K-12 Mathematics Framework

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The following teaching and learning practices are designed to guide professional learning to ensure all Spring Lake Park teachers of mathematics possess the following qualities of an expert mathematics teacher (adapted from Taking Action (National Council of Teachers of Mathematics).

Teachers of mathematics:

- Believe each student is capable of making sense of mathematical ideas and is able to use understanding and reasoning to solve authentic problems.
- Attend to and value each students' thinking, including emergent understanding and mistakes.
- Engage students in challenging tasks and collaborative inquiry.
- Challenge and support each student in developing conceptual understanding and proficiency in mathematics.
- Observe and listen as students work in order to provide an appropriate level of feedback and support.
- Believe that each student can succeed in doing meaningful, high-quality mathematics learning and work, not simply execute procedures with speed and accuracy.

Teaching and Learning Practices for Mathematics

Build a Mathematical Mindframe

All teachers need to believe that each student can do mathematics at deep levels.

Building a mathematical mindframe is about helping each student develop an identity as a confident and competent member of a "math community" -- using mathematics in powerful ways across content areas and the contexts of their lives. Students learning of mathematics involves: sense-making, flexible thinking, questioning, seeing and using patterns, problem solving, communicating, creative thinking about space, data, and numbers, and making and analyzing connections and relationships. Mathematics is not about memorization, rote practice, and definitive answers.

We utilize students' knowledge and experiences as resources for learning mathematics. We promote student participation and value contributions of each student. Conditions that teachers create to build a mathematical mindframe include:

- Recognizing and finding the value in mistakes and using mistakes as integral parts of learning.
- Creating the conditions for students to think creatively about mathematics.
- Encouraging and allowing for multiple ways to solve.
- Building a deep conceptual understanding prior to procedural.
- Creating conditions for students to persist through difficulties.

Mental models that not everyone can learn mathematics and unintentional messaging can create fear, anxiety, and a lack of confidence in our students.

To build a mathematical mindframe, we can shift our (unintentional) messaging from:	To an intentional message of:
This is hard/may be hard … I was bad at math … I didn't like math … Math is about solving problems quickly … Math is about practicing and memorizing … Math is only about a correct answer …	Let's embrace challenges. We'll learn from mistakes. Questions are important. Math is about sense-making. Math is about creativity. Math is about the process. Math is about learning, connecting, and applying, not just memorizing procedures. Math is about depth, building to fluency

Implement “Rich” Tasks

All students need to learn mathematics through rich tasks that focus on conceptual mathematical understanding. Rich tasks require complex non-algorithmic thinking and reasoning. A predictable path to a solution is not explicit, which causes students to have to explore and access mathematical concepts, processes, and relationships. Students need to draw on their own relevant knowledge and experiences as they work through the task. Rich tasks require cognitive effort and persistence because of the unpredictable nature of the thinking process required. In order to keep the cognitive demands and learning high, teachers resist rescuing students by offering answers or specific strategies to solve. Instead, the teacher asks about student thinking, encourages continued effort on the task, and provides time for students to linger in their thinking.

Rich tasks often involve inquiry. Students are first allowed to explore concepts, develop questions, and develop their own reasoning and multiple representations prior to teachers providing procedural strategies and algorithms to increase efficiency in solving problems.

When designing rich mathematical tasks (from procedural tasks) for students:

- Is the teacher or student doing the thinking and the work?
- Are you rescuing the student or carefully coaching them to think on their own?
- Can I open the task to encourage multiple methods, pathways, and representations?
- Can I make it an inquiry task?
- Can I ask the problem before teaching the method?
- Can I add a visual component?
- Can I make it low floor and high ceiling? (accessible and challenging)
- Can I add the requirement to convince and reason?
- Can I open up the task?
- Can I provide two problems/examples/solutions and have students identify the similarities and/or differences?

Also consider:

- Is the task purposeful and relevant across content areas?
- Is the task purposeful and relevant in the real world?

To reflect real-world application of 21st century teamwork, students should have opportunities to collaborate with others to work on rich mathematical tasks. Through collaboration with others, students propose ideas without being afraid of making mistakes and respect and connect to the thinking of others.

Build Conceptual Understanding Prior to Procedural Fluency

Fluency in mathematics builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of procedures as tools in solving problems. Effective teaching of mathematics builds fluency with procedures after building a foundation of conceptual understanding; over time students become skillful in using procedures flexibly as they solve contextual and other mathematical problems. When students gain conceptual understanding prior to procedural fluency, they learn to use procedures mindfully with strong connections to meaning and sense-making.

Both conceptual understanding and procedural fluency are important in math learning. It is important to move to procedures when students have become so familiar with the meaning of the quantities and operations that they draw naturally on what they know to solve problems. When knowledge of procedures builds from conceptual understanding, students see mathematics as making sense and are able to use mathematical procedures meaningfully and appropriately. Moving to fluency with procedures should not be rushed.

Teaching procedures and formulas before building conceptual understanding creates conditions in which:	Teaching conceptual understanding followed by procedures and formulas builds number sense (mathematical fluency) and leads to:
• “Drill and practice” of procedures don’t help to build conceptual understanding. • Students are more likely to forget or mix up critical steps and apply procedures in ways that are inappropriate to the task. • Students may get lost in extraneous steps. • Students may not be able to represent their thinking. • Students may not be able to explain or justify their thinking. • Students may not be able to judge the reasonableness of their answer.	• **Flexible thinking.** Students choose approaches appropriate to the numbers in the task. • **Understanding and explaining approaches and strategies**, including computational algorithms. Students’ thinking should be based on mathematical ideas. At the foundational level this includes knowledge of base 10, properties of multiplication and division, and relationships between numbers. • **Accuracy**. Students not only get the “right” answer, but consider the application of the operation. They record work carefully. They can estimate and judge the reasonableness of the answer (this can only occur when fluency is integrated with understanding). • **Efficiency**. Students use strategic thinking to see and understand a clear path to a solution (they do not get lost in lots of extraneous steps).

Questioning

Purposeful questioning makes students' mathematics thinking visible. Setting up the conditions for students to get engaged in real-world problems and contextual mathematical learning allows for students to be curious about math. This creates the conditions for students to wonder and ask their own questions. As students try to resolve their questions, it is important for teachers to pose purposeful questions that deepen student thinking.

Posing the right purposeful question at the right time (throughout the learning experience and work) requires depth of knowledge of mathematics, depth of knowledge of student learning of mathematics, and knowledge of the students in the class. Teachers use purposeful questions and probing to assess and advance each students' reasoning and sense-making about mathematical concepts and mathematical connections. Posing purposeful questions gives students "math authority" as they become the thinkers who must make sense of the mathematics. It's not only the questioning that makes the impact, but the students' explanations of their mathematical understandings and reasonings (both the conceptual and procedural).

When posing purposeful questions:

- Am I allowing for wait time (think-time) after questions and after student responses? (makes a difference in student learning)
- Am I carefully selecting who responds to questions and the order in which they respond so that conceptual understanding builds? (instead of relying on traditional hand raising)
- Am I observing student thinking and understanding and asking the "right" question at the "right" time?
- Am I asking more probing and higher-level questions and fewer recall questions and questions that lead students down a specific mathematics pathway? (With purposeful questions and wait time, students give longer responses at higher levels of cognitive learning and responses more connected to others' responses.)
- Am I creating a mathematical community where the focus is less about "right" answers and more about conceptual understanding so students feel more confident volunteering and are less likely to respond, "I don't know?"
- Are students having opportunity to pose mathematical questions?

Utilize and Connect Multiple Mathematical Representations

Representations help students understand the abstract concepts of mathematics and provide a way to explore, analyze, and discuss math with others. Representations help students remember and make connections to previously learned mathematical concepts, learn new mathematical concepts, help them deepen their understanding, and support their sense-making when they are solving contextual or other mathematical problems.

Learning needs to focus on interconnections among all five types of representations (physical, visual, symbolic, verbal, contextual) so that students are able to move easily from one type of representation to another. It is important that students learn that utilizing representations is a tool for understanding and solving problems; the goal is not simply knowing the types of representations.

Physical Representations	Visual Representations	Symbolic Representations	Verbal Representations	Contextual Representations
Use concrete objects to show, study or manipulate mathematical thinking.	Show mathematical thinking using pictures, diagrams, number lines, graphs, and other drawings.	Record or work with mathematical concepts using numerals, variables, tables and other symbols.	Use language (oral or written) to define, describe, or discuss mathematical thinking.	Mathematical concepts occur within everyday, real world or imaginary situations.

When students use mathematical representations their conceptual understanding is revealed. Students reflect on their understanding and learning of mathematics and share why they used their chosen representations. Students learn from each other as they engage in dialogue, make connections and comparisons among their chosen representations, and make decisions on which representations to use in solving problems. Through analyzing a student's use of multiple representations, teachers notice strengths, conceptual gaps, and what is needed next in student learning.

Facilitate Meaningful Mathematical Discourse

Mathematical discourse engages students in sense-making and connecting mathematical ideas. Discourse is focused on conceptual learning, reasoning, and problem solving. Discourse needs to go beyond "show and tell" of procedures. Mathematical discourse gives the teacher valuable insight and information about student thinking, learning, and identification of gaps in conceptual understanding. Students naturally incorporate mathematical representations to deepen their understanding and make connections between other strategies. Mathematical discourse is not exclusive to verbal explanations, but includes summarizing and explaining thinking through writing.

Mathematical discourse needs to have learning goals in mind and be sequential. Facilitation of discourse helps students move through a coherent math storyline. Prior to facilitating mathematical discourse, teachers need to anticipate student thinking and possible ways students will approach the learning and work. While students are engaging in rich mathematical tasks, the teacher is listening and noting the learning and work, and is selecting students who will share their thinking at different stages with varied strategies and representations (sequenced in a logical order). Student discourse should help students connect, compare, and contrast the multiple strategies, which build upon and strengthen mathematical fluency.

Teachers facilitate conversations between students and encourage them to ask each other questions. Teachers ask probing questions to help students make connections and deepen learning. This leads to students asking more questions and clarifying their own thinking. As students develop their thinking, they more often ask “why” and for justification of answers, leading to students asking initial questions about mathematics.

Teachers need to work together with students to establish agreed upon norms and routines for collaboration and mathematical discourse that allow students to interact and respond to each other.

When facilitating mathematical discourse:

- Am I expecting students to justify and reason in the explanations vs “show and tell?”
- Am I providing students opportunities for oral and written discourse?
- Prior to the discourse, have I anticipated possible student approaches?
- Am I allowing the time for students to develop deep discourse about their mathematical thinking?
- Am I carefully observing and noting student thinking and work?
- Am I asking probing questions to help students make connections and deepen learning?
- Have I created norms and routines that allow for all voices and mathematical ideas to be heard?

Collaboration and Grouping

Collaborating with others is a tool to help students learn math. Working with a partner, explaining and writing to learn have great impact on student learning. Students need some time to think individually, time to talk with a partner, and time to explain and/or write about their learning.

Every student benefits from heterogeneous grouping and a collaborative approach to mathematical learning, including students who perform procedural tasks more quickly. Each student needs opportunities to think deeply and reason about conceptual mathematical ideas, explain and justify their work, and see mathematics from varying perspectives. Through discussing their thinking with others, students gain deeper conceptual knowledge and accelerate their learning. In a heterogeneous group the conversation rises to the highest thinking in the group.

Within a heterogeneous grouping, there may at times be opportunity for teachers to work with small groups of students who have a similar conceptual understanding that needs further development. More often, students need to be grouped heterogeneously to allow for deep learning through rich tasks.

All teachers need to believe that each student can do mathematics at deep levels. Heterogeneous grouping of students is equitable and helps students believe that they can achieve. Student efficacy increases engagement and persistence which leads to increased student achievement.

Feedback and Assessment

Teachers use daily ongoing formative assessment to make decisions and give feedback in the moment, day-to-day and minute-to-minute, in response to student understandings and misconceptions. Teachers need to examine the student work to notice strengths and conceptual gaps and determine next steps in student learning. Formative assessment of students' learning needs to communicate information about students' mathematical conceptual understanding and mathematical reasoning, rather than an emphasis on the "right" answer. Students and teachers value making mistakes as a part of the learning process.

Ongoing, specific feedback improves student achievement and motivation. Teachers set up conditions for students to take ownership of their learning through understanding what they have learned, where they are now in their learning, and where they are going. They work together to set goals and next steps in their learning.

Over-utilizing summative assessments and emphasizing grading impacts students' learning of mathematics and the mindframe with which they approach mathematics. Students see scores and grades as indicators of whether they are good or bad at math. A focus on formative assessment helps keep student mindframe on what they have learned and what is next for them to learn.

Homework

Empirical research has shown that at the elementary level, homework can be unnecessary, not effective, or damaging to students' thinking about mathematics. Students are often given homework that emphasizes procedural mathematical practices. If a student has not yet developed conceptual understanding, this type of homework will reinforce the use of mathematics as procedural processes and not build conceptual understandings. If homework is given, it needs an emphasis on conceptual understanding of mathematical concepts. It also needs to include teacher feedback, rather than simply being corrected in class or a score given.

At the secondary level, homework can have some benefit if chosen purposefully and completion schedules are flexible. Homework is most effective when it is at the appropriate difficulty level and is differentiated for each student. Homework is a piece of formative assessment that requires teacher feedback, rather than simply being corrected in class or a score given. Purposes for homework may include presenting new content in a flipped format, practicing a skill or process for which students have built conceptual understanding but aren't yet fluent, summarizing and reflecting on conceptual knowledge and providing opportunities to explore topics of students' own interest that connect to their mathematical learning.

Below are some suggestions for purposeful homework across all grades K-12. Homework should include only a few problems or be a reflection on learning that occurred during the school day:

- What were the main mathematical concepts or ideas that you learned today or that we discussed in class today?
- Describe a mistake or misconception that you or a classmate had in class today. What did you learn from this mistake or misconception?
- How did you or your group approach today's problem(s)? Was your approach successful? What did you learn from your approach?
- Describe in detail how someone else in class approached a problem. How is their approach similar or different to the way you approached the problem?
- What new vocabulary words or terms were introduced today? What do you believe each new word means? Give an example/picture of each word.
- How is … similar or different to …?

When assigning homework:

- What is the purpose of this homework?
- Is this homework building conceptual understanding?
- Have I differentiated this homework based on my knowledge of students' conceptual understanding?
- Will students be able to complete the homework independently?
- How will I give individual students feedback on homework?

Resources (with examples) for deepening teacher learning and implementation include:
Taking Action: Implementing Effective Mathematics (NCTM) K-5, 6-8, 9-12
Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching (Boaler)