones go on the trip Write an expression to determine the cost per student/chaperone. Evaluate your expression for 50 and then 100 students/chaperones. What is the domain of this expression?

    B. Students said they wanted to also attend an IMAX movie (see earlier part A of this problem) That was not part of the package. The discounted cost will be an additional $4 a student/chaperone. Write a new expression for the cost per person for this situation. See if you can take your expression and by manipulating it come up with an equivalent yet different expression.

C. Students thought it was unfair to have chaperones pay for the trip or movie. If 5 chaperones are needed, write an expression for the cost per student for trip and movie when chaperones pay $0.

D. Use your expression in part C to determine the cost per student when 50 students and 5 chaperones go on the trip, when 100 students and 5 chaperones go on the trip.

E. Students wanted to pay for the chaperones but thought most of them could afford no more than 12 dollars to go on the trip and see the movie. How many students would need to sign up to keep the cost per ticket no more than $12. Explain how you solved this problem.

Activity 4.5.1 Rational Equations

Section I

You have used the Multiplication Property of Equality at least since Algebra 1 when you solved a linear equation such as 2x + 4 = 12. Here is the statement of the Multiplication Property of Equality:

If a, b, and c are real numbers with $c \neq 0$, then a = b is equivalent to ac = bc. This means that if each side of an equation is multiplied by the same non zero number, the new equation is equivalent to the old one.

For example:
(1) 8x = 35
(2) 8x (1/8) = 35(1/8). Since $1/8 \neq 0$ the solution 35/8 is not only the solution to equation (2) but it is also the solution of equation (1).

Note if:
(3) 8x = 35 and we multiply both sides by 0, we get
(4) 0 = 0 an equation for which all Real numbers are solutions, not just 35/8.

We introduced extra solutions known as extraneous solutions. An extraneous solution is a solution of a transformed version of an equation that does not satisfy the original equation because it was excluded from the domain of the original equation.

When a variable expression is in the denominator of a fraction, we need to multiply by the variable expression. We have no way of knowing whether the expression will equal zero or not. Thus the original equation and the new equation may not be equivalent.

Thus the check is not just a good thing to do, it is essential because a solution to the new equation may not be a solution of the original equation. Consider:

(5) $\frac{1}{x-2} + \frac{1}{x+2} = \frac{4}{x^2-4}$ Multiply both sides by $x^2 - 4$ in its factored form (x-2)(x+2)

(6) $\frac{1(x+2)(x-2)}{x-2} + \frac{1(x-2)(x+2)}{x+2} = \frac{4(x-2)(x+2)}{x^2-4}$

Work out the solution and you should get
(7) (x + 2) + (x – 2) = 4
(8) 2x = 4
(9) Hence x = 2.

But 2 does not satisfy the original equation. So the original has no solution even though (x + 2) + (x – 2) = 4 and 2x = 4 are satisfied by 2 and thus have a solution. Two is not on the domain of the original equation and 2 is an extraneous solution.

We introduced extra solutions known as extraneous solutions. An extraneous solution is a solution of a transformed version of an equation that does not satisfy the original equation because it was excluded from the domain of the original equation. For our example, 2 is not in the domain of the original expressions making up the equation.

Try the next four problems with your neighbor:

1. $\frac{3}{x+7} = \frac{8}{x}$

2. $\frac{1}{x} + \frac{7}{x} = 9$

3. $\frac{5}{y+2} = \frac{7}{y+9}$

4. $\frac{x}{x^2-9} = \frac{3}{x^2-9}$

Your teacher now has an activity for you.

Activity 4.5.2 Solving Equations with Fractions and Rational Expressions

Solve each equation or problem.

1. $\frac{4+x}{x+7} = \frac{2}{3}$

2. $\frac{5}{x+9} = \frac{7}{x-12}$

3. $\frac{2}{5x^2-4} = \frac{4}{10x^2-3x}$

4. $\frac{4x}{4x+4} = -\frac{1}{4x+4}$

5. $\frac{2x}{3} + \frac{5x}{2} = \frac{11}{6}$

6. $\frac{1}{x+9} - \frac{6}{x^2} = \frac{1}{x^3+9x^2}$

7. $\frac{2}{x-5} + \frac{x}{x+5} = \frac{x^2-2x+5}{x^2-25}$

8. A rectangle that has a perimeter of 22 meters has a width that is four less than one half its length. Find the length and the width of the rectangle.

9. A parallelogram has a base whose measure is given by 2a/(a +2) and the measure of the adjacent side is a/(a +1).
   a. Find the value of a if the perimeter is 4 meters.

   b. Round the value to the nearest hundredth.

   c. Using the rounded value in part b, find the measures of the base and adjacent side if the perimeter is 4 meters.

10. In unit 1, you combined functions finding f(x) + g(x) and f(x) – g(x). If $f(x) = \frac{x}{2x-1}$ and $g(x) = \frac{8x^2}{4x^2-1}$
   a. Find h(x) = f(x) + g(x) and find the domain of h

   b. Find k(x) = f(x) – g(x) and find the domain of k

   c. Now solve f(x) – g(x) = 0

11. If $f(x) = \frac{2x+4}{x^2+6x+9}$ and $g(x) = \frac{x^2+3x}{x^2+2x}$ find k(x) where k is the product of f and g.

   a. Domain of f is

   b. Domain of g is

c. Domain of k is

d. Does the graph of k cross the x axis? Explain.

e. Solve f(x)g(x) = 0

12. A boat can travel 10 miles downstream (with the current) in the same amount of time it can travel 8 miles upstream (against the current).If the speed of the current is 2mph, what is the speed of the boat in still water? Hint: D = RT

13. Jayden wants to help his grandfather stain the deck. Last year he did it for his grandfather and it took 6 hours. His grandfather can do it by himself in 5 hours. This year they will do it together so it will not take long. How long will it take this year?

14. To make a spray solution to kill poison ivy, Allen needed to mix 5 parts of water to 2 parts of the liquid ivy killer. If he used 15 more quarts of water than quarts of the liquid ivy killer, how much water did he use?

Activity 4.5.3 More Equations and Problems

Solve:

1. $5(x+7)-2(5x-8)= -2x-3(x-7)$

2. $x^2+10x+28=4$

3. Working together it takes 3 hours for a new teacher and an experienced teacher to set up a bulletin board. It the new teacher did it alone it would take her 7 hours. How long would it take the experienced teacher to set up the bulletin board by herself?

4. $\sqrt{x+8}=4$

5. $\sqrt{4x+8} = x-1$

6. A beaker contains 10 ounces of a liquid that is 12 % alcohol. You need to add water to make the alcohol concentration 9%. How much water must you add?

7. $(x-9)(x+3) = -5$

8. Find the zeros (Real and Complex) of $f(x) = x^3 + 3x^2 - x - 3$.

9. Find the zeros (Real and Complex) of $g(x) = x^3 - 3x^2 + x - 3$.

10. You have probably had a rushing ambulance come toward you with loud sirens while in the car. The pitch of the siren sound (measure of the frequency) is high getting louder but as soon as the ambulance passes you, it is less. This phenomenon is the Doppler Effect—the apparent shift in frequency of sound waves produced by a moving source. The observed frequency of the sound, f_0 when a source is traveling toward you is given by the formula $f_0 = (f_s c/(c - v_s)$ where f_s is the emitted frequency; c, the velocity of the sound and v_s is the velocity of the source coming toward you.

    If a vehicle is coming toward you at 65 mph or 104.6 km/hr and it sounds its horn at 8000Hz, what is the frequency of the sound you hear when the speed of the sound is 340 m/sec (1115ft/sec)? Hint: Use metric and make sure all your units match. Look up Hz if you need to.

11. A farmer wants to enclose a rectangular field by a fence and divide it into two smaller rectangular fields by a fence parallel to one side of the field. He has 3600 feet of fencing . Find the dimensions of the field so that so that the total enclosed area is a maximum.