Algebra I

Unit 2: Equations and Inequalities

Supplemental Integer Arithmetic
Supplemental Order of Operations
Supplemental Simplifying Expressions
Quiz – Prerequisite Skills

2.1.2 Representing Expressions with Stories and Flowcharts
2.1.3 Representing Expressions with Algebra Arrows
2.1 Exit Slip

2.2.1 Solving Linear Equations with Flowcharts
2.2.3 Solving One-Step Equations
Supplemental Two-step equations (WS 2.3B)
2.2.6 Stations Activity
2.2.7 Solving Two-Step Equations
2.2 Exit Slip
Quiz – Solving One and Two Step Equations

2.3.2 Solving Equations that Contain Like Terms
2.3.3 Solving Equations with Variables on Both Sides
2.3.1 Exit Slip
2.3.4 Practice Solving Equations
2.3.6 How Many Solutions?
2.3.2 Exit Slip

Mid-Unit 2 Test

2.4.1 Solving Problems using the Distributive Property
2.4.1 Exit Slip
2.4.3 Using the Distributive Property
2.4.2 Exit Slip
2.4.4 Walk-A-Thon
2.4.5a Practice Solving Equations
2.4.6a Pizza Party
2.4.7 Multi-Step Equation Challenge
2.4.3 Exit Slip
2.4.8 Fraction Busters
Quiz – Solving Multi-Step Equations

2.5.3 Transforming Literal Equations with Flowcharts
2.5 Exit Slip
Supplemental Literal Equations
2.5.2 More Literal Equations
Quiz – Literal Equations

2.6.1 Representing Inequalities
2.6.2 Equations and Inequalities
2.6.1 Exit Slip
2.6.4 Working with Inequalities
2.6.5 Practice Solving Inequalities
2.6.8 Inequalities in the Real World
2.6.2 Exit Slip

Unit 2 Test

Name_____ ID: 1

Date_____ Period____

Integer Operations ALL

Find each product.

1) $8 \cdot -8$

2) $6 \cdot -2$

3) $-9 \cdot 3$

4) $-8 \cdot -9$

5) $10 \cdot -7$

6) $10 \cdot -5$

Find each quotient.

7) $-15 \div 3$

8) $-81 \div -9$

9) $56 \div 8$

10) $-16 \div -8$

11) $-81 \div 9$

12) $30 \div 5$

Evaluate each expression.

13) $4 - 7$

14) $(-1) + 4$

15) $6 - 8$

16) $(-4) - (-8)$

17) $5 - (-8)$

18) $6 - 3$

19) $5 + (-8)$

20) $4 - (-3)$

21) $3 - 5$

22) $(-6) - (-2)$

23) $7 + (-5)$

24) $(-2) - (-3)$

25) $(-8) - (-3)$

26) $7 - 1$

27) $7 + (-4)$

28) $(-2) + (-1)$

29) $7 + (-1)$

30) $6 - 2$

31) $(-6) + (-5)$

32) $7 - (-8)$

33) $(-5) - 7$

34) $8 - 1$

35) $1 - 8$

36) $(-4) - (-3)$

37) $8 - (-2)$

38) $(-2) - 2$

39) $(-6) + 3$

40) $(-2) + 4$

41) $8 - (-7)$

42) $(-7) + (-6)$

43) $(-4) + 1$

44) $(-5) + 1$

45) $(-6) - (-6)$

46) $(-8) - (-7)$

47) $(-2) + 7$

48) $(-1) - 4$

49) $(-2) - (-1)$

50) $7 - 2$

Integer Order of Operations Worksheet

All work must be shown for credit.

1. $6 - 15 \div 3$

2. $-10 \div 2 + 1$

3. $3(4 - 7) - (-6)$

4. $1 - (9 - 4) \div 5$

5. $7 - (-2)^3$

6. $(-2)^3 - (-5)$

7. $2(-6 + 2) \div 4$

8. $7 - 3(4 - 5)$

9. $8 - (-4)^2 - 5$

10. $-7 + 1^2 + 2$

11. $-3^3 - 6(-2) - 2$

12. $5 \cdot 3 - (-3)^3$

13. $-8(2 - 6) \div 2$

14. $4(6 - 9) \div 6$

15. $-8(2 - 5) \div (-4)$

16. $8 - 3 \cdot 2 - 33 \div 11$

17. $9 - 3(6 \div 2)$

18. $(-3)^2 - (-2)^2 - 1$

19. $7 \cdot 2 - 5 \cdot 3$

20. $20 \div 4 - 14 \div 2$

21. $2^3 - 6 \cdot 2 + 3$

22. $(-3)^2 \cdot (5 - 7)^2 - (-9) \div 3$

23. $1^3 - 6 \div (-3)$

24. $4 \cdot 5 - 10 - 2(1 - 2) + 5$

25. $(-1) \cdot (2-6)^2 \div 8 + 8 - 3 \cdot 4$ **26.** $5 - (-3)^2 - 6$ **27.** $10 \div 5 - (-2)^2$

28. $20 - 2 \cdot 7 + 1 - (-3) + 10$

Given $w = -1$, $x = 6$, $y = 3$, and $z = -2$; evaluate the following:

29. $4w + 2y$ **30.** $x - 3(-z)$ **31.** $xy \div z$

32. $9z \div x$ **33.** $x^2 - y^2$ **34.** $y^2 - z^2$

35. $\dfrac{2x+y}{z+w}$ **36.** $\dfrac{3x-z}{-w}$ **37.** $\dfrac{x+w}{y-z}$

38. $\dfrac{xy}{z} \div w$ **39.** $(-x+z)^2 \div 8$ **40.** $(y+z)^2 + (w-x)^2$

Algebra 1   Name_____ ID: 1

Simplifying Expressions

Date_____ Period____

Simplify each expression.

1) $-2k + k$

2) $x - 6 + 2x - 5$

3) $8n - 1 - 2 - 3n$

4) $4r + 5 + 2r$

5) $r - 1 + 10r$

6) $3n + 3n$

7) $4 - 3x + x + 3$

8) $-6a - 7a$

9) $-2 - 4p + 7p$

10) $1 - 10n - 8$

11) $9b - 8 + 3b$

12) $5x - 8 - 7 + 6x$

13) $-5a - 10a$

14) $-6k + 7k$

15) $a + 2 - 1$

16) $-5 - 10v - 10$

17) $-7n + 7 + n - 2$

18) $-7n - 5n$

19) $-9a + a$

20) $-7p - 9p$

Representing Expressions with Stories and Flowcharts

When we think of stories we usually don't think of algebra, but mathematical expressions tell stories too! When we see an expression involving a variable, something is happening to that variable. In other words, something is *being done* to the variable. Let's first use a story to represent what is being done to the variable. Use the order of operations to decide what steps are taken to evaluate the expression.

Expression	Story of x
$x + 6$	
$x/3$	
$8x$	
$x - 5$	
$2x + 6$	
$-6x + 3$	
$\dfrac{x}{2} - 4$	
$\dfrac{x + 5}{2}$	
$8 - x$	
$\dfrac{-7x + 2}{8}$	
$\dfrac{2x + 7}{3}$	
$\dfrac{3 - x}{5}$	

We can also represent the *story of x* by a *flowchart*. The flowchart below displays the story of x for the expression $6x + 5$.

| x | ⇒ | **Multiply by 6** | ⇒ | **Add 5** |
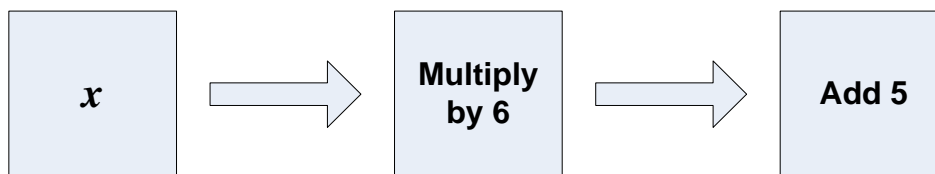
1. Use a flowchart to represent the following expressions involving two operations:

    A. $7x + 1.75$

    | x | ⇒ | | ⇒ | |

    B. $x/3 + 12$

    | x | ⇒ | | ⇒ | |

    C. $5x - 11$

    | x | ⇒ | | ⇒ | |

    D. $-x + 5$

    | x | ⇒ | | ⇒ | |
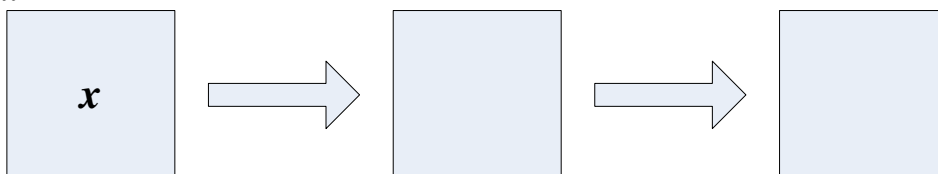
    E. $-5 - 3x$

    | x | ⇒ | | ⇒ | |

2. Use a flowchart to represent the following expressions involving three operations:

A. $\dfrac{2x + 5}{6}$

x ⇒ ⬜ ⇒ ⬜ ⇒ ⬜

B. $\dfrac{-3x + 5}{11}$

x ⇒ ⬜ ⇒ ⬜ ⇒ ⬜

C. $\dfrac{4x - 0.5}{-3}$

x ⇒ ⬜ ⇒ ⬜ ⇒ ⬜

D. $\dfrac{6 - 2x}{5}$

x ⇒ ⬜ ⇒ ⬜ ⇒ ⬜
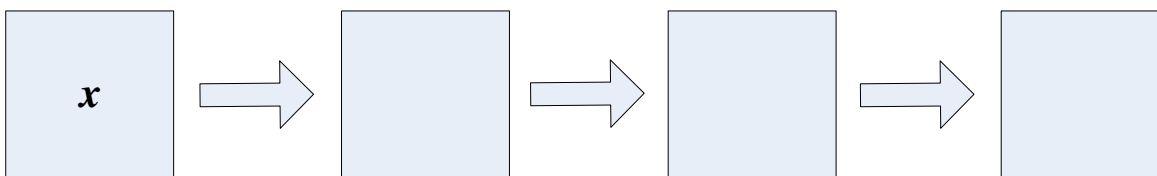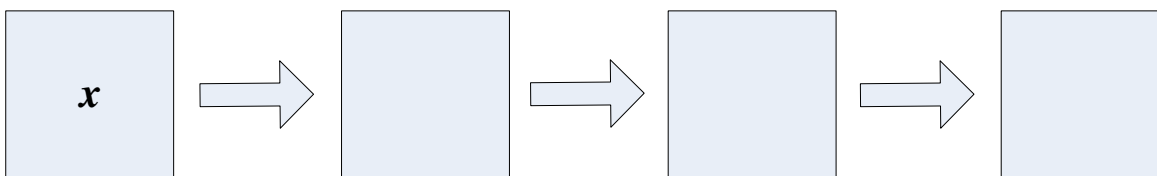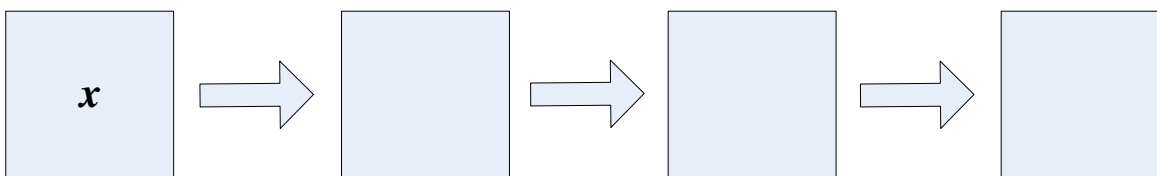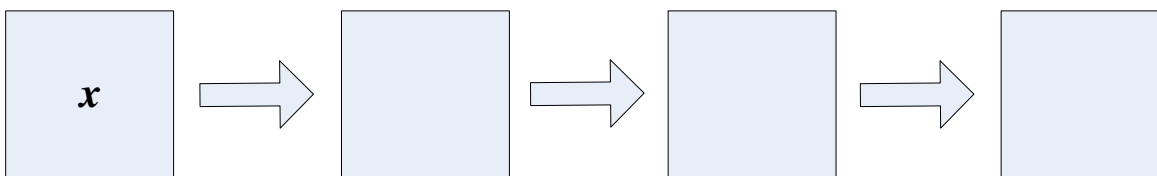
3. Let's now convert stories to mathematical expressions. Given the story on *x*, write a mathematical expression that describes the story. Use *x* to represent the unknown number.

   A. Multiply a number by 7.

   B. Add 14 to a number.

   C. Subtract a number from 12.

   D. Subtract 6 from a number.

   E. Divide a number by -3

   F. Divide -8 by a number.

   G. Multiple a number by 4, then add 3.

   H. Multiply a number by 8, then add −11.

   I. Subtract 4 from a number, then multiply by 6.

   J. Add -1 to a number, then divide by 3.

   K. Divide a number by 2, then subtract 13.

Representing Expressions with Algebra Arrows

Write the "story of *n*" for each of the following algebraic expressions. Carefully identify the order in which the operations occur on the variable term. Then build each story using the *Algebra Arrows* applet on the Freudenthal Institute website*. Sketch a chain of arrows for each expression and evaluate each expression when *n* = 3.

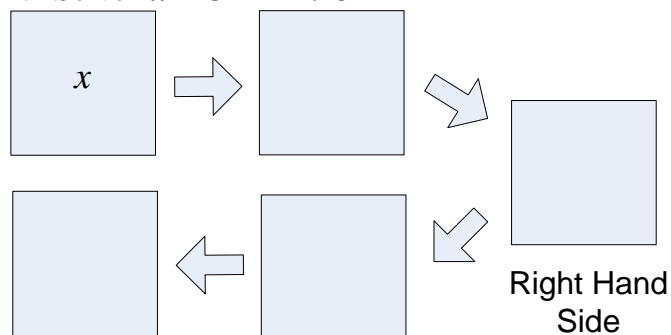| Expression	Story of *n*	Algebra Arrows Chain	Evaluate when *n* = 3
$3n + 7$			
$\dfrac{n}{3} - 10$			
$2(n - 5)$			
$\dfrac{n - 8}{4}$			
$7(n^2 - 1)$			
$6n^2 + 2$			
$(n + 3)^2 - 6$			
$\dfrac{3n^2 + 2}{5}$			

* http://www.fi.uu.nl/wisweb/en/

Fill in the expression created by each algebra arrow chain, and then tell the story of *x*. Then evaluate the expression when *x* = 5.

Algebra Arrows Chain	Story of *x*	Evaluate when *x* = 5
x → x2 → - 7.5 → ☐		
x → + 1 → / 4 → ☐		
x → ...² → - 9 → ☐		
x → + 3 → x 8 → ☐		
x → / 5 → - 6 → ☐		
x → x 2 → ...² → ☐		
x → ...² → / 5 → + 3 → ☐		
x → - 8 → / 3 → + 4 → ☐		
x → x 3 → + 1 → ...² → ☐		

Solving Linear Equations using Flowcharts

You will now use flowcharts to solve one-step and two-step linear equations. Apply the corresponding steps to the equation on the right side. Check your solution.
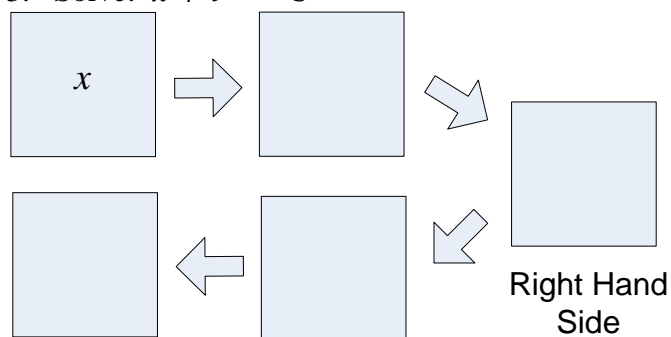
1. Solve $x - 5.4 = 19.8$

$$x - 5.4 = 19.8$$

x →

← ← Right Hand Side

Check:

2. Solve: $x/4.6 = 3.5$

$$x/4.6 = 3.5$$

x →

← ← Right Hand Side
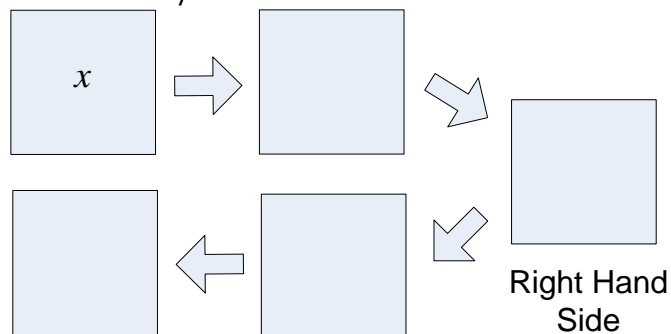
Check:

3. Solve: $x + 9 = -5$

$$x + 9 = -5$$

x →

← ← Right Hand Side

Check:

4. Solve $3x + 4 = -11$

$$3x + 4 = -11$$

x			

Right Hand Side
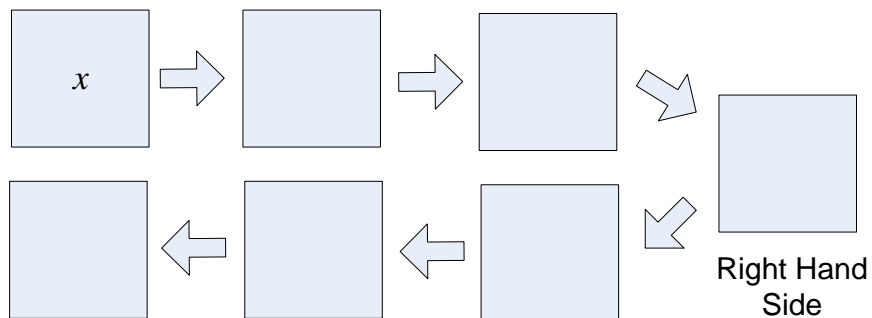
Check:

5. Solve $2x - 8 = 14$

$$2x - 8 = 14$$

x			

Right Hand Side
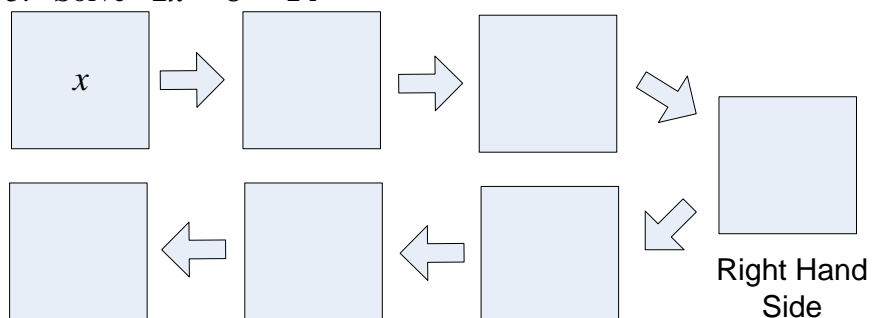
Check:

6. Solve $-2x + 12 = -20$

$$-2x + 12 = -20$$

x			

Right Hand Side
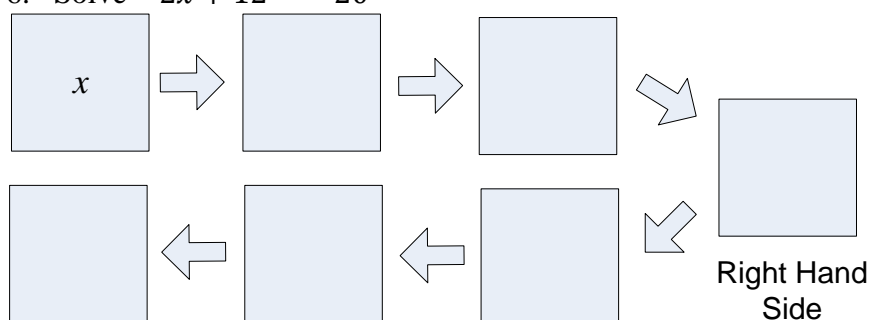
Check:

Model the situations below with a linear equation. For each problem, identify the unknown, create an equation, solve the equation using a flowchart, and then check your solution.

7. Kevin bought seven tickets to the Haunted Graveyard at Lake Compounce for $209.93. How much does one ticket cost?
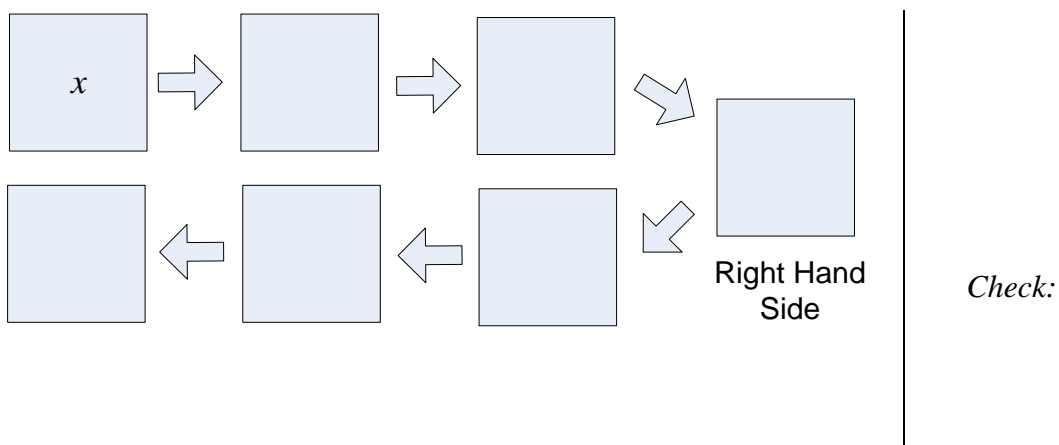
Identify the unknown:

Equation:

x

Right Hand Side   *Check:*
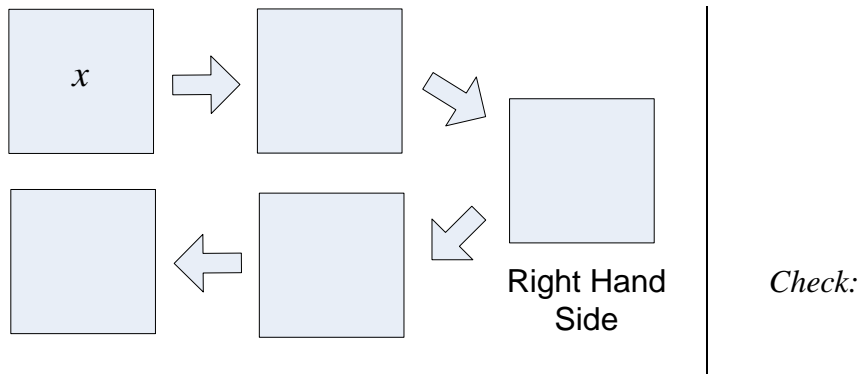
8. Verizon charges $18.75 per month for phone service and $0.08 per minute. Last month my bill was $33.63. How many minutes did I use?

Identify the unknown:

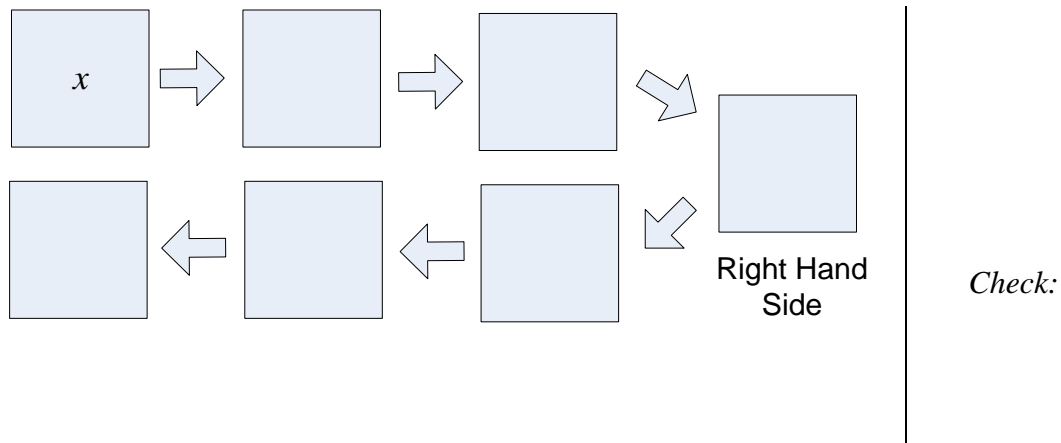Equation:

x

Right Hand Side   *Check:*

9. Jose spent $177.69 of his birthday money. He bought an iPod for $159 and 21 songs from iTunes. How much did each song cost?
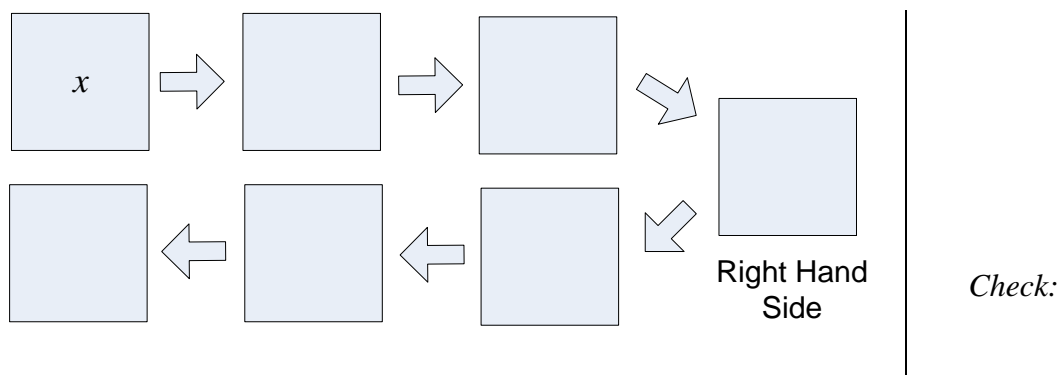
Identify the unknown:

Equation:

x	→		→		→		

	←		←		←	Right Hand Side

Check:

10. Your school band needs to buy new recording equipment. The equipment will cost $3000. The band has collected $1200 from previous fundraisers. If the band sells sandwiches at $5 each, how many sandwiches must they to sell to raise the remaining funds?

Identify the unknown:

Equation:

x	→		→		→		

	←		←		←	Right Hand Side

Check:

Solving One-Step Equations

Solve each problem by defining the unknown, writing an equation, solving it, and checking your answer. Show your work!

Problem	Identify the unknown	Write and solve an equation	Check it
Four more than a number is negative six. Find the number.			
The product of a number and 2.5 is 375. Find the number.			
A number divided by negative six is five. Find the number.			
Three less than a number is negative ten. Find the number.			
A number divided by $\frac{2}{3}$ is $\frac{15}{16}$. Find the number.			

Problem	Identify the unknown	Write and solve an equation	Check
A corn stalk grew 3.5 inches in one week in July. If the stalk was 42.1 inches at the end of the week, how tall was it at the start of the week?			
A box of 24 candles is packed in rows, and the rows are stacked three deep. How many candles are in each row?			
Beth ran a 400 meter dash in 57.2 seconds. This was 2.4 sec faster than her previous time. What was her previous time?			
Four friends share the cost of a meal equally. If each pays $12.50, what was the cost of the meal?			
Dan bought 6 boxes of candy for his teachers at holiday time. If he spent $41.70, how much did each box cost?			

Algebra I   Name _____ ID: 1

Two step equations

Date _____ Period ____

Solve each equation.

1) $7 + \dfrac{x}{2} = 4$

2) $5 - 4n = 45$

3) $\dfrac{x}{4} + 8 = 6$

4) $\dfrac{x}{-4} - 10 = -14$

5) $-4 + \dfrac{p}{-4} = -5$

6) $2 + \dfrac{x}{14} = 3$

7) $2 - 3n = -46$

8) $-4 + 8x = 4$

9) $\dfrac{a}{4} - 2 = -5$

10) $-6x + 7 = -77$

11) $-4 - 4n = 28$

12) $-9 + \dfrac{k}{3} = -13$

13) $-2 + \dfrac{r}{-10} = -3$

14) $-8 + \dfrac{m}{2} = -11$

15) $6 + \dfrac{v}{-3} = 10$

16) $-8 + 3b = -32$

17) $-5 + 2m = 17$

18) $10m + 6 = 186$

19) $10 + \dfrac{b}{5} = 12$

20) $2 + 2k = 20$

Solving Two-Step Equations

In the equation $4x - 5 = 23$, the "story of x" contains the operations done to x to get 23 (following the order of operations). In this activity, you will work backwards to solve each equation. When working backwards, you will perform an inverse operation to undo each operation in the story of x. The operation done first in the story of x gets undone last. Remember that what you do to one side of the equation must be done to the other side!

Equation	Story of x	Work it backwards	Solution	Check
$4x - 5 = 23$	Multiply by 4, then subtract 5.	Add 5, then divide by 4.	$4x - 5 + 5 = 23 + 5$ $4x = 28$ $\frac{4x}{4} = \frac{28}{4}$ $x = 7$	$4(7) - 5 = 23$ $28 - 5 = 23$ $23 = 23$
$\frac{x}{3} + 8 = -2$	Divide by 3, then add 8.	Subtract 8, then multiply by 3.	$\frac{x}{3} + 8 - 8 = -2 - 8$ $\frac{x}{3} = -10$ $(3)\frac{x}{3} = (3)(-10)$ $x = -30$	$\frac{(-30)}{3} + 8 = -2$ $-10 + 8 = -2$ $-2 = -2$
$-3x + 6 = 24$				

Equation	Story of x	Work it backwards	Solution	Check
$\dfrac{2}{3}x - 1 = 19$				
$\dfrac{x + 3}{-2} = 10$				
$-14 + 5x = 31$ Rearrange:				
$25 - \dfrac{x}{4} = 49$ Rearrange:				

Complete the following chart, filling-in the missing steps in the process of solving two-step equations.

Equation	Story of x	Work it backwards	Solution	Check
$-2x + 5 = 37$				
	Divide by 4, then add 7, to get 12			
$3x - \dfrac{2}{3} = \dfrac{1}{6}$				
	Multiply by -5, then add 6, to get -9			

Equation	Story of x	Work it backwards	Solution	Check
			$(9)\dfrac{x-3}{9} = (9)(-2)$ $x - 3 = \text{-}18$ $x - 3 + 3 = \text{-}18 + 3$ $x = -15$	
$7.5 + 3.1x = 32.3$ Rearrange:				
$-17 + \dfrac{x}{7} = 4$ Rearrange:				

Solving Equations that Contain Like Terms

1. Two computer technicians are upgrading software on 51 computers in a school. On average, Marissa upgrades 5 computers in 1 hour and Ryan upgrades 7 computers in 1 hour. We want to know how long it will take for both of them to upgrade the 51 computers. We can use several strategies to solve this problem.

    a) What is unknown in this problem?

    b) Complete the table that shows the number of computers upgraded by each technician and the total number of computers upgraded after 0, 1, 2, 3, 4, and 5 hours.

Number of Hours	Number of Computers Marissa Upgraded	Number of Computers Ryan Upgraded	Total Computers Upgraded
0			
1			
2			
3			
4			
t			

    c) Make a graph that represents the total number of computers upgraded after each hour.
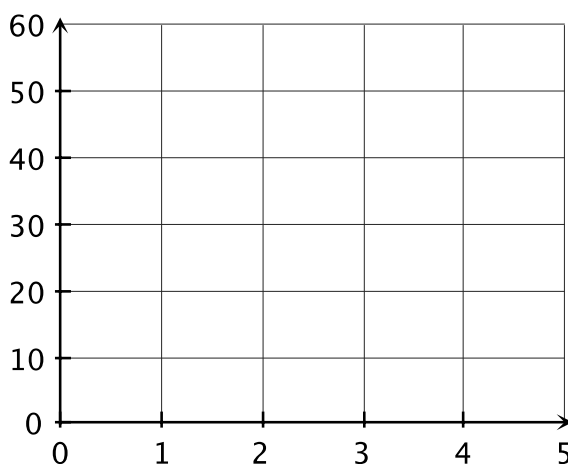
    d) Use the graph to estimate the number of hours it took to upgrade all the computers.

e) Let *t* equal the number of hours that Marissa and Ryan upgrade computers. Write an expression for the total number of computers upgraded in *t* hours.

f) Write an equation to determine the amount of time it will take Marissa and Ryan to upgrade 51 computers. Then solve the equation.

g) Write your answer in a sentence.

h) In your own words, why do you think we were able to combine the technicians' work?

2. A skateboard park charges $7 per session to skate and $4 per session to rent safety equipment. Jared skates safely and rents safety equipment every time he skates. He bought a new skateboard for $125 in the spring. During the year, he spent $224 for his skateboard, skating charges, and equipment rentals. How many skating sessions did he attend?

    a) Define your variable(s).

    b) Write an equation that can be used to find the number of sessions Jared attended.

    c) Solve the equation. List your steps and show your work.

    d) Write your answer in a sentence.

    e) When Jared's friend Rocco tried to solve this problem he took the following steps.

    $$7s + 4s + 125 = 225$$
    $$136s = 224$$
    $$s = 1.65$$

    Rocco concluded that since Jared can't skate 0.65 times, he skated once.

    Explain to Rocco what he did wrong and why it is mathematically illegal. Also explain what he should he have done to solve the problem. Write your explanation below.

    Dear Rocco,

3. The crazy game show called "Mud-Tower" begins with a pile of mud that weighs 20 pounds. The Blue-Team has to add mud to that pile, while the Red-Team has to remove mud from the same pile at the same time. The Red-Team is scooping-away an average of 0.75 lbs of mud every second. The Blue-Team is piling-on an average 0.55 lbs of mud every second.

   a) Write an expression that represents the final weight of the mud pile based on how many seconds the two teams have been competing.

   b) When the game ended there was a total of 10 pounds of mud left in the pile. Write and solve an equation to determine the number of seconds it took for the game to finish. List your steps, show your work, and write your answer in a complete sentence.

   c) Rocco tried to solve this problem he took the following steps.

$$20 - 0.75s + 0.55s = 10$$
$$20 - 1.3s = 10$$
$$-1.3s = -10$$
$$s = 7.69$$

   Rocco concluded that the game lasted 7.69 seconds.

   Explain to Rocco what he did wrong and why it is mathematically illegal. Explain what he should have done to solve the problem.

   Dear Rocco,

4. Julie purchased tickets to a Mets baseball game from the Mets website. Each ticket costs $19 and the website charged a convenience fee of $5.75 per ticket. To celebrate going to the game, Julie also bought a new jersey from the website for $60.

a) Write an expression that represents the total amount of money she spent on the website based on the number of tickets she bought. First define your variable.

b) If her bill came to $159.00, how many tickets did she buy? Write and solve an equation, list your steps, show you work, and then write your answer in a complete sentence.

Solving Equations with Variables on Both Sides
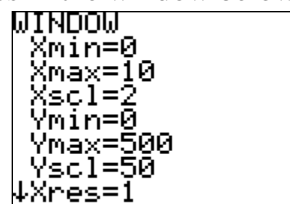
1. You are looking for a dog spa to care for your dog. Each time you visit a spa, you ask them to give your dog a bath. There are two companies you are considering. Golden Dog Care charges $20 for each visit plus $5 for each bath. Super Dog Delight charges $15 for each visit and a $60 fee for an unlimited number of baths. You need to know how many visits will make the two companies charge the same amount.

   a) Write an expression for the amount that Golden Dog Care will charge for *n* visits.

   b) Write an expression for the amount that Super Dog Delight will charge for *n* visits.

   c) Write an equation which determines when these companies charge the same amount.

   d) We can solve this equation in three different ways. Let's first use a **table**. Complete the table below.

Number of Visits	Golden Dog Care	Super Dog Delight
0		
1		
2		
3		
4		
5		
6		

   e) What are some disadvantages to using a table to find the solution of an equation?

f) Now solve the equation using the **properties of equality**. List your steps, and show all of your work. Check your solution.

g) Now solve the equation by making a **graph** on your graphing calculator. Set your graphing window using the values in the window below:

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=2
 Ymin=0
 Ymax=500
 Yscl=50
↓Xres=1
```

In the Y= menu, enter the left side expression into Y_1 and the right side expression into Y_2. Press GRAPH and find the point where the two lines intersect.

h) What are some disadvantages to using a graph to find the solution?

2. Dennis is collecting aluminum cans to raise funds for a local animal shelter. He needs to determine whether he should return the cans to the local supermarket or return the cans to the recycling center. If he brings the cans to the supermarket, he receives 5 cents per can. If he brings the cans to the recycling center, he receives 6 cents per can but is forced to pay a $15 recycling fee. How many aluminum cans would he have to return to receive the same amount of money at the supermarket and the recycling center? Write an equation that describes this problem, and then solve it.

3. Create your own problem by filling in the blanks below.

> A **membership** to a rock-climbing gym allows you to climb as much as you want for a fee of $_____ but you also pay $_____ per day for equipment rental. **Nonmembers** pay $_____ per day to use the gym and $_____ per day for equipment rental**.** Find the *number of days* in which the total cost for the members and nonmembers *are the same*.

> Write an equation that solves the problem you created above. Then solve the equation and check your solution.

Practice Solving Equations

Solve the following equations. Show your work below each equation. Check your solution.

1. $4c + 8c = -55 + 3c$

2. $4f - 24 + 4f = -8$

3. $5w - 7 = 2w + 1$

4. $x + 6x + 49 = 5x + 59$

5. $40 + 14y = -20 - 8y - 6$

6. $3 + 5z - 1 = -4z + 3 + 10z$

How Many Solutions?

A linear equation can have one solution, no solution, or an infinite number of solutions. Solve each multi-step equation and identify the number of solutions.

1. $4x + 5 - x = 10 + 3x + 4$

2. $x + 8 = 15 + 4x - 7$

3. $6x - 3x + 9 = 2 + 8x$

4. $10 - x - 6 = 9x + 4 - 10x$

5. $x + x + 3 = -3 + 2x + 6$

6. $7 - 2x - 3x = 10 - 5x$

7. In solving a multi-step equation, a student ended up with $0x = 4$. She decided to divide both sides of the equation by 0 and got $x = 0$. How would you convince her that she has made a mistake?

8. What values of x satisfy the equation $0x = 0$?   Explain your reasoning.

Solving Problems using the Distributive Property

Class description of the **Distributive Property**:

When you distribute, you _____ a term _____ of the grouping symbols by each term inside the grouping symbols.

Formal definition:

1. You and three friends go to the local fair. You each buy a $3 food ticket and a stamp for unlimited rides. If the total cost for the four of you is $32, how much does the stamp for unlimited rides cost?

   a) What is the unknown cost in the problem?

   b) What is the known cost in the problem?

   c) Write an expression showing the cost for one person.

   d) Write an expression showing the total cost for everyone.

   e) The total cost equals $32. Write an equation that models this situation.

   f) Now we want to solve this equation to find the cost of the stamp. If you followed the steps correctly, your equation should have a pair or parentheses in it. Up until now, we have not solved any equations with parentheses. How we are going to get rid of the parentheses? Explain below.

g) Use the distributive property to eliminate the parentheses in the equation.

h) Now solve the equation. Show your work and circle your answer.

2. Mr. Matthews organized a field trip to Lake Compounce for his ninth grade students who had perfect attendance. He bought the admission ticket for each student, plus a gift for each student. The gifts for the girls cost $5 each and the gifts for the boys cost $4 each. 12 boys and 15 girls attended the field trip. The total cost of the tickets and gifts was $798. After Mr. Matthews spends the money, the Principal demands to know much each admission ticket cost. Mr. Matthews has never been good at math, so he needs your help. How much did each ticket cost?

a) Write an expression modeling the cost for just the girls.

b) Write an expression modeling the cost for just the boys.

c) Using the two expressions, write an expression modeling the total cost.

d) Use the distributive property to simplify the expression.

e) Write an equation that you can use to find the ticket cost. Solve the equation and check your solution.

3. Your bank charges a monthly fee of $2.25 for your checking account and an additional $1.25 for each transaction you make with your debit card. Your May bill is for $13.50. How many transactions did you make with your debit card in May?

    a) Assign a variable for the number of transactions that you made in May.

    b) Write an equation to find the number of transactions you made in May.

    c) Solve the equation. Does your answer seem reasonable? Why or why not?

4. Because you always pay your monthly bill on time, the bank says that in June, the first two transactions that you make with your debit card will not be charged a $1.25 fee.

    a) Let x equal the number of transactions you made in June. Write an expression for the number of transactions that you will be charged for in June.

    b) Now write an expression for the cost of the transactions in June.

    c) Suppose that your June bill is $13.50. Write an equation to find the number of transactions you made in June.

    d) Solve the equation. Does your answer seem reasonable? Why or why not?

5. Something went wrong in July! Your monthly fee increased to $2.45 and you are again being charged for each transaction. Your July bill is $8.95, and you know that you only made five transactions. What were you charged for each transaction? Write an equation and solve.

6. The bank wants to keep your business, so they change their policy. In August, your first two transactions are not charged a fee. You suspect that the transaction fee changed. Your monthly fee is $2.45, and your August bill is $11.65. You know that you made 10 transactions. What were you charged for each transaction? Write an equation and solve.

Using the Distributive Property

If three people buy the same laptop computer and same phone (from the same store at the same time), the total cost for the three people can be represented in two different ways. Since the total cost is unique, the two *representations* must be equal.

$$Total\ Cost = Cost\ of\ 3\ laptops + Cost\ of\ 3\ phones$$
or
$$Total\ Cost = 3\ (Cost\ of\ a\ laptop + Cost\ of\ a\ phone)$$

This idea can be applied to algebraic expressions. Recall that multiplication can be thought of as repeated addition, so $5(x + 3)$ means there are five groups of $(x + 3)$. So, there are five x's and five 3's, so all together, there is $5x + 15$. This is represented below.
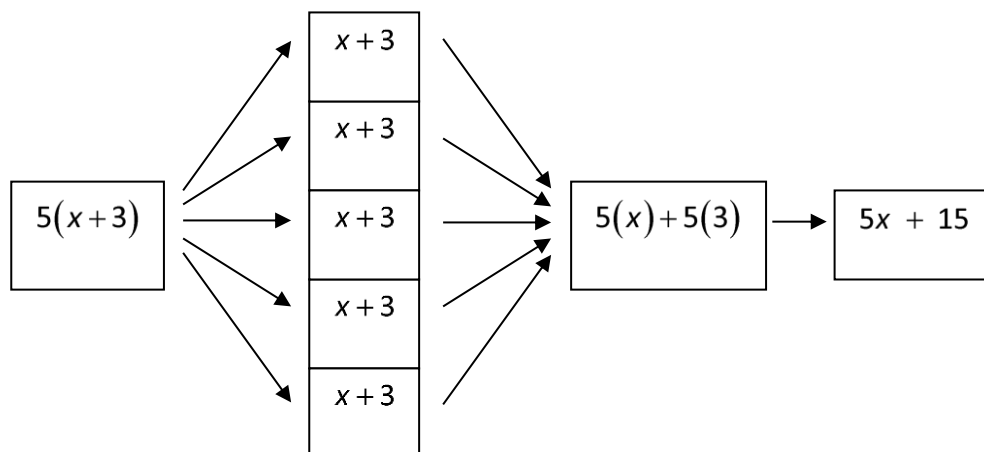
Write equivalent expressions without parentheses:

1. $3(a + 6)$ 2. $5(n - 4)$ 3. $4(c + d)$

Do you notice a pattern? In each case, you end up with the products of the "outside" number with each of the "inside" numbers. This pattern is called the **distributive property** and it can be expressed with the following rules:

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

Sometimes it's helpful to draw arrows from the outside factor to the inside factors like this:

$$7\ (\ a\ +\ 4\ ) = 7a + 28 \qquad\qquad -8\ (2v\ +\ 3\ ) = -16v + (-24)$$
$$= -16v - 24$$

Simplify the following expressions using the distributive property.

4. $9(5 + y) =$

5. $-5(-p + 6) =$

6. $11(3c - 4) =$

7. $-2(4m - 7) =$

8. $3(10x + 4) =$

9. $-1(p + 8) =$

10. $2(3a + 4c - 5d) =$

11. $0.5(8m - 4n) =$

12. $6(1 - 5m) =$

13. $-2(1 - 5v) =$

14. $3(4 + 3r) =$

15. $3(6r + 8) =$

16. $-6(7k + 11) =$

17. $-3(7n + 1) =$

18. $-6(1 + 11b) =$

19. $-10(a - 5) =$

20. $(3 - 7k)(-2) =$

21. $-20(8x + 20) =$

22. $(7 + 19b)(-15) =$

23. $(x + 1)(14) =$

A Walk-a-Thon

Raul's school decided to participate in a walk-a-thon to raise money for a local charity. His homework last night was to get pledges and bring them back to class. Three of Raul's aunts said they would each pledge $4.50 for his participation and then $1.50 for every mile that Raul walked. His classmate Casey got her grandfather to pledge $200 for the walk-a-thon. His friend Thanoj got his mother and sister to each pledge $8.00 for every mile that he completed.

1. If we let m = miles walked, then the expression: $3(4.50 + 1.50m)$ describes the amount of money pledged to Raul. Briefly explain why this expression models his pledge amount.

2. Write expressions for Casey's and Thanoj's pledges.

3. In your opinion, which of the three students received the "best" pledge? Briefly justify your answer.

4. Anyone who earns more than $100 receives an iTunes gift card. How many miles would Raul need to walk to earn the gift card? Explain how your answer.

5. How many miles would Thanoj need to walk to earn the gift card? Does either Raul or Thanoj have a realistic chance?

6. How many miles would Raul need to walk to collect the same amount in pledges as Casey? Briefly explain your strategy.

7. How many miles would Thanoj and Raul have to walk to both earn the same amount of money?

8. The graphing calculator can also be used to explore the three students' pledges. Put Raul's pledge expression into Y_1, Casey's into Y_2, and Thanoj's into Y_3. Use the table feature of the graphing calculator to solve problems 3 – 7. Copy values from the tables in the graphing calculator into the tables below. Then explain how the values in the tables may be used to answer these problems.

Raul		Casey		Thanoj	
x	y	x	y	x	y
0		0		0	
1		1		1	
2		2		2	
3		3		3	
4		4		4	
5		5		5	

9. Graph the three equations together in the same window. Use the window settings below. Does the graph verify your predictions? How is using a graph similar to using a table? How is it different?

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=2
 Ymin=0
 Ymax=250
 Yscl=50
↓Xres=1
```

Pizza Party

1. For Raul's birthday, Jessica invited Raul and some of their friends to a fair. They got a picnic table and she ordered 6 large pizzas. Luckily, she had a coupon for $3 off each pizza. If the bill came to $38.94, what was the price of a large pizza?

   a) What is the unknown in this problem?

   b) What is known about the actual cost of one pizza?

   c) Write an expression for the cost of one pizza.

   d) Write an expression for the cost of all six pizzas.

   e) The total cost for all the pizzas was $38.94. Write an equation that models this situation.

   f) Solve the equation. What was the price of each large pizza *before* the discount?

2. What if Jessica bought small pizzas instead? She decides to buy 7 small pizzas, but she only has four coupons. Each coupon reduces the cost by $2. She bought four small pizzas at the discounted price and paid full price for the other three. If the bill came to $44.50, how much was each small pizza?

   a) Write an expression for the cost of the 3 small pizzas that Jessica bought at full price.

   b) Write an expression for the cost of the 4 small pizzas that Jessica bought at a discounted price.

c) Write an equation that models this situation.

d) Solve the equation. What was the price of each small pizza *before* the discount?

3. Raul, Jessica, and their friends enjoyed eating the pizza on a rectangular picnic table. The dimensions of the picnic table are presented in the figure below. The perimeter of the rectangular table is 96 in. Find the length of each side.

$$3(x + 5)$$

$$2(3x + 3)$$
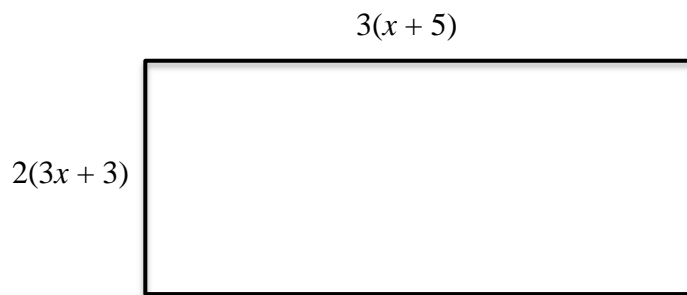
a) Write an equation modeling the perimeter of the rectangle.

b) Solve the equation.

c) List the lengths of each side: Side 1: _____   Side 2: _____

                                     Side 3: _____   Side 4: _____

d) Examine your answers in question c. What is this kind of rectangle called?

Multi-Step Equation Challenge

Let's look at some challenging equations. We will do the first one together.

1. Solve $7(2a + 3) - 15 = 3(4a + 7) - 3$

Distribute:

Combine like terms:

Get all variable terms on one side:

Get all constant terms on opposite side:

Solve for a:

Solve the following equations.

2. $2(2b + 1) + 5 = 3(2b + 1) - b$

3. $9(c + 2) - 11 = 5(2c + 3) - 2c$

4. $5(3d + 2) + 15 = 4(2d + 5) + 7d + 5$

5. $4(2e + 6) - 14 = 2(5e + 5) - 2e$

6. $8(3f + 4) - 9f = 7(3f + 2) + 10$

7. $5(3g + 2) - 15 = 5(3g + 5) + 3g$

8. The graphing calculator may also be used to explore question 3 above. Put the expression on the left side into Y_1, and put the expression on the right side into Y_2. Graph the two equations together in the same window. Does the graph verify your answer? Explain.

9. The graphing calculator may also be used to explore question 4 above. Put the expression on the left side into Y_1, and put the expression on the right side into Y_2. Graph the two equations together in the same window. Does the graph verify your answer? Explain.

10. The graphing calculator may also be used to explore question 5 above. Put the expression on the left side into Y_1, and put the expression on the right side into Y_2. Graph the two equations together in the same window. Does the graph verify your answer? Explain.

11. The graphing calculator may also be used to explore question 6 above. Put the expression on the left side into Y_1, and put the expression on the right side into Y_2. Graph the two equations together in the same window. Does the graph verify your answer? Explain.

Fraction Busters

To eliminate fractions from an equation, multiply both sides of the equation by the *least common denominator* of all the fractions in the equation. Practice this method on the following equations.

1. $-\dfrac{3}{8}p + \dfrac{7}{8} = \dfrac{19}{8}$

2. $\dfrac{5}{6}x + \dfrac{7}{6} = \dfrac{2}{6}$

3. $\dfrac{5}{2}x + \dfrac{2}{3} = \dfrac{5}{4}$

4. $\dfrac{1}{5}x + \dfrac{3}{10} = -4$

5. $\dfrac{1}{6}x + 4 = \dfrac{5}{6}$

6. $\dfrac{12 + x}{2} = 8$

7. $\dfrac{3}{4}a - \dfrac{1}{2} = a + \dfrac{2}{3}$

8. $\dfrac{1}{8} + \dfrac{5}{8}x - \dfrac{1}{2} = \dfrac{1}{2}x + \dfrac{7}{8}$

9. $4x - 5 = \dfrac{1}{5}(5x + 20)$

10. $8y - 6 = \dfrac{2}{3}(6y + 15)$

11. $-\dfrac{2}{3}x + \dfrac{4}{3} = -\dfrac{10}{3}$

12. $\dfrac{4}{5} + 3x = \dfrac{3}{5}$

13. $\dfrac{2}{7}x + \dfrac{4}{7} = -3$

14. $\dfrac{2 + w}{3} = 10$

15. $\dfrac{2}{3}y - \dfrac{11}{6} = 28$

16. $-\dfrac{1}{6}x + 1 = \dfrac{7}{9}$

17. $\dfrac{2}{7}y + 15 = \dfrac{12}{14}y + 9$

18. $\dfrac{5}{6}x - \dfrac{3}{8}x = \dfrac{1}{2}x - 2$

Transforming Literal Equations with Flowcharts

You will now use flowcharts to solve for variables in literal equations. Apply the corresponding steps to the equation on the right side.

1. Solve $24 + x = z$ for x. $\qquad\qquad\qquad\qquad$ $24 + x = z$

x		

Right Hand Side

2. Solve $2\pi r = C$ for r. $\qquad\qquad\qquad\qquad$ $2\pi r = C$

r		

Right Hand Side
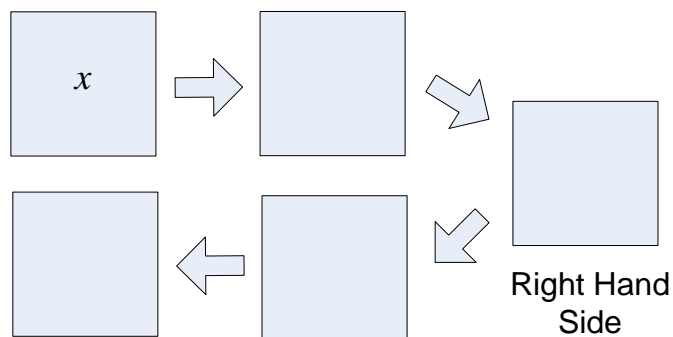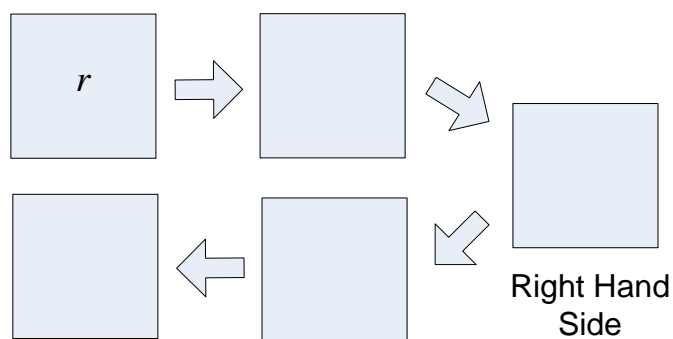
3. Solve $\frac{bh}{2} = A$ for b. $\qquad\qquad\qquad\qquad$ $\frac{bh}{2} = A$

b		

Right Hand Side
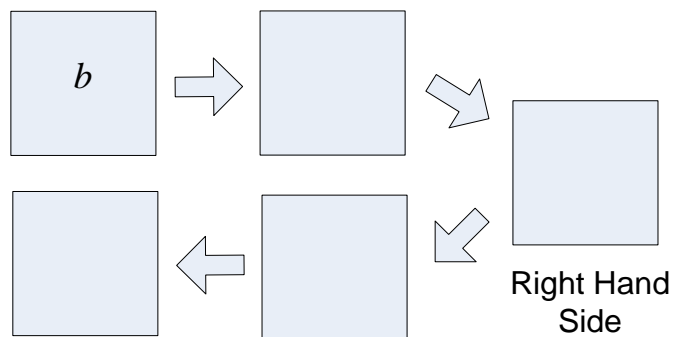
4. Solve $8x + 2y = 2$ for y. $8x + 2y = 2$

y	→		→		↘	

Right Hand Side
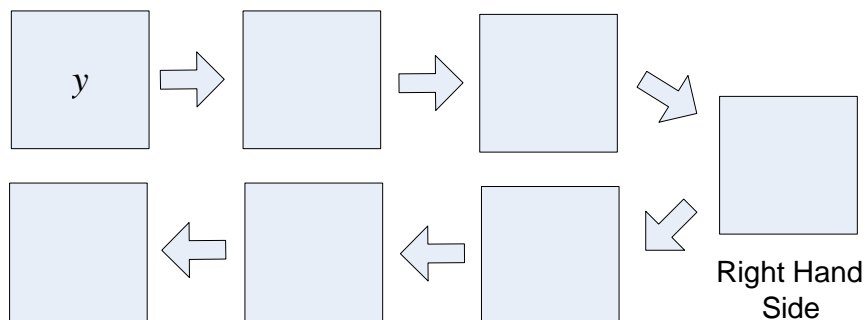
5. Solve $A + B + C = 180$ for A. $A + B + C = 180$

A	→		→		↘	

Right Hand Side
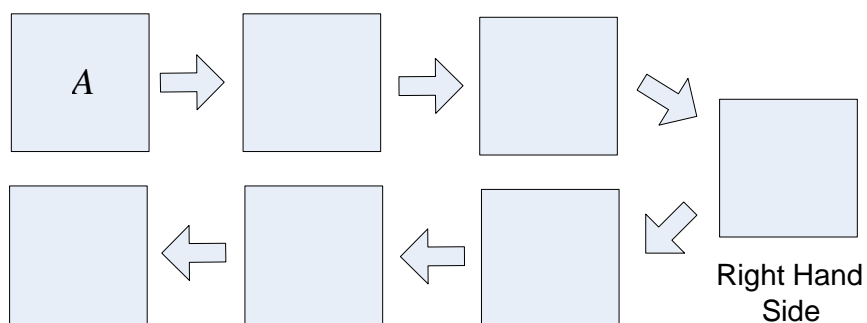
6. Solve $prt = I$ for t. $prt = I$

t	→		→		↘	

Right Hand Side
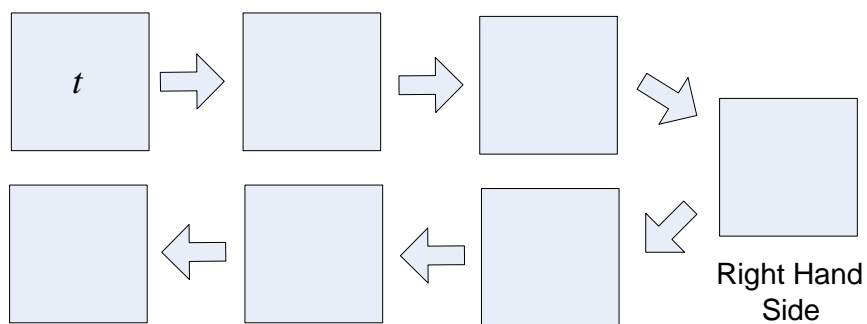
7. Solve $2x + 5y = 20$ for x. $2x + 5y = 20$

x		

Right Hand
Side
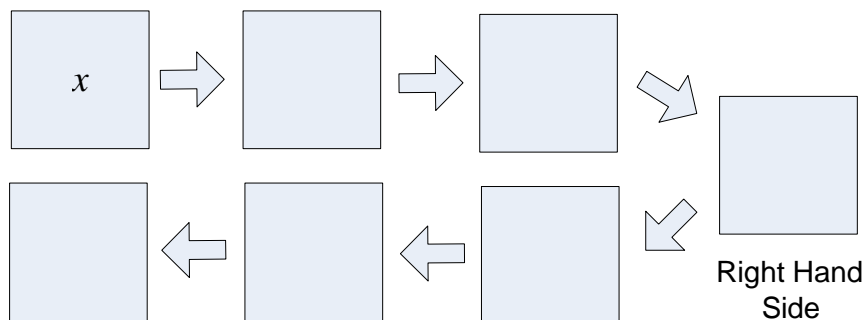
8. Solve $12x - 3y = -6$ for y. $12x - 3y = -6$

y		

Right Hand
Side
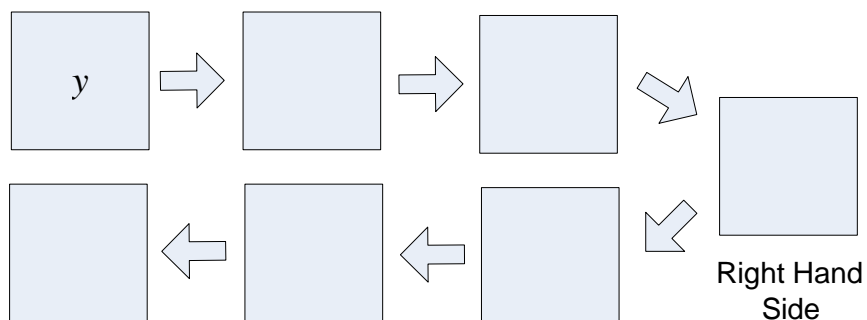
9. Solve $x - 4y = 28$ for y. $x - 4y = 28$

y		

Right Hand
Side
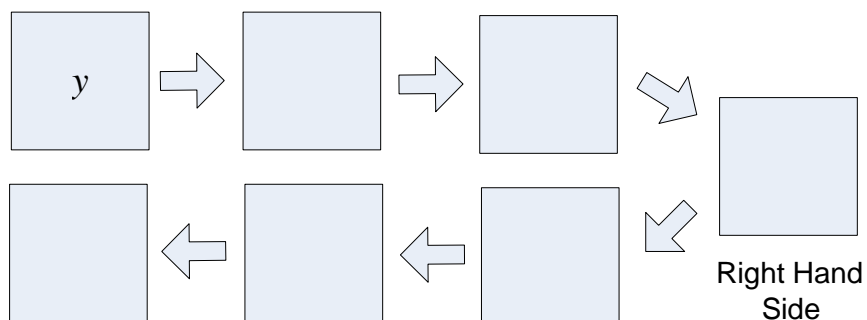
LITERAL EQUATIONS WORKSHEET

Solve for the indicated variable in the parenthesis ON A SEPARATE SHEET OF PAPER!

1) $P = IRT$ (T)

2) $A = 2(L + W)$ (W)

3) $y = 5x - 6$ (x)

4) $2x - 3y = 8$ (y)

5) $\frac{x+y}{3} = 5$ (x)

6) $y = mx + b$ (b)

7) $ax + by = c$ (y)

8) $A = h(b + c)$ (b)

9) $V = LWH$ (L)

10) $A = 4r^2$ (r^2)

11) $V = \pi r^2 h$ (h)

12) $7x - y = 14$ (x)

13) $A = \frac{x+y}{2}$ (y)

14) $R = \frac{E}{I}$ (I)

15) $x = \frac{yz}{6}$ (z)

16) $A = \frac{r}{2L}$ (L)

17) $A = \frac{a+b+c}{3}$ (b)

18) $12x - 4y = 20$ (y)

19) $x = \frac{2y-z}{4}$ (z)

20) $P = \frac{R-C}{N}$ (R)

More Literal Equations

1. $d = rt$ Solve for *t*.

2. $p = 2l + 2w$ Solve for *w*.

3. $4a + 2b = 14$ Solve for *a*.

4. $y - y_1 = m(x - x_1)$ Solve for *y*.

5. $V = \pi r^2 h$ Solve for *h*.

6. $V = \frac{4}{3}r^3$ Solve for *r*.

7. $A = \frac{h}{2}(a + b)$ Solve for *a*.

8. What do the *p*, *l,* and *w* represent in the formula in question 2? What geometric figure does this formula relate to?

9. What do the *V*, *r*, and *h* represent in the formula in question 5? What geometric figure does this formula relate to?

10. What do the *V* and *r* represent in the formula in question 6? What geometric figure does this formula relate to?

11. What do the *A*, *a*, *b*, and *h* represent in the formula in question 7? What geometric figure does this formula relate to?

Representing Inequalities

Graph each situation on the number line.

1. To serve as president of the United States you must be at least 35 years old.

```
←——+——+——+——+——+——+——+——+——+——+——+——+——+——→
   0   5  10  15  20  25  30  35  40  45  50  55  60
```
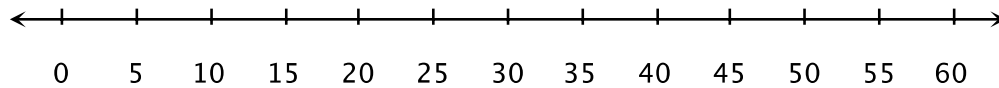
2. To compete in the junior Olympics you must be under 17 years of age.

```
←——+——+——+——+——+——+——+——+——→
   0    5   10   15   20   25   30   35   40
```
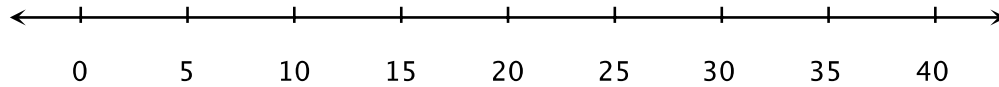
3. For Thanksgiving dinner we will need a turkey weighing more than 20 pounds.

```
←——+——+——+——+——+——+——+——+——→
   0    5   10   15   20   25   30   35   40
```
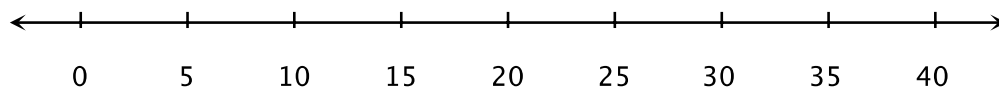
4. At the amusement park, only children less than 48 inches tall may use the kiddie cars.

```
←——+——+——+——+——+——+——+——+——+——+——+——+——+——+——+——→
   0    10    20    30    40    50    60    70    80
```
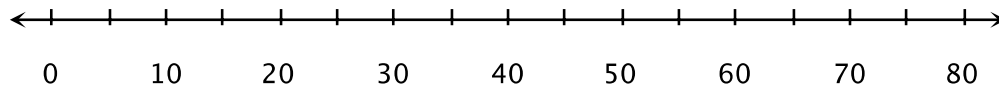
5. If you want to ride the roller coaster your height must be 54 inches or more.

```
←——+——+——+——+——+——+——+——+——+——+——+——+——+——+——+——→
   0    10    20    30    40    50    60    70    80
```
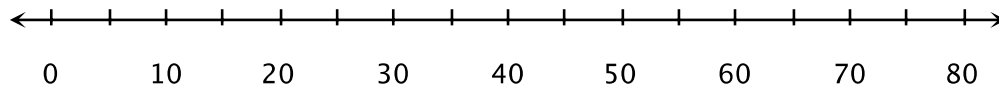
6. To be eligible to vote your age must be greater than or equal to 18 years.

```
←——+——+——+——+——+——+——+——+——→
   0    5   10   15   20   25   30   35   40
```
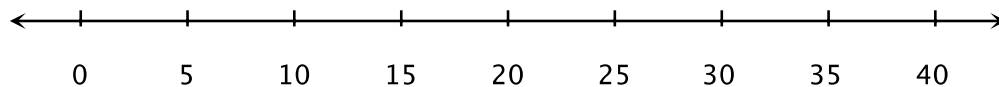
7. You will fail the course if your average is below 60.

```
←——+——+——+——+——+——+——+——+——+——+——+——+——+——+——+——→
   0    10    20    30    40    50    60    70    80
```
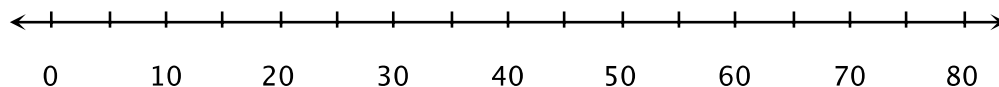
8. Water will freeze when the temperature is less than or equal to 32 degrees Fahrenheit.

```
←——+——+——+——+——+——+——+——+——+——+——+——+——+——→
   0   5  10  15  20  25  30  35  40  45  50  55  60
```
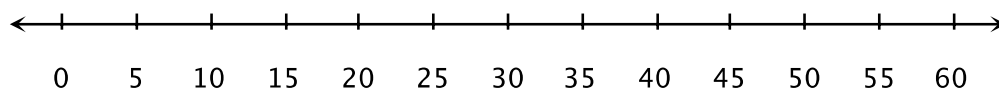
9. Students at Greenfield High School may have no more than 4 unexcused absences in a year.

```
←——+——+——+——+——+——+——+——+——+——→
   0   1   2   3   4   5   6   7   8
```
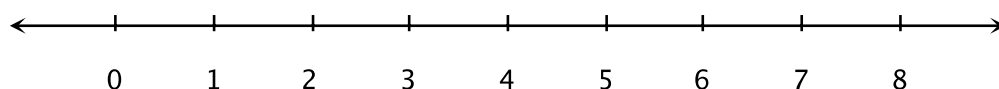
Equations and Inequalities

1. Explain the difference between an equation and an inequality.

2. How many solutions does an equation have?

3. How many solutions does an inequality have?

Use the number lines A – E below to answer the following questions:
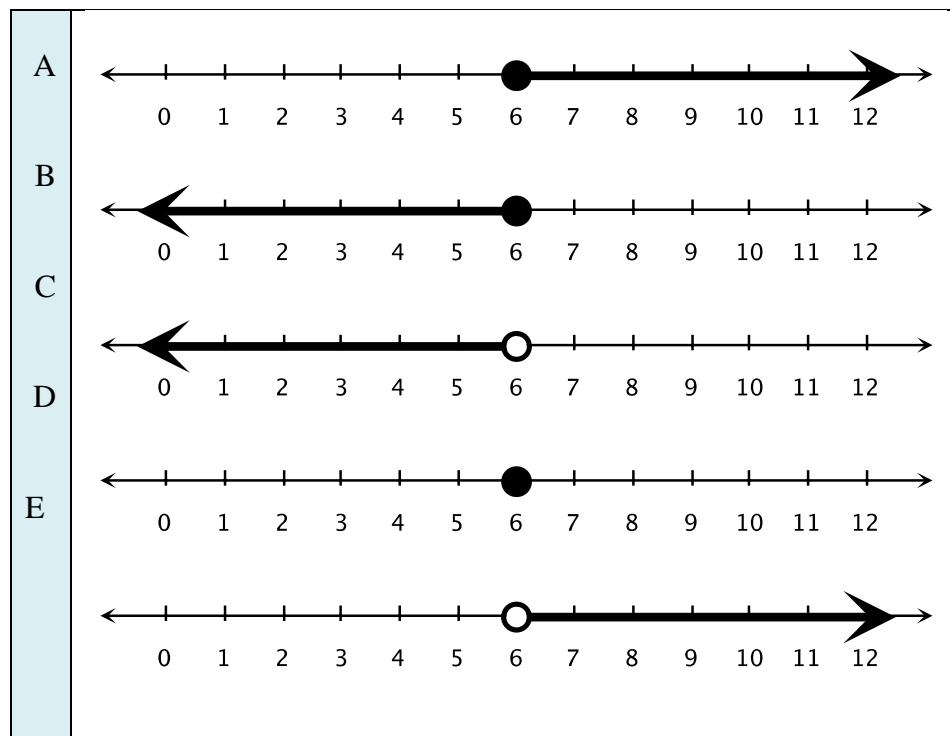
4. For which number lines is $x = 6$ included in the shaded set?

5. For which number lines is $x = 4$ included in the shaded set?

6. For which number lines is $x = 7$ included in the shaded set?

7. Which of the number lines (A – E) show an equality?

8. Which of the number lines (A – E) show inequalities?

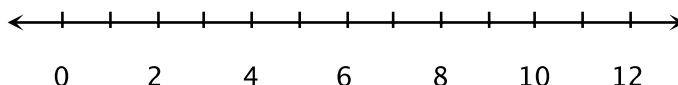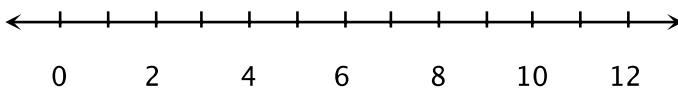9. $x = 5$ Say it in words. _____

     Is 4 a possible solution for x? Why / why not? _____

     Is 5 a possible solution for x? Why / why not? _____

     Is 6 a possible solution for x? Why/ why not? _____

     How many possible solutions are there for x? _____

     Graph solutions on this number line.

```
  <------+--+--+--+--+--+--+--+--+--+--+--+--+-->
         0     2     4     6     8    10    12
```

10. $x \leq 5$ Say it in words. _____

     Is 4 a possible solution for x? Why / why not? _____

     Is 5 a possible solution for x? Why / why not? _____

     Is 6 a possible solution for x? Why/ why not? _____

     How many possible solutions are there for x? _____

     Graph solutions on this number line.

```
  <------+--+--+--+--+--+--+--+--+--+--+--+--+-->
         0     2     4     6     8    10    12
```
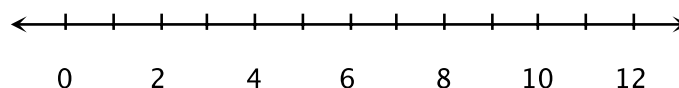
11. $x \geq 5$ Say it in words. _____

     Is 4 a possible solution for x? Why / why not? _____

     Is 5 a possible solution for x? Why / why not? _____

     Is 6 a possible solution for x? Why/ why not? _____

     How many possible solutions are there for x? _____

     Graph solutions on this number line.

```
  <------+--+--+--+--+--+--+--+--+--+--+--+--+-->
         0     2     4     6     8    10    12
```

12. $x < 5$ Say it in words. _____

   Is 4.5 a possible solution for x? Why / why not? _____

   Is 5 a possible solution for x? Why / why not? _____

   Is 6 a possible solution for x? Why/ why not? _____

   How many possible solutions are there for x? _____

   Graph solutions on this number line.

   0   2   4   6   8   10   12

13. $x > 5$ Say it in words. _____

   Is 4.5 a possible solution for x? Why / why not? _____

   Is 5 a possible solution for x? Why / why not? _____

   Is 6 a possible solution for x? Why/ why not? _____

   How many possible solutions are there for x? _____

   Graph solutions on this number line.

   0   2   4   6   8   10   12

14. $x \neq 5$ Say it in words. _____

   Is 4.5 a possible solution for x? Why / why not? _____

   Is 5 a possible solution for x? Why / why not? _____

   Is 6 a possible solution for x? Why/ why not? _____

   How many possible solutions are there for x? _____

   Graph solutions on this number line.

   0   2   4   6   8   10   12

15. **$5 < x$**   What does $5 < x$ mean? _____

    Is 4 ½ a possible solution for x? Why / why not? _____

    Is 5 a possible solution for x? Why / why not? _____

    Is 5 ½ a possible solution for x? Why / why not? _____

    Write another inequality that looks different than $5 < x$ but *means the same thing.*

    _____

    Graph the solution on a number line.

    0   2   4   6   8   10   12

16. **$5 > x$**   What does $5 > x$ mean? _____

    Is 4.75 a possible solution for x? Why / why not? _____

    Is 5 a possible solution for x? Why / why not? _____

    Is $5\frac{2}{5}$ a possible solution for x? Why / why not? _____

    Write another inequality that looks different than $5 > x$ but *means the same thing*

    _____

    Graph the solution on a number line.

    0   2   4   6   8   10   12
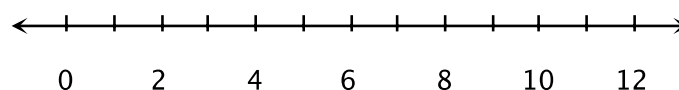
17. **Vocabulary**: For each symbol listed below, write down EVERY word and/or phrase that means the same thing as the symbol. See how many you can come up with!

=   _____

       _____

>   _____

       _____

≥   _____

       _____

<   _____

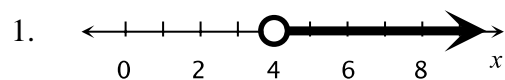   _____

≤   _____

       _____

≠   _____

       _____

Working with Inequalities

Complete the following table.

Inequality	Rewrite if necessary	Say it in words	Graph it	Name a point in the solution set	Check that it works (substitute)
$x > 5$					
$y \leq 2$					
$8 > a$					
$-3 \leq m$					

You should also be able to work backwards. Write the inequality represented by each graph, using any variable you choose.

1.

2.

3.

4.

5.

6.
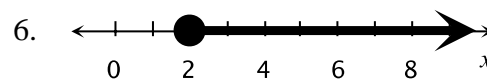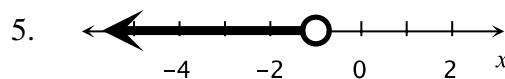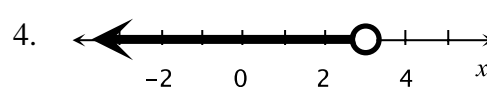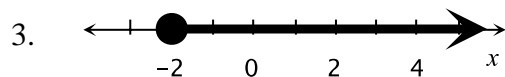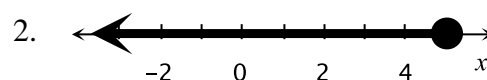
Often you will need to solve an inequality before graphing it. This is almost like solving an equation - "undo" what's been done to the variable.

Inequality	Solve it	Rewrite if necessary	Say it in words	Graph it	Pick a point	Check it
$3x > 12$						
$\dfrac{a}{5} \leq -4$						
$9 > n - 1$						
$-3 \leq m + 4$						
$2x + 5 < 17$						
$4 < \dfrac{w}{2} - 6$						

Practice Solving Inequalities

Solve the following equations: (Review)

3. $2x - 7 = 5$ 2. $-4 + 6x = 17$ 3. $23 = 19 - x$
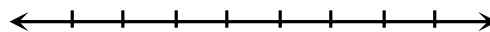
4. $5(x - 3) = 15$ 5. $5x - 3 = 4x + 19$ 6. $7 + 2x = -4x - 11$

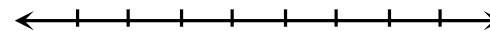**Solve the inequalities and graph the solutions on a number line.**

7. $x - 9 < -3$ 8. $10 - x > 5$

9. $6x - 5 \geq 19$ 10. $-2x + 9 > 15$

11. $-3 \le 2x - 13$

12. $-x - 3 \ge -10x - 12$

13. $4x + 8 \le 2(x - 6)$

14. $-(x + 7) > 8x - 25$

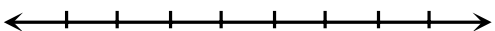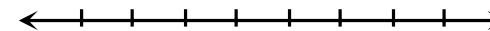15. $\quad 7x - 30 < 19$

16. $-7 - 4x < 13$

17. $2x + 5 \leq 9$

18. $-3x - 5 < 16$

<----+----+----+----+----+----+----+----+---->

<----+----+----+----+----+----+----+----+---->

19. $2x + 3 < 6x - 1$

20. $3x - 2 \geq 7x - 10$

<----+----+----+----+----+----+----+----+---->

<----+----+----+----+----+----+----+----+---->

21. $-2(x + 4) > 6x - 4$

22. $7x - 2 \leq -3(x - 2)$

<----+----+----+----+----+----+----+----+---->

<----+----+----+----+----+----+----+----+---->

Inequalities in the Real World

1. Chloe and Charlie are taking a trip to the pet store to buy some things for their new puppy. They know that they need a bag of food that costs $7, and they also want to buy some new toys for the puppy. They find a bargain barrel containing toys that cost $2 each.

   a. Write an <u>expression</u> for the amount of money they will spend if they purchase a bag of food and t toys.

   b. Together, Chloe and Charlie can spend no more than $40. Use this information and the expression you wrote in part (a) to write an <u>inequality</u> for finding the number of toys they can buy.

   c. Solve the inequality and graph the solution on the number line below.


   d. Explain what the graph of the solution means?

2. Valley Video charges a $15 annual membership fee plus $3 for each movie rental. Tanya puts aside $100 for renting movies for the year. How many movies can Tanya rent from Valley Video? Use an inequality to solve this problem. Graph your solution on the number line and explain the meaning of your graph in a sentence.
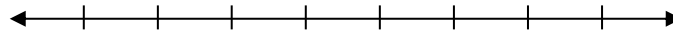
3. You are a salesperson at Nissan. Each month you earn $2,200 plus one-fifteenth of your sales. You want to earn more than $4000 this month. How much must you sell this month in order to earn more than $4000? Use an inequality to solve this problem. Graph your solution on the number line and explain the meaning of your graph in a sentence.
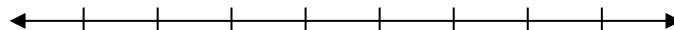
4. Joe's car needs work. The mechanic charges $140 for parts plus $48 per hour for labor. The mechanic said the bill will be at least $300. What is the possible number of hours that the mechanic will work on Joe's car? Use an inequality to solve this problem. Graph your solution on the number line and explain the meaning of your graph in a sentence.
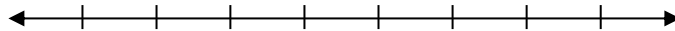
5. A popular cellular phone family plan provides 1,500 minutes. It charges $89.99/month for the first two lines and $9.99/month for every line after that. Unlimited text messages for all phone lines costs $30.00/month, and Internet costs $10.00/month per phone line. If a family with a $200 monthly budget buys this plan and signs up for unlimited text messaging and Internet on each phone line, how many cell phone lines can they afford? Use an inequality to solve this problem. Graph your solution on the number line and explain the meaning of your graph in a sentence.