Algebra I

Unit 3: Functions

3.1.1a Representing Relations I3.1.1b Representing Relations II3.1.2 Is it a Function?Supplemental Function Practice3.1 Exit Slip

3.2.1 Bottled Water
3.2 Exit Slip
3.2.2a Hartford Precipitation
3.2.3 Functions Everywhere – Identifying Independent and Dependent Variables

3.3.1 Function Machines
3.3.2 Introduction to Function Notation
3.3.1 Exit Slip
3.3.4 Hot Air Balloon
3.3.2 Exit Slip
Quiz – Relations and Function Notation

3.4.1a Highway Driving3.4.3 Free Throws3.4 Exit Slip

Unit 3 Test

Representing Relations I

A *relation* is a connection between an input and an output. Relations can be expressed in five different ways: mapping diagrams, tables, ordered pairs, graphs, and equations. However, not all relations are easily expressed using all five methods. The following table shows artists, their average ticket price and their gross income for the tour. Use the data to express the relations in various ways.

2011 Top Concert Tours

Artist	Ticket Price	Income (in millions)		
U2	97	232		
Bon Jovi	94	149		
Taylor Swift	71	104		
Elton John	120	103		
Rihanna	75	90		
Kenny Chesney	73	85		

1. Mapping Diagram

(Ticket Price, Income in millions)

Ticket Price	Income
97	103
94	104
71	149
120	90
75	85
73	232

Source: Pollstar

A relation can also describe a connection between things that aren't numbers, like the artist and the income for the tour, or the artist and the stage manager.

2. Ordered Pairs

(Artist, Income in millions)



3. Graph (Ticket Price, Income in millions)



- 4. Map five students to their English teacher
- 5. Map a number of siblings to five students

Student	English Teacher	# of Sibling

# of Siblings	Student

6. Make tables for the relations above.

Student	English Teacher

# of Siblings	Student

7. List ordered pairs with famous athletes as the inputs and the sports they play as the outputs.



9. Which of the relations in 1–8 are functions? Which are not? Explain.

Representing Relations II

A *relation* is a connection between an input and an output. The following table gives the toprated NFL quarterbacks in terms of the percentage (PCT) of their completed passes (as of 11/13/10). The measures used to find the percentages are passes attempted (ATT) and passes completed (COMP). Use the data to express the relations in a various ways.

NFL Quarterback Passing Statistics - 2010

PLAYER	TEAM	ATT	COMP	PCT
Drew Brees	Saints	374	261	70
Tony Romo	Cowboys	213	148	70
David Garrard	Jaguars	149	101	68
Eli Manning	Giants	271	178	66
Philip Rivers	Chargers	329	215	65
Peyton Manning	Colts	350	228	65
Matt Schaub	Texans	267	170	64
Tom Brady	Patriots	261	166	64
http://espn.go.com/nfl/s	tatistics/player/	/stat/pass	ing/sort/cor	npletionP



Player	PCT	
D. Brees		
T. Romo		70
D. Garrard		68
E. Manning		66
P. Rivers		
P. Manning		65
M. Schaub		64
T. Brady		

2. Ordered Pairs (Player, PCT Passes Complete)



3. Graph (Attempted passes, Completed passes)



4. Map the gender of your teachers to the subject 5. Map the subjects of your classes to the gender they teach

they teach.		of the teacher.					
Gender	Subject	Subject	Gender				

6. Make a table for each relation. You may need to look things up in an atlas, an almanac, or online.

State	Capital City
Connecticut	
Massachusetts	
Rhode Island	
Maine	
New Hampshire	
Vermont	

State	Ocean it Borders
Maine	
Oregon	
Virginia	
Florida	
Washington	
California	

7. List ordered pairs with famous actors as the inputs, and the movies they were in as the outputs.



- 8. List the ordered pairs (x, y) that are plotted on the graph to the right.
 - A (_____, ____) B (_____, ____)
 - C(_____, ____) D(_____, ____)
 - E(_____, ____) F(_____, ____)
- 9. Which of the relations in 1–8 are functions? Which are not? Explain.



Is it a Function?



1. Identify whether or not each relation is a function.

(a)	x	-2	-1	0	1	2	(b) x	-2	-1	0	1	-2
	У	0	5	6	0	3	У	4	-1	3	2	1

(c)

State	Maine	New	Vermont	Massachusetts	Rhode	Connecticut
		Hampshire			Island	
Capital	Augusta	Concord	Montpelier	Boston	Providence	Hartford





(f)	Tiger Woods	Football
	Eli Manning	Golf
	Serena Williams ——	 Tennis

(g) $\{(2, 3), (4, 3), (-1, 0), (6, 1), (-2, 8)\}$

(h) $\{(3, 4), (5, -2), (7, -1), (3, 3), (1, 5)\}$

Domain and Range	-
The domain of a function is	
The range of a function is	

2. For each function in Exercise 1, identify the domain and range.

Letter of the function	Domain	Range
A		
В		
C		
D		
Е		
F		
G		
Н		

3. Create your own function below and explain why it is a function.

4. Give an example of a relation that is not a function and explain why not.

- (a) (b) (d) (d) (d)
- 5. Identify whether or not each graph represents a function.

6. For each of the graphs in Exercise 5, find three ordered pairs and display them in a table.

(d)

(a)

x	у

(b)		
	x	у

(c)

x	у

x	у

- 7. You are at an altitude of 250 feet in a hot-air balloon. You turn the burner on high and rise at a rate of 20 feet per minute for 5 minutes. Your altitude *h* after you have risen for *t* minutes is given by the function h = 250 + 20t.
 - (a) Make a table to show the altitude as a function of the number of minutes you have traveled.
 - (b) Graph your data points—make sure to label the units on your axes.
 - (c) Does it make sense to connect the points on the graph? Explain.

# of minutes	Altitude (in feet)	350
0		300
1		250
1.5		200
2		150
3		100
4		50
5		

- (d) After 5 minutes, you turn the burner to low. This gives just enough heat to keep the balloon from falling but not enough to make it rise any higher. Plot a point on the graph to show how high the balloon is after 6 minutes.
- (e) Does it make sense to connect the point plotted in part (d) to the rest of the graph? Explain.
- (f) What is the domain of the function?
- (g) What is the range of the function?

- 8. As a scuba diver dives deeper and deeper into the ocean, the pressure of the water on his body steadily increases. The pressure at the surface of the water is 14.7 pounds per square inch (psi). The pressure increases at a rate of 0.445 psi for each foot you descend.
 - (a) Write an equation that represents the pressure P as a function of the depth d.
 - (b) Complete the table below. Show all of your work in the space below.

Depth (# of feet)	Pressure (psi)
0	
20	
40	
60	
80	
100	

(c) Make a graph of the function. Make sure you label your axes with the correct units.



(d) Describe in words why this is a function.

Name Relations and Functions				
Relations Expressed as Ordered Pairs Determine if the following relations are functions. Then state the domain and range.				
1. {(1, -2), (-2, 0), (-1, 2), (1, 3)}	2. {(1, 1), (2, 2), (3, 5), (4, 10), (5, 15)}			
Function:	Function:			
Domain:	Domain:			
Range:	Range:			
$3. \left\{ \left(17, \frac{15}{4}\right), \left(\frac{15}{4}, 17\right), \left(15, \frac{17}{4}\right), \left(\frac{17}{4}, 15\right) \right\}$	$4.\left\{ \left(-3,\frac{2}{5} \right), \left(-3,\frac{3}{5} \right), \left(\frac{3}{2}, -5 \right), \left(5,\frac{2}{5} \right) \right\}$			
Function:	Function:			
Domain:	Domain:			
Range:	Range:			

Relations Expressed as Graphing Write each of the following as a relation, state the domain and range, then determine if it is a function. 5. 6.



		j.	
-2	-2 -2		x

Relation:	Relation:
Domain:	Domain:
Range:	Range:
Function:	Function:

Name:	Date:	Page 13 of 29
7.	8. -2 0 2 • -2 •	
-2		н Нас
Relation:	Relation:	
Domain:	Domain:	
Range:	Range:	
Function:	Function:	

Relations Expressed as Mappings

Express the following relations as a mapping, state the domain and range, then determine if is a function.

9. {(-2, -1), (0, 3), (5, 4), (-2, 3)}

10. $\{(-1, 5), (0, 3), (2, 3), (3, -1)\}$

Domain:	

Range:		
	 _	

Function:

11.	{(-1,	7),	(0,	-3),	(1,	10),	(0,	7)}
-----	-------	-----	-----	------	-----	------	-----	-----

Domain:
Range:
Function:
12. $\left\{ \left(\frac{1}{2}, 2\right), \left(\frac{1}{4}, 2\right), \left(\frac{1}{8}, 2\right), \left(\frac{-1}{2}, 2\right) \right\}$

Domain:		

Range: _____

Function:

Domain:
Range:
Function:

. _ _ . .

Determine if the graph is a function, then state the domain and range.

13. 	14.
Domain:	Domain:
Range:	Range:
Function:	Function:
15.	16.
Domain:	Domain:
Range:	Range:
Function:	Function:
17.	18. $21 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + $
Domain:	Domain:
Range:	Range:
Function:	Function:

Bottled Water

Some cities have banned the use of bottled water. There are concerns about chemicals in the plastic containers leeching into the bottled water, the amount of trash generated by plastic bottles, and the fact that plastic bottles do not readily decompose. The cost to our society needs to be weighed against the convenience of bottled water. Bottled water is healthier than carbonated soft drinks and does not require refrigeration during storage.

In the April/March 2009 edition of the Bottled Water Reporter, the Beverage Marketing Corporation lists the amount of bottled water sold during the years 2000 – 2008.

Table 1 shows the number of gallons of bottled water sold during each year.

- Since for any one year there will be only one total volume, Table 1 describes a special type of relation called a
- 2. Write as a number how many gallons of bottled water were sold in 2000.
- 3. What trend do you notice over the nine years?
- 4. Looking at the data in Table 1, select a scale for the axes and label the graph.
- 5. Plot the ordered pairs.
- 6. Describe this graph.

Table 1
Bottled Water Sold Annually

Year	Billions of Gallons
Independent	Dependent
Variable	Variable
2000	4.7
2001	5.2
2002	5.9
2003	6.3
2004	6.8
2005	7.5
2006	8.3
2007	8.8
2008	8.7



Year	Revenue (Billions of \$)
Independent	Dependent
Variable	Variable
2000	6.1
2001	6.9
2002	7.9
2003	8.5
2004	9.2
2005	10.0
2006	10.9
2007	11.5
2008	11.2

7. Table 2 also describes a relationship. What quantities does it relate?

- 8. As you examine Table 2, notice that for each year the producers reported revenue in dollars. Why is this relationship also a function?
- 9. What trend do you notice over the nine years?
- 10. Looking at the data in Table 2, select a scale for the axes and label the graph below.
- 11. Plot the ordered pairs.
- 12. Do you think the table or the graph does a better job helping you to see trends in bottled water annual revenue over the nine year period? Why?
- 13. Write the revenue for 2008 as a number (without using the word "billion").
- 14. How did you decide how many zeroes and commas to put in your answer to #10?

ĺ					

Hartford Precipitation

A function is a	between two sets, a first set and a second set, where each			
element from the first set is paired	l with	element from the second set.		
The values we use for the first set	are called	and are the same as the		
variable. T	he values in the second set a	re often called		
and are th	e same as the	variable.		
The set of all possible inputs is ca	lled the	of the function.		
The set of all possible outputs is c	alled the	of the function.		
When we present a relation in a table the left column always contains the inputs, also called the				
	The depender	nt variable is always in the right		
column.				

- 1. Look at Table 1 below. Does the data in the table describe a function? Why or why not?
- 2. The independent variable is:
- 3. The dependent variable is:

Table 1				
Average Precipitation for				
Hartford, C	Connecticut			
Month	Inches			
1	3.66			
2	2.65			
3	3.61			
4	3.82			
5	3.99			
6	3.83			
7	3.93			
8	3.83			
9	3.83			
10	3.91			
11	3.79			
12	3.44			

4. Plot the data on the graph below.



Suppose we change the table. The independent variable and dependent variable have been interchanged. In the previous relation, the input is the month and the output is the average precipitation. In the new relation, the input is the average precipitation and the output is the month.

5. (a) Do we ever get nearly 4 inches of rain in a month?

(b) If so, in what month does that happen?

Usually we organize our tables so the values of the <u>domain</u>, the inputs, are listed in increasing order. We list an input only once. Fill in the data into Table 3.

- 6. Does Tables 2 and 3 represent a function? Explain.
- 7. Plot the data in (Table 2 or Table 3) on the graph below.



Table 2				
Average Precipitation				
for Hartford, CT				
Inches	Month			
3.66	1			
2.65	2			
3.61	3			
3.82	4			
3.99	5			
3.83	6			
3.93	7			
3.83	8			
3.83	9			
3.91	10			
3.79	11			
3.44	12			

Table 3				
Average l	Average Precipitation			
for Ha	rtford, CT			
Inches	Month(s)			
2.65				
3.44				
3.61				
3.66				
3.79				
3.82				
3.83				
3.91				
3.93				
3.99				

8. How does this graph differ from the graph of the data in Table 1?

Functions Everywhere - Identifying Independent and Dependent Variables

The	variable corresponds to the input value	S.	
The	variable corresponds to the output val	lues.	
We say that the	variable is a function of the	variable.	
This means that the dependent variable's value is dependent on the independent variable's value.			

Example: The pressure exerted by water on a diver increases as the diver's depth increases. The independent variable is *diver's depth*. The dependent variable is *pressure*. *Pressure* is a function of *diver's depth*.

Now consider the following scenarios and complete each sentence.

1.	The circumference of a circle increases when the radius of a circle is increased. The independent variable is
	The dependent variable is
2.	The height of liquid in a 50-gallon tank continues to decrease when it leaks all day.
	The independent variable is
	The dependent variable is
	is a function of
3.	World record times for each year that a record is set for running the 100-meter dash. The independent variable is
	The dependent variable is
	is a function of
4.	The median age of the U.S. population has been increasing since 1850. The independent variable is The dependent variable is
	is a function of
5.	The amount of points scored by a player depends upon the number of baskets she makes. The independent variable is The dependent variable is

 The amount of money spent on DVDs depends on the number of DVDs you buy. The independent variable is ______
 The dependent variable is ______

_____ is a function of ______

Often relationships are described verbally. Consider each of the following scenarios and provide a response and explanation in the space provided.

- 7. The independent variable is the time you ride in a car at 55 mph (using cruise control) and the dependent variable is the distance traveled. Is this relationship a function? Explain your response.
- 8. The independent variable is the cost of a taxable item and the dependent variable is the sales tax owed on the item. Is this relationship a function? Explain your response.
- 9. The independent variable is the gender of your teacher and the dependent variable is the subject they teach. Is this relationship a function? Explain your response.
- 10. The independent variable is the length of the side of a cube and the dependent variable is the volume of the cube. Is this relationship a function? Explain your response.
- 11. The independent variable is a student's age and the dependent variable is the number of siblings the student has. Is this relationship a function? Explain your response.
- 12. The independent variable is time and the dependent variable is the world population. Is this relationship a function? Explain your response.
- 13. The independent variable is the day of the year and the dependent variable is the time of low tide on that day of the year. Is this relationship a function? Explain your response.

High & Low Tides at Old Saybrook, CT					
November 2012					
Day Low High Low I				High	
Tuon 12	2:50	9:12	3:38	9:38	
Tues 15	AM	AM	PM	PM	
Wed 14	3:40	10:02	4:28	10:29	
weu 14	AM	AM	PM	PM	
Thung 15	4:31	10:55	5:20	11:24	
	AM	AM	PM	PM	

Function Machines

Here is a function machine: It takes one number as an **input** and produces another number as the **output**. A **function** is a rule that tells the machine what output to produce for any input.



Example 1: The doubling function. The rule here is that the output is twice the input.



Here are some examples of how this function works.



Questions on Example 1:

- 1. If the input of the doubling function is 5, what is the output?
- 2. Doubling (6) = ____?
- 3. Doubling (-9) = ____?
- 4. When the input of the doubling function is a positive number, what kind of number (positive or negative) is the output? _____
- 5. When the input of the doubling function is a negative number, what kind of number (positive or negative) is the output?

- 6. Another way of describing the doubling function is that the output is _____ times the input.
- 7. What is the domain of the doubling function? What is the range?
- 8. Make a table of values for the doubling function. Chose your own values for the last three inputs.

Input	Output
-9	
-4	
0	
3	
6	
7	

Example 2: The square function. The rule here is that the output is found by multiplying the input by itself.

Here are some examples of how this function works.



Questions on Example 2:

9. If the input of the square function is 5, what is the output?

- 10. Square (6) = ____?
- 11. Square (-9) = ____?
- 12. When the input of the square function is a positive number, what kind of number (positive or negative) is the output? _____

- 13. When the input of the square function is a negative number, what kind of number (positive or negative) is the output?
- 14. If the input of the square function is zero, what is the output?_____
- 15. If *x* is the input of the square function, what is the output?
- 16. What is the domain of the square function? What is the range?
- 17. Make a table of values for the square function. Chose your own values for the last three inputs.

Input	Output
-9	
-4	
0	
3	
6	

Function Notation:

We have observed that functions may be given names, such as "doubling" or "square." Sometimes we just use a letter of the alphabet for the function name. Since the word "function" begins with the letter "f" we most often use f. Sometimes we use g or h since they come after f in the alphabet.

Since the input and output of the function may take on many values they are **variables.** The input is sometimes called the **independent variable** and the output the **dependent variable**. (Here's how to remember this: the output **depends**

upon the input.)

The letter x is often use for the input (independent) variable. The letter y is used for the output (dependent) variable. Thus, functions in general may be represented by this machine.



Example 3

Functions may be defined by algebraic rules. Suppose f(x), g(x), and h(x) are defined as follows:

f(x) = x + 10 g(x) = 3x h(x) = 4x - 5

Questions on Example 3:

- 18. For the function *f*, when the input is 6, the output is _____.
- 19. For the function g, when the input is -5, the output is _____.
- 20. For the function *h*, when the input is 2, the output is _____.
- 21. Find *f*(−3).
- 22. Find *g*(7).
- 23. Find *h*(7).
- 24. Fill in the tables below:

x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	

x	g(x)
-3	
-2	
-1	
0	
1	
2	
3	

x	h(x)
-3	
-2	
-1	
0	
1	
2	
3	

25. Describe any patterns you see in the table above.

26. Does f(x) mean "*f* times x"? Explain.

Introduction to Function Notation

Jane works as a travel agent. Every week she gets paid \$900 (her base salary) plus \$100 for each cruise that she books (her commission). We can use a function to describe the relationship between her weekly salary and the number of cruises that she books.

1. Fill in the blanks below:

Jane's	depends on	, and every
number of	gives only one	•
Therefore, we can say her	is a function of	

- 2. Write a recursive rule to describe Jane's weekly salary.
- 3. Let **S** be her weekly salary, and **c** be the number of cruises she books. Write an explicit rule to model Jane's weekly salary based on the number of cruises she books.
- 4. Jane's weekly salary is a function of number of cruises she books. We can use a special notation called function notation to label it. Instead of writing S alone as we did in the explicit rule, we write S(c). Write S(c) below.
- 5. Let's look at the parts of that notation, *S*(c), which is read "S of c." Fill in the table below.

S is the:		c is the:	
S is the	_variable	c is the	_variable
The parentheses hold the place for the input value, they do not signify multiplication!			

6. Complete the table below.

с	S(c)
0	
1	
2	
3	
4	
5	

7. Graph the function on the axes below. Label the axes appropriately.



8. Fill in the mapping diagram below.

0	1100
1	900
2	1400
3	1000
4	1200
5	1300

9. What are the domain and range of this function?

Hot Air Balloon

The graph below shows the altitude during a hot air balloon ride with Berkshire Balloons. The altitude of the hot air balloon is a function of time.



1. Find the domain and range of the function graphed above.

Domain:

Range:

- 2. Find f(30) and explain what it means in the context of the problem.
- 3. If f(x) = 100, find all values of x and explain what they mean in the context of the problem.
- 4. When is the balloon at 200 feet?
- 5. For how long are you flying at an altitude at or above 200 feet?
- 6. If f(x) = 300, find all values of *x*.

Function Applications – Highway Driving

Leon went to his Grandma's home for Thanksgiving. His dad set the cruise control at 60 miles per hour while they drove on the highway. They travelled on the highway for 4 hours. **Create a function that models the distance traveled on the highway**, *d* (*in miles*), **after they have driven on the highway for** *t* **hours**.

(a) Independent variable:

(i) Complete the table below.

$(b) \Sigma$	Dependen	t variable	•

- (c) Write the equation for this function.
- (d) Use function notation to express the function.
- (e) We can say _____ is a

function of ______.

- (f) Find the distance Leon's family travelled after driving for 3.2 hours on the highway. Use function notation.
- (g) Find the time it took for Leon's family to travel 175 miles on the highway.
- (h) What are the domain and range of this function?
- (k) Identify the shape of this graph using the Parent Function Reference Sheet.

Input	Output

(j) Graph the function on the axes below. Scale and label the axes.

 Image: selection of the se

Function Applications – Free Throws

Ben is trying out for the school's Basketball team. He has been shooting free throws after school for an entire week, trying to get better. He shot 100 free throws on the first day, and then each day after that he shot 10 more free throws than the day before. **Create a function that models** the number of free throws f taken each day according to the day number d.

(a) Independent variable:

(i) Complete the table below.

- (b) Dependent variable:
- (c) Write the equation for this function:
- (d) Use function notation to express the function:
- (e) We can say _____ is a

function of ______.

- (f) Find the number of free throws Ben shot on day 4. Use function notation.
- (g) Find the day that Ben shot 160 free throws.

Input	Output

(j) Graph the function on the axes below. Scale and label the axes.

- (h) What are the domain and range of this function?
- (k) Write a sentence to express the meaning of the following equation. f(0) = 100
- (1) Identify the shape of this graph using the Parent Function Reference Sheet.