MATHEMATICS AND COMPUTER SCIENCE

The quantitative nature of our world makes it imperative for students to achieve literacy and proficiency with mathematical concepts, methods, and problem-solving strategies. Mathematics as a system of thought is considered a core part of the human cultural experience. Our mission is to help students appreciate this discipline in its pure and applied forms.

The department focuses on the development of logical, analytical, and critical thinking skills. To this end, teachers expect clear, cogent arguments in students' written work and increasingly emphasize the ability to read and discuss mathematics as students mature. Through a mixture of standard and novel problem types, students also develop self-confidence and creativity, thus enabling them to reach the highest level in our curriculum that their ability and discipline will allow.

The mathematics requirement consists of Algebra I, Geometry, and Algebra II. Most students continue beyond the required courses. The honors sequence puts a relatively greater emphasis on formal mathematical theory and rigor. For new students, math placement is based on the student's previous mathematics history and standardized test scores, a questionnaire completed by a former mathematics teacher, and a diagnostic assessment test. For returning students, placement is made by teacher recommendation and the approval of the department head.

Extensive course offerings enable students to master traditional topics as well as to gain an awareness of current developments in pure and applied mathematics. Incorporating advances in technology, our courses allow students to use computing to solve problems and examine data. Computer Science courses emphasize structured programs and techniques and develop organizational and problem-solving skills. Students may compete in regional and national mathematics, robotics, and computer science contests, and participate in the peer tutoring program.

Given the many options available in the mathematics curriculum, students may consult with their teacher, adviser, form dean, or the department head to help plan their mathematics programs.

– Matthew K. Bardoe, Department Head

Note: The use of a graphing calculator is required from Algebra II through calculus. The department supports the use of the TI-84 series calculators.

ALGEBRA I, ACCELERATED
Year; 3 course credits
MA125

This algebra course focuses on the development of precise and accurate habits of mathematical expression and the application of the tools of algebra to problem-solving. Students explore linear relationships in depth, as they graph and solve linear equations and inequalities, and solve systems of linear equations both graphically and analytically. Students also begin to learn the language and notation of functions. Other topics include properties of exponents, radicals, informal geometry, probability, data analysis, matrices, and the introduction of quadratic relationships. Students are also challenged to become more independent learners and to make connections between different mathematical concepts. Open to students by the recommendation of the department.

GEOMETRY
Year; 3 course credits
MA200

In this course, students are given a thorough introduction to Euclidean geometry. In addition to learning to write deductive arguments, they are involved in discovering and exploring concepts that relate geometry to the real world and to other disciplines. Students also review and strengthen algebra skills and develop greater facility solving numerical problems. Open to students who have completed Algebra I.

GEOMETRY, HONORS
Year; 3 course credits
MA250HO

This course offers a formal deductive approach to the study of Euclidean geometry in two and three dimensions. From the outset, students focus on writing rigorous geometric arguments using a variety of techniques. In addition, coordinate geometry, transformations, and elementary geometric probability are studied informally throughout the course. Students also continue to review algebraic techniques and strengthen their ability to solve numerical problems. Open to students who have completed Algebra I and have the permission of the department.

ALGEBRA II
Three terms; 1 course credit (each term)
MA301 (Fall); MA302 (Winter); MA303 (Spring)

This second-year algebra sequence begins with a review of basic algebra skills followed by a more in-depth study of linear and quadratic equations and inequalities, including an introduction to complex numbers. Matrices are also studied, from basic operations and determinants to inverses and matrix equations. Higher order polynomials and rational expressions are examined, and a discussion of exponents leads to the study of radical expressions, equations, and functions. Exponential and logarithmic equations and functions are also introduced. Throughout the course function notation, mathematical models, and graphing techniques are emphasized. The graphing calculator is used to both enhance understanding of these topics and increase knowledge of its use. Open to students who have completed Algebra I and Geometry.

ALGEBRA II, ACCELERATED
Three terms; 1 course credit (each term)
MA331 (Fall); MA332 (Winter); MA333 (Spring)

This second-year algebra sequence covers all of the topics in the MA301—MA303 sequence, but at a faster pace and in more detail. The course also includes a study of sequences and series. Emphasis is placed on developing sound analytical and problem-solving techniques. Open to students who have completed Algebra I, Geometry, and have the permission of the department.

ALGEBRA II, HONORS
Three terms; 1 course credit (each term)
MA351HO (Fall); MA352HO (Winter); MA353HO (Spring)

A brief review of algebraic expressions, inequalities, absolute value and elementary probability is followed by a study of linear equations, functions, and systems. Matrix algebra is introduced, including applications. A thorough study of quadratic functions is undertaken followed by polynomial functions and associated theorems, including the Binomial Theorem. Other topics include permutations, combinations, radical functions, rational exponents, and inverses. The course continues with an in-depth study of exponential and logarithmic functions. Students analyze the conic sections, and the course concludes with a study of sequences and series. Students are encouraged throughout to think independently and are expected to be able to expand their knowledge by applying the basic concepts in new and productive ways. The graphing calculator is required and is used regularly to analyze data and to discover new concepts by identifying patterns and by making and testing conjectures. Open to students who have completed Algebra I, Geometry, and have the permission of the department.

ELECTIVE COURSES

VISUAL MATHEMATICS
Spring term; 1 course credit
MA417

Mathematical concepts are embedded in many art forms—whether they were intended by the artist or as a result of what was aesthetically pleasing to the eye. This course offers students the opportunity to explore these connections from a mathematical perspective. Topics from geometry, such as constructions, proofs, the Golden Ratio, similarity, and polygons are explored. From analytic geometry and algebra, students work with transformations, compositions, vectors, matrices, fractals, and the Fibonacci sequence. Three dimensional constructs, including polyhedra, Platonic and Archimedean solids are included. Students complete art projects in relation to each topic throughout the term. Creations include mandalas, friezes, and tessellations, as well as three-dimensional and computer-generated pieces. Open to students who have completed Geometry.

STRATEGIC DECISIONMAKING
All terms; 1 course credit
MA418

This case study-driven course examines leadership, strategy, and negotiation techniques responsible for successful boardroom turnarounds and perilous Mt. Everest summit failures. Simulating a practical environment, student teams apply a problem-based learning system to research and present conclusions throughout the term. Another course component exposes students to basic financial statement analysis and an introduction to industry-recognized corporate valuation techniques. As the course concludes, teams apply these skills through an exploration of the energy markets, specifically targeting alternative and renewable energy companies. Specific case studies may vary across sections. Open to students who have completed Algebra II. *This course earns one quantitative credit.*

FUNDAMENTALS OF PRECALCULUS
Three terms; 1 course credit (each term)
MA421 (Fall); MA422 (Winter); MA423 (Spring)

This three-term course is designed for the student who wants to continue beyond math department diploma requirements, but for whom the precalculus or honors precalculus courses are not appropriate. Functions are studied extensively, including polynomial, trigonometric, exponential, and logarithmic functions. Additional topics include statistics and data analysis, probability, and sequences and series. Students may receive course credit for the first one or two terms without completing the sequence, but they must begin in the fall term. Open to students who have completed Algebra II.

Note: Students who complete MA423 must have the permission of the department to enroll in MA436AD.

DATA SCIENCE
Three terms; 1 course credit (each term)
MA441 (Fall); MA442 (Winter); MA443 (Spring)

This three-term course is designed to offer background and preparation for college statistics courses required for many majors. The course focuses on gathering, organizing, simplifying, analyzing, and interpreting data. Topics include descriptive statistics and exploratory data analysis, design of experiments, sampling distributions and estimation, inference and decision making, and fitting models to data. A thorough introduction to probability is also included. Randomness, independence and conditional probability, discrete and continuous probability models, and counting methods are covered. Measuring the probability of an event, interpreting probability, and using probability in decision making are central themes of this course. Applications to games of chance, business, medicine, the natural and social sciences, policy making, and sports are introduced and explored. Students may receive course credit for the first one or two terms without completing the sequence but must begin in the fall term. Open to students who have completed Algebra II.

ADVANCED ELECTIVES: PRECALCULUS, CALCULUS, AND HONORS STATISTICS

PRECALCULUS
Three terms; 1 course credit (each term)
MA436AD (Fall); MA437AD (Winter); MA438AD (Spring)

This course focuses on the study of functions. Topics include polynomial, rational, exponential, and logarithmic functions with emphasis on their graphs and applications. Students engage in an in-depth study of trigonometric functions, including the laws of sines and cosines, trigonometric identities, inverse trigonometric functions, and techniques for solving trigonometric equations. Sequences and series, while introduced in Algebra II, are studied in a formal manner during this course. An introduction to displays of data, measures of variance, and the Normal distribution give students a basic background in statistics. The course finishes with an introduction to limits and the derivative. This course is designed to prepare students for either MA507AD, MA531HO, MA521HO, or CS550HO. Open to students who have completed Algebra II and have the permission of the department.

PRECALCULUS, HONORS
Two terms; 1 course credit (each term)
MA450HO (Fall); MA460HO (Winter)

The topics of MA436AD and MA437AD are studied in greater depth, including a more intensive study of graphs of combinations of functions, the inverse trigonometric functions, identities, and trigonometric equations. In addition, students are regularly expected to solve more challenging and complex problems. Polar coordinates, polar graphs, and the polar form of complex numbers are studied. Other topics include DeMoivre's Theorem, sequences and series, limits of sequences, limits of functions of a real variable, vectors, and parametric equations in the plane. A review of conic sections is included, as needed. This course is designed to prepare students for the Calculus BC, Honors sequence (MA561HO—MA564HO). Open to students who have completed Algebra II, Honors and have the permission of the department.

CALCULUS
Three terms; 1 course credit (each term)
MA507AD (Fall); MA508AD (Winter); MA509AD (Spring)

This three-term sequence covers several of the topics included in an introductory college-level calculus course but without the rigorous emphasis on theory or symbolic manipulation. The sequence incorporates more practical applications than Calculus AB, Honors. Use of a graphing calculator is essential, both in class and on assignments. Open to students who have completed Precalculus and have the permission of the department.

Note: Students who have received credit for a parallel term of honors calculus may elect this sequence, but they may not enter MA508AD without successfully completing MA507AD or MA531HO and may not enter MA509AD without successfully completing MA508AD or MA532HO.

CALCULUS AB, HONORS
Three terms; 1 course credit (each term)
MA531HO (Fall); MA532HO (Winter); MA533HO (Spring)

An initial study of functions, limits, and analytical geometry leads to the study of the derivative and differentiation techniques. Functions are explored symbolically, graphically, and numerically, and the relationship between a function and its derivative is carefully developed. Applications of the derivative include related rates of change, a formal analysis of local and absolute extreme values, and optimization problems. The concepts of the antiderivative and slope fields are introduced. Students learn to use a Riemann Sums to approximate the area under a curve, after which the concept of the integral is formally defined. Elementary techniques of integration are studied, including integration by substitution and by parts. The integral is studied as an area accumulator, and the Fundamental Theorem of Calculus is explored and applied, as are applications of definite integrals, including finding volumes, arc lengths, and average values of functions. Differential equations are studied. Solutions are considered graphically with slope fields and analytically with separable differential equations. Exponential growth and decay problems are revisited in the context of modeling with differential equations, and the logistic curve is explored. Open to students who have completed Precalculus and have the permission of the department.

CALCULUS BC, HONORS
Four terms; 1 course credit (each term)
MA561HO (Spring); MA562HO (Fall); MA563HO (Winter); MA564HO (Spring)

This four-term sequence introduces students to differential and integral calculus. This course includes the basic topics covered in MA531HO—MA533HO, as well as techniques of integration using trigonometric substitutions and partial fractions, improper integrals, and Euler's Method. This course includes a study of infinite sequences and series, Taylor polynomials with the Lagrange form of the remainder, and various techniques of operating on power series. The study of parametrically defined vector functions and polar functions, with applications, are also included. Open to students who have completed Precalculus, Honors and have the permission of the department.

Note: Students who have taken Calculus AB, Honors (MA514HO or MA533HO) and have the permission of the department may take this course, beginning with either MA562HO or MA563HO.

STATISTICS, HONORS
Three terms; 1 course credit (each term)
MA521HO (Fall); MA522HO (Winter); MA523HO (Spring)

This sequence encompasses the material covered in a first-year college-level statistics course that is required for many majors. Using case studies from a variety of disciplines, students explore in detail the background, concepts, and tools for studying data and its variability. The focus of the course is on problem-solving (defining the problem, collecting and analyzing the data, interpreting and then drawing conclusions from the results and taking action), and statistical reasoning (understanding that randomness is not haphazard, utilizing distributions to explain uncertainty, selecting models to increase precision and to reduce bias, and recognizing that correlation is necessary but not sufficient to demonstrate causality). Modern technology, especially the graphing calculator, is used extensively to manipulate, simulate, and visually display the data. Students may receive course credit for the first one or two terms without completing the sequence, but they must begin in the fall term. Open to students who have completed Precalculus and have the permission of the department.

GAME THEORY, HONORS
Winter, Spring terms; 1 course credit
MA575HO

This honors elective is an introduction to game theory and strategic thinking. The course covers mathematics – logic, probability and statistics – with an emphasis on application. Beginning with representations and basic assumptions, including the extensive form, strategies and the normal form, beliefs, mixed strategies, and expected payoffs, and general assumptions and methodology, the course moves on to analyzing behavior in static settings. Dominance and best response, rationalizability and iterated dominance, location and partnership, and Nash equilibrium are covered. Game theory concepts are highly applicable in fields such as economics and business, political science, biology, computer science, and philosophy. Open to students who have completed Precalculus and have the permission of the department.

LINEAR ALGEBRA, HONORS
Fall term; 1 course credit
MA650HO

This course introduces students to the study of vector spaces over the real numbers, linear mappings between vector spaces, and their matrix representations. Topics include an investigation of ways to represent and analyze lines and planes in space, with frequent use of the scalar product and cross product, the study of subspaces, bases and dimension, the kernel and image of a linear mapping, and determinants. Students are also exposed to examples of more general vector spaces. The theory is applied to the solution of systems of linear equations; other applications (e.g., eigenvalue problems, difference equations, Markov chains) may be considered as time permits. Open to students who have completed at least Calculus AB, Honors (MA514HO or MA533HO) and have the permission of the department.

MULTIVARIABLE CALCULUS, HONORS
Winter and Spring terms; 2 course credits
MA670HO

This two-term course builds on the foundation laid in the single-variable calculus and linear algebra courses. Students study the calculus of vector functions, with emphasis on functions defining curves in the plane, as well as curves and surfaces in space. The course treats explicit, parametric, and implicit representations of curves and surfaces, along with their tangent lines and planes. The uses of partial derivatives, directional derivatives, and the gradient are explored. The study of integration includes iterated integrals and multiple integrals, with Fubini's Theorem tying them together, along with line and surface integrals, culminating with the important theorems of Green and Stokes. Applications include extrema problems (with Lagrange multipliers), volume and surface area, and physical interpretations of the vector field theory. Open to students who have completed MA564HO, MA650HO, and have the permission of the department.

SEMINARS IN MODERN MATHEMATICS

These term courses are designed to provide highly advanced students with an introduction to mathematics beyond the typical secondary curriculum. Students develop an appreciation of formal methods and improve their skill at understanding and constructing proofs. Each seminar includes an investigation of relevant problems and theorems, with some introduction to the mathematicians who posited or proved them. Students may take the first one or two terms without completing the sequence, but they must begin in the fall term.

FOUNDATIONS OF MODERN MATHEMATICS, HONORS
Fall term; 1 course credit
MA681HO

This seminar introduces the development of set theory, including the influence of Dedekind, Cantor, Hilbert, Russell, Gödel, Zermelo, Skolem, and von Neumann. The foundations of the axiomatic structure of mathematics are examined. The power and the fundamental and inescapable faults of this system are also explored, as well as the issue of the connection between reality and mathematics. Topics covered include first-order languages, orderings, cardinality, models, the axiom of choice, and constructing the real number system. Open to students who have completed MA670HO or are currently enrolled in MA650HO and have the permission of the department.

TOPICS IN MODERN MATHEMATICS, HONORS
Winter term; 1 course credit
MA682HO

This seminar provides a survey of mathematical structures and systems focused on those areas that have given rise to the greatest leaps in understanding in mathematics in the last 100 years. The main concepts of abstract algebra, topology, combinatorics, and dynamical systems are introduced. Students investigate mathematical structures and properties such as groups, rings, metric space, neighborhood, compactness, open sets, generating functions, recursive functions, and Markov Chains. Open to students who have completed MA681HO and have the permission of the department.

RESEARCH TOPICS IN MODERN MATHEMATICS, HONORS
Spring term; 1 course credit
MA683HO

In this seminar students work collaboratively toward a solution to an outstanding research problem in mathematics, specifically in the area of combinatorics. Several easy-to-comprehend research topics are introduced from which students select a problem that most interests them. Working with other students and faculty, they generate ideas to pursue toward the solution of that problem. The computer software Sage is used to help students make and test hypotheses. Throughout the term students read current research papers in mathematics relevant to their problem and make presentations on these papers in class to their peers. Each student creates a final report on the status of their research, which may be used by future students as a starting point for solving the problem. Open to students who have completed MA682HO and have the permission of the department.

COMPUTER SCIENCE

Computer Science encourages students to see the world through the lens of algorithms and abstraction. These courses offer students the opportunity to learn the tools that allow them to create relevant and powerful programs. These offerings are designed for students who have a strong interest in exploring how computers work internally and how to write programs that are efficient and effective.

INTRODUCTION TO PROGRAMMING
All terms; 1 course credit
CS200

This course is meant for students who have little to no experience in programming computers. Students learn the basics of algorithmic thinking, and design programs to solve simple problems. A number of different programming environments are used from highly structured programming environments such as Alice and Scratch. Students eventually gain familiarity with at least one high level programming language such as Python. Students learn the basics of variables, loops, conditional statements, and standard data structures such as arrays and dictionaries. Students will also gain familiarity with how computers work and their architecture. Ultimately, the goal will be for students to use their computers to create and manipulate data sets and to automate tasks. Open to all students.

COMPUTER SCIENCE, HONORS
Fall and Winter terms; 2 course credits
CS550HO

This two-term course introduces students to the principles of computer science, particularly algorithm and program design. Using the Python language, students work in an object-oriented paradigm as they study such traditional topics as conditional statements, loop structures, methods, arrays, recursion, and introductory graphics. Efficient, elegant design is emphasized throughout. Additionally, students learn to use inheritance, interfaces, and polymorphism as they design more complex programs. Following an examination of basic sorts, searches, and data structures, the course concludes with each student designing and completing a major programming project. Open to students who have completed CS200 or CS300 and are at least concurrently enrolled in Precalculus, or who have the permission of the department.

APPLICATION DEVELOPMENT, HONORS
Spring term; 1 course credit
CS555HO

For students with extensive programming experience, this course introduces the basics of mobile-platform application development. The programming language Objective-C is used as a basis for programming applications for iOS devices such as iPhones and iPads. Students complete both individual and group projects, which vary depending on student interests. Open to students who have completed CS550HO or who have the permission of the department.

INTRODUCTION TO ROBOTICS
Fall, Spring terms; 1 course credit
CS300

This course introduces students to the fundamentals of robotics using the VEX platform. Students learn to design and build robots, to program autonomous behaviors, and to use sensors to improve their robots' abilities to interact with their environment. This course serves as an introduction to aspects of computer programming and covers such fundamental topics as program design and control, looping, and Boolean logic. Prior programming experience is not necessary. Open to all students.

ROBOTICS II
Spring term; 1 course credit
CS310

This course in robotics builds on the fundamentals introduced in CS300, but students are expected to complete more advanced projects with greater autonomy. The design development process is further explored, and students learn to use more complex programming structures. In addition to the VEX platform, students may also be introduced to other robotics systems. Open to students who have completed CS300 or who have the permission of the department.

ROBOTICS DESIGN AND FABRICATION, HONORS
Fall term; 1 course credit
CS450HO

This course teaches students how to manage and design mechanical, electrical and programmable systems. Rather than working in predefined robotic environments (such as VEX), students use a variety of materials and systems to create their own robots. Programmable microcontrollers provide a flexible environment that can be applied to many robotics projects and will be the main source of control logic. Students are expected to design and fabricate custom components. Important skills practiced in this class include: soldering, testing circuitry, CAD design, wiring, electrical prototyping with a breadboard, fabrication with power tools, and programming with appropriate languages. Open to students who are in ARC or, as space is available, to students who have completed CS310, or who have the permission of the department.

COMPETITION ROBITICS, HONORS
Winter term; 1 course credit
CS560HO

This course prepares ARC students to compete in the FIRST Robotics Competition (FRC). Initially, students familiarize themselves with the typical components of a competition robot, while exploring how to best organize themselves into an effectively functioning team. Once the FRC game is revealed in January, the intensive build period begins. Students strategize how to approach the game and rapidly begin to prototype mechanisms for specific tasks. Testing and refining their designs, as well as adding and debugging appropriate programming, are continuously done until the end of the build period. Throughout the term, students, as a group, fully document their build and design process and, individually, record their progress and reflections in their personal journals. Students practice communicating their ideas, in words, drawings, or orally, throughout their work in the lab and at competitions. Open to students who are in ARC.

Note 1: Students in CS560HO are usually required to concurrently be enrolled in the accompanying afternoon lab activity as their winter term activity/sport.

Note 2: Although the FRC build season occurs entirely during the winter term, the actual competition events that the students attend will typically occur during the spring term. Students would be expected to attend at least one event in the spring.

Note 3: Students in ARC may repeat CS560HO for another course credit in subsequent years that they are in the program.

AUTONOMOUS ROBOTICS, HONORS
Spring term; 1 course credit
CS570HO

This course builds upon the work completed in CS450HO. Students use a robot explore autonomous tasks, including navigating, mapping, and solving mazes. Students design automatic control and management systems which analyze and interpret feedback and provide programmed responses in robot behavior. A focus is placed on using organized and clear structure in programming as well as careful debugging of work. Open to students who are in ARC or, as space is available, to students who have completed CS450HO, CS550HO, or have the permission of the department.

MACHINE LEARNING, HONORS
Spring term; 1 course credit
CS580HO

Machine learning (ML) is the process by which computers train themselves to make predictions and ultimately, decisions based upon data. In this course, students will use powerful software libraries to construct adaptive and predictive models. While recent developments in hardware have made possible, incredible increases of computational power, simultaneous advances in approaches to the software development of machine learning have also provided powerful new ways for us to make sense of the immense data generated by today's connected devices. Students learn the basic concepts, tools, and techniques of adaptive computation and machine learning; for example, building datasets, training neural networks, using matrix multiplication, calculating error functions, error propagation, gradient descent, and supervised and unsupervised learning. With a hands-on introduction to the necessary mathematics, this course proceeds through a series of increasingly complex projects, culminating in a project where students program and train their own intelligent systems. Students also gain important experience working directly with programming environments that support ongoing artificial intelligence and ML development. Open to students who have completed CS550HO, CS570HO, or have the permission of the department.