Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Sixth Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**
- **f. Justify their conclusions, communicate them to others, and respond to the arguments of others.**

- **g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.**

4. Model with mathematics.

Mathematically proficient students:

- **a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.**
- **b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.**
- **c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.**
- **d. Analyze mathematical relationships to draw conclusions.**

5. Use appropriate tools strategically.

Mathematically proficient students:

- **a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)**
- **b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.**

6. Attend to precision.

Mathematically proficient students:

- **a. Communicate their understanding of mathematics to others.**
- **b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.**
- **c. Specify units of measure and use label parts of graphs and charts**
- **d. Strive for accuracy.**

7. Look for and make use of structure.

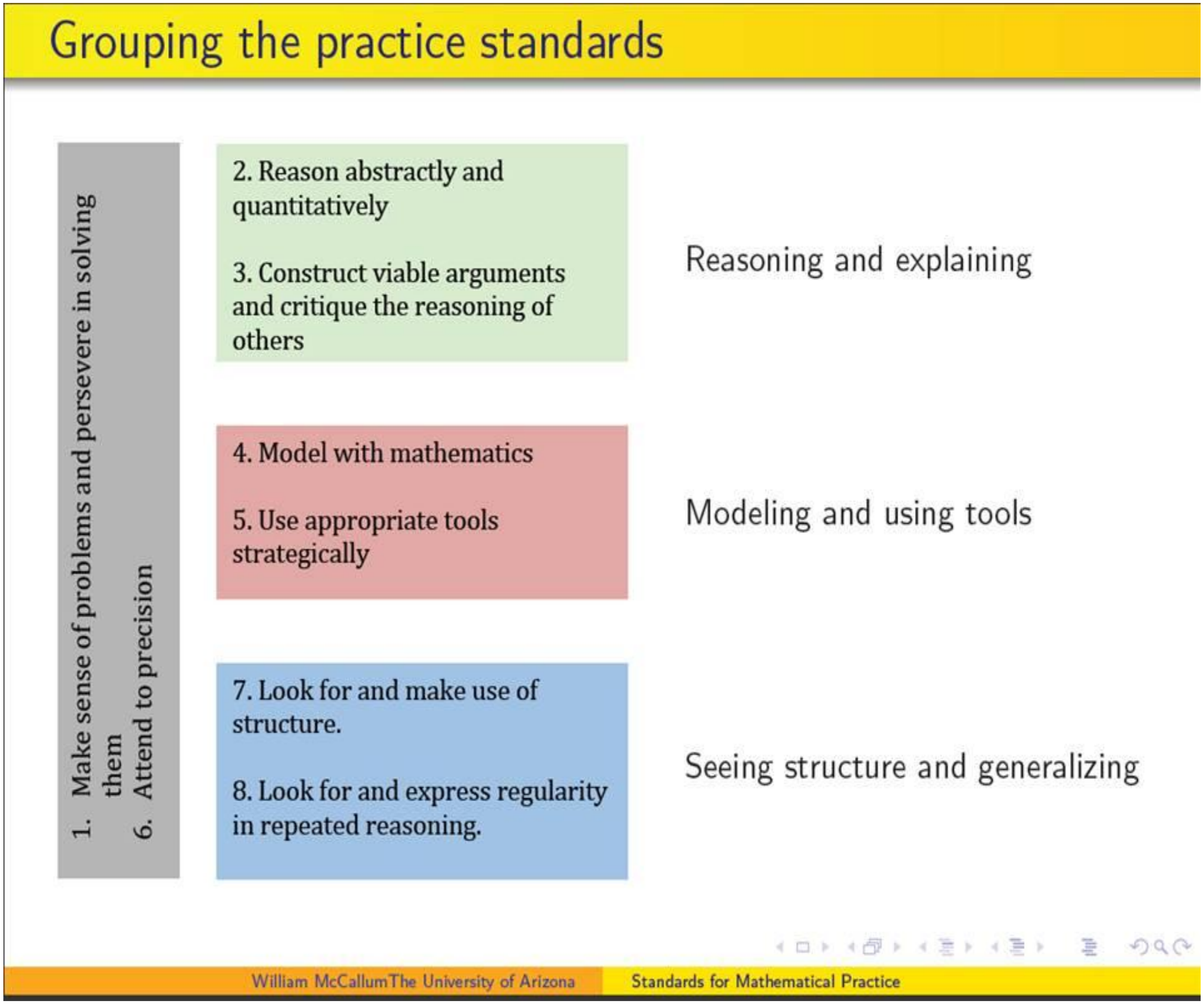
Mathematically proficient students:

- **a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.**
- **b. Apply and discuss properties.**

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- **a. Look for mathematically sound shortcuts.**
- **b. Use repeated applications to generalize properties.**



Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

Reasoning and explaining

4. Model with mathematics

5. Use appropriate tools strategically

Modeling and using tools

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Seeing structure and generalizing

William McCallumThe University of Arizona

Standards for Mathematical Practice

Standards for Mathematical Practice: Grade 6 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
6.MP.1. Make sense of problems and persevere in solving them.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
6.MP.2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
6.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
6.MP.4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
6.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6.MP.6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
6.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2\ (3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
6.MP.8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.• Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

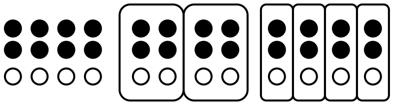
- Develop understanding of statistical variability.
- Summarize and describe distributions.

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-towhole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

Standards/Learning Objectives		***Explanations and Examples***
6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”* Write ratio notation-a:b, a to b, a/b ▪ Know order matters when writing a ratio ▪ Know ratios can be simplified ▪ Know ratios compare two quantities; the quantities do not have to be the same unit of measure ▪ Recognize that ratios appear in a variety of different contexts; part-to-whole, part-to-part, and rates ▪ Generalize that all ratios relate two quantities or measures within a given situation in a multiplicative relationship ▪ Analyze your context to determine which type of ratio is represented		A ratio is a comparison of two quantities which can be written as a to b, $\frac{a}{b}$, or a:b. A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically. A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and 2 black circles to 1 white circle (2:1).  Students should be able to identify all these ratios and describe them using “For every…., there are …”

Ratios and Proportional Relationships **6.RP**

Understand ratio concepts and use ratio reasoning to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-towhole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

Standards/Learning Objectives

6.RP.2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (Expectations for unit rates in this grade are limited to non-complex fractions.)

- Identify and calculate a unit rate
- Use appropriate math terminology as related to rate
- Analyze the relationship between a ratio $a:b$ and a unit rate a/b where $b \neq 0$

Explanations and Examples

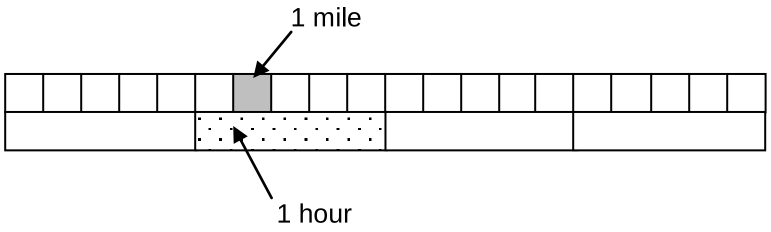
A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates.

In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

Examples:

- On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?

  Solution: You can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\,hr}{1\,mi}$. Students can represent the relationship between 20 miles and 4 hours.



- A simple modeling clay recipe calls for 1 cup corn starch, 2 cups salt, and 2 cups boiling water. How many cups of corn starch are needed to mix with each cup of salt?

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-towhole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

Standards/Learning Objectives

6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?*

Explanations and Examples

Examples:

- Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

There are several strategies that students could use to determine the solution to this problem.

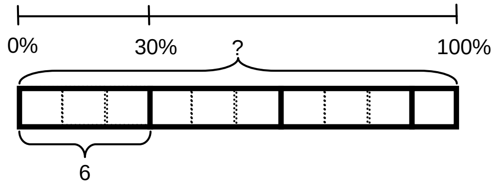
  - Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards must be 8 yards (3 yards and 5 yards).
  - Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

- Compare the number of black to white circles. If the ratio remains the same, how many black circles will you have if you have 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

- If 6 is 30% of a value, what is that value? (Solution: 20)



Continued on next page

Ratios and Proportional Relationships **6.RP**

Understand ratio concepts and use ratio reasoning to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-towhole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

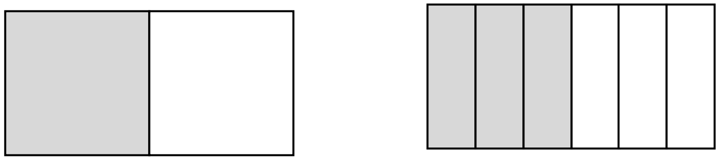
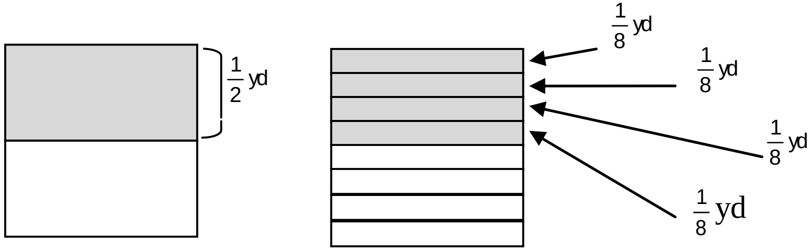
Standards/Learning Objectives	***Explanations and Examples***
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	• A credit card company charges 17% interest on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If your bill totals $450 for this month, how much interest would you have to pay if you let the balance carry to the next month? Show the relationship on a graph and use the graph to predict the interest charges for a $300 balance.

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **reciprocal, multiplicative inverses, visual fraction model**

Standards/Learning Objectives	***Explanations and Examples***
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* ▪ Compute quotients of fractions divided by fractions (including mixed numbers) ▪ Interpret quotients of fractions ▪ Solve word problems involving division of fractions by fractions	Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions build upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer generated models. Examples: • 3 people share $\frac{1}{2}$ pound of chocolate. How much of a pound of chocolate does each person get? Solution: Each person gets $\frac{1}{6}$ lb of chocolate.  • Manny has $\frac{1}{2}$ yard of fabric to make book covers. Each book is made from $\frac{1}{8}$ yard of fabric. How many book covers can Manny make? Solution: Manny can make 4 book covers.  • Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

The Number System **6.NS**

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **reciprocal, multiplicative inverses, visual fraction model**

Standards/Learning Objectives		***Explanations and Examples***

Context: You are making a recipe that calls for $\frac{2}{3}$ cup of yogurt. You have $\frac{1}{2}$ cup of yogurt from a snack pack. How much of the recipe can you make?

Continued on next page

Explanation of Model:

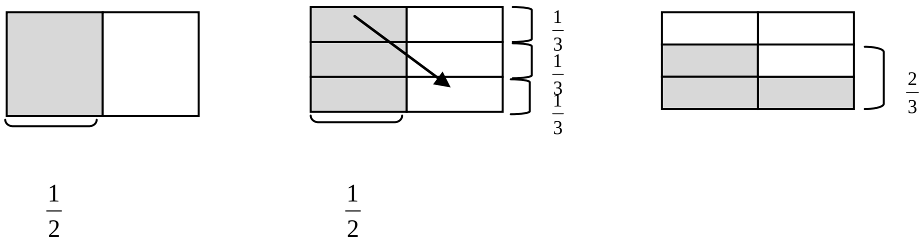
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so you can only make ¾ of the recipe.



The Number System **6.NS**

Compute fluently with multi-digit numbers and find common factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multi-digit**

Standards/Learning Objectives		***Explanations and Examples***
6.NS.2. Fluently divide multi-digit numbers using the standard algorithm.		Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level. As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "there are 200 thirty-twos in 8456 " and could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ -\underline{6400} \\ \underline{2056} \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -\underline{6400} \\ 2056 \end{array}$	There are 60 thirty twos in 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -\underline{6400} \\ 2056 \\ -\underline{1920} \\ \underline{136} \end{array}$	60 times 32 is 1920. 2056 minus 1920 is 136.

Continued on next page

The Number System **6.NS**
Compute fluently with multi-digit numbers and find common factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multi-digit**

Standards/Learning Objectives | ***Explanations and Examples***

$\begin{array}{r} 264 \\ 32\overline{)8456} \\ -6400 \\ 2056 \\ -1920 \\ \hline 136 \\ -128 \\ \hline \end{array}$	There are 4 thirty twos in 136. 4 times 32 is 128.
$\begin{array}{r} 264 \\ 32\overline{)8456} \\ \underline{-6400} \\ 2056 \\ \underline{-1920} \\ 136 \\ \underline{-128} \\ 8 \end{array}$	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8

6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. .

The use of estimation strategies supports student understanding of operating on decimals.

Example:

- First, students estimate the sum and then find the exact sum of 14.4 and 8.75. An estimate of the sum might be 14 + 9 or 23. Students may also state if their estimate is low or high. They would expect their answer to be greater than 23. They can use their estimates to self-correct.

  Answers of 10.19 or 101.9 indicate that students are not considering the concept of place value when adding (adding tenths to tenths or hundredths to hundredths) whereas answers like 22.125 or 22.79 indicate that students are having difficulty understanding how the four-tenths and seventy-five hundredths fit together to make one whole and 25 hundredths.

  Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals.

The Number System **6.NS**
Compute fluently with multi-digit numbers and find common factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multi-digit**

Standards/Learning Objectives		***Explanations and Examples***
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4*(*9+2*). ▪ Fluently identify the factors of two whole numbers less than or equal to 100 and determine the Greatest Common Factor ▪ Fluently identify the multiples of two whole numbers less than or equal to 12 and determine the Least Common Factor ▪ Apply the Distributive Property to rewrite addition problems by factoring out the Greatest Common Factor		Examples: • What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF? Solution: $2^2 \ast 3 = 12$. Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus 2 x 2 x 3 is the greatest common factor.) • What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM? Solution: $2^3 \ast 3 = 24$. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3 (2 x 2 x 3). To be a multiple of 8, a number must have 3 factors of 2 (2 x 2 x 2). Thus the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 (2 x 2 x 2 x 3). • Rewrite 84 + 28 by using the distributive property. Have you divided by the largest common factor? How do you know? • Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions. ○ 27 + 36 = 9 (3 + 4) 63 = 9 x 7 63 = 63 ○ 31 + 80 There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because 2 x 31 is 62 and 3 x 31 is 93.

The Number System **6.NS**

Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

<u>Standards/Learning Objectives</u>		***<u>Explanations and Examples</u>***
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.		Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. Example 1: a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level

The Number System **6.NS**

Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

Standards/Learning Objectives

6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

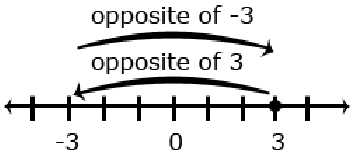
a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Explanations and Examples

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.



Example:

- Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

$\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, -0.75)$

The Number System 6.NS

Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

Standards/Learning Objectives

6.NS.7. Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.*

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

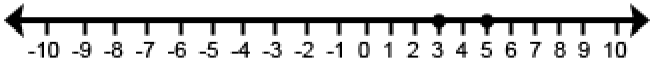
d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Explanations and Examples

Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers.

In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers.

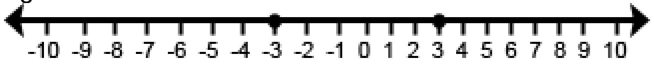
Case 1: Two positive numbers



5 > 3

5 is greater than 3

Case 2: One positive and one negative number

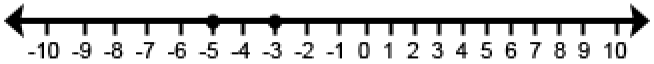


3 > -3

positive 3 is greater than negative 3

negative 3 is less than positive 3

Case 3: Two negative numbers



-3 > -5

negative 3 is greater than negative 5

negative 5 is less than negative 3

Continued on next page

The Number System **6.NS**

Apply and extend previous understandings of numbers to the system of rational numbers.

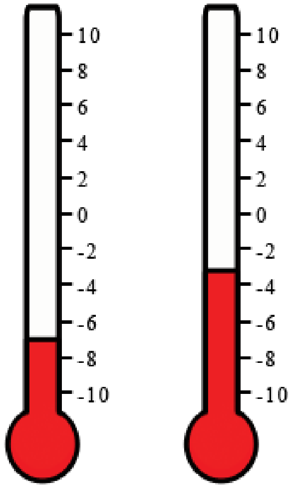
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

Standards/Learning Objectives	***Explanations and Examples***

Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in grade 7.

Example:

- One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.



Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

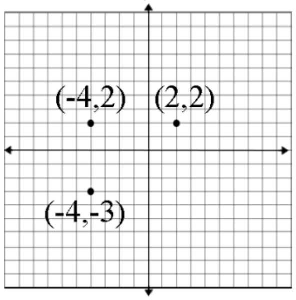
Example:

- The Great Barrier Reef is the world’s largest reef system and is located off the coast of Australia. It reaches from the surface of the ocean to a depth of 150 meters. Students could represent this value as less than 150 meters or a depth no greater than 150 meters below sea level.

The Number System **6.NS**

Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

Standards/Learning Objectives	***Explanations and Examples***
6.NS.8. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Example: • If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?  To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is $\lvert -4 \rvert$ or 4 units to the left of 0 and 2 is $\lvert 2 \rvert$ or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, $\lvert -4 \rvert$ + $\lvert 2 \rvert$.

Expressions and Equations **6.EE**

Apply and extend previous understandings of arithmetic to algebraic expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables**

Standards/Learning Objectives		***Explanations and Examples***
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.		Examples: • Write the following as a numerical expressions using exponential notation. ○ The area of a square with a side length of 8 m (Solution: $8^2 m^2$) ○ The volume of a cube with a side length of 5 ft: (Solution: $5^3 ft^3$) ○ Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) • Evaluate: ○ 4^3 (Solution: 64) ○ $5+2^4 \bullet 6$ (Solution: 101) ○ $7^2 - 24 \div 3 + 26$ (Solution: 67)
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8+7) as a product of two factors; view (8+7) as both a single entity and a sum of two terms*		It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. • r + 21 as "some number plus 21 as well as "r plus 21" • n • 6 as "some number times 6 as well as "n times 6" • $\frac{s}{6}$ and s ÷ 6 as "as some number divided by 6" as well as "s divided by 6" Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes. Continued on next page

Expressions and Equations **6.EE**

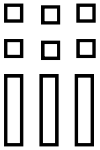
Apply and extend previous understandings of arithmetic to algebraic expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables**

Standards/Learning Objectives		***Explanations and Examples***
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V=s^3$ *and* $A=6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s=1/2$.		Consider the following expression: $x^2+5y+3x+6$ The variables are x and y. There are 4 terms, x^2, 5y, 3x, and 6. There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1\ x^2$. The term 5y represent 5 y's or 5 * y. There is one constant term, 6. The expression shows a sum of all four terms. Examples: • 7 more than 3 times a number (Solution: $3x+7$) • 3 times the sum of a number and 5 (Solution: $3(x+5)$ • 7 less than the product of 2 and a number (Solution: $2x-7$) • Twice the difference between a number and 5 (Solution: $2(z-5)$) • Evaluate $5(n + 3) – 7n$, when $n = \frac{1}{2}$. • The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where *c* is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25. • The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.

Expressions and Equations **6.EE**
Apply and extend previous understandings of arithmetic to algebraic expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables**

Standards/Learning Objectives		***Explanations and Examples***
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3 (2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6 (4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.		Students use their understanding of multiplication to interpret $3 (2 + x)$. *For example, 3 groups of* $(2 + x)$. They use a model to represent x, and make an array to show the meaning of $3(2 + x)$. They can explain why it makes sense that $3(2 + x)$ is equal to $6 + 3x$. An array with 3 columns and $x + 2$ in each column:  Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y ***must be*** $3y$. They also the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$: $y + y + y = y \text{ x } 1 + y \text{ x } 1 + y \text{ x } 1 = y \text{ x } (1 + 1 + 1) = y \text{ x } 3 = 3y$

Expressions and Equations **6.EE**

Apply and extend previous understandings of arithmetic to algebraic expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables**

Standards/Learning Objectives

6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

.

Explanations and Examples

Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example:

- Are the expressions equivalent? How do you know?

4m + 8 *4(m+2)* *3m + 8 + m* *2 + 2m + m + 6 + m*

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *(3m + m) + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2 + 2m + m + 6 + m* *2 + 6 + 2m + m + m* *(2 + 6) + (2m + m + m)* *8 + 4m* *4m + 8*	*Combined like terms*

Expressions and Equations **6.EE**

Reason about and solve one-variable equations and inequalities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**

Standards/Learning Objectives	***Explanations and Examples***
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	See below

Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities.

Consider the following situation: Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as “some number was added to 26 and the result was 100”. Students ask themselves “What number was added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem.

- Reasoning: 26 + 70 is 96. 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations:
  $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps you find n easily.
- Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

100	
26	n

Examples:

- The equation $0.44s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to determine your answer. Show that your solution is correct using substitution.
- Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?

Expressions and Equations Reason about and solve one-variable equations and inequalities.		**6.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**		
Standards/Learning Objectives		***Explanations and Examples***
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.		Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. Examples: • Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. (Solution: $2c + 3$ where c represents the number of crayons that Elizabeth has.) • An amusement park charges \$28 to enter and \$0.35 per ticket. Write an algebraic expression to represent the total amount spent. (Solution: $28 + 0.35t$ where t represents the number of tickets purchased) • Andrew has a summer job doing yard work. He is paid \$15 per hour and a \$20 bonus when he completes the yard. He was paid \$85 for completing one yard. Write an equation to represent the amount of money he earned. (Solution: $15h + 20 = 85$ where h is the number of hours worked) • Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where c represents the cost of an item • Bill earned \$5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. (Solution: $\$5.00 + n$)

Expressions and Equations 6.EE

Reason about and solve one-variable equations and inequalities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**

Standards/Learning Objectives

6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers

- Define inverse operation
- Know how inverse operations can be used in solving one-variable equations
- Apply rules of the form $x + p = q$ and $px = q$, for cases in which p, q, and x are all nonnegative rational numbers, to solve real-world and mathematical problems; with only one unknown quantity
- Develop a rule for solving one-step equations using inverse operations with nonnegative rational coefficients

Explanations and Examples

Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Example:

- Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

\$56.58		
J	J	J

Sample Solution: Students might say: “I created the bar model to show the cost of the three pairs of jeans. Each bar labeled *J* is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10 x 3 is only 30 but less than \$20 each because 20 x 3 is 60. If I start with \$15 each, I am up to \$45. I have \$11.58 left. I then give each pair of jeans \$3. That’s \$9 more dollars. I only have \$2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.86. Each pair of jeans costs \$18.86 (15+3+0.86). I double check that the jeans cost \$18.86 each because \$18.86 x 3 is \$56.58.”

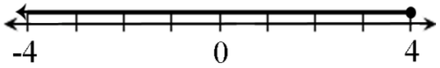
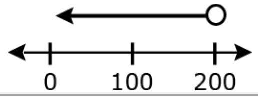
- Julio gets paid \$20 for babysitting. He spends \$1.99 on a package of trading cards and \$6.50 on lunch. Write and solve an equation to show how much money Julio has left.
  (Solution: $20 = 1.99 + 6.50 + x$, $x = \$11.51$)

20		
1.99	6.50	money left over (m)

Expressions and Equations 6.EE

Reason about and solve one-variable equations and inequalities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**

Standards/Learning Objectives		***Explanations and Examples***
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.		Examples: - Graph $x \le 4$.  - Jonas spent more than $50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line. - Less than $200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line. Solution: $200 > x$ 

Expressions and Equations **6.EE**

Represent and analyze quantitative relationships between dependent and independent variables.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **dependent variables, independent variables, discrete data, continuous data**

Standards/Learning Objectives

6.EE.9. Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Explanations and Examples

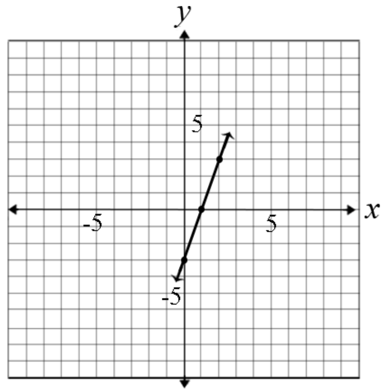
Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.

Examples:

- What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

- Use the graph below to describe the change in *y* as *x* increases by 1.



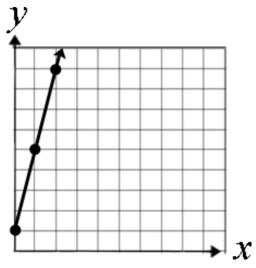
Continued on next page

Expressions and Equations Represent and analyze quantitative relationships between dependent and independent variables.		**6.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **dependent variables, independent variables, discrete data, continuous data**		
Standards/Learning Objectives		***Explanations and Examples***

- Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table and graph to demonstrate the relationship between the number of weeks that pass and the amount in her savings account.
  - Language: Susan has $1 in her savings account. She is going to save $4 each week.
  - Equation: $y = 4x + 1$
  - Table:

x	y
0	1
1	5
2	9

  - Graph:



Geometry **6.G**

Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism**

__Standards/Learning Objectives__

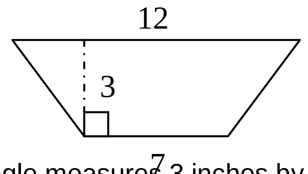
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians. For example, use Montana American Indian designs to decompose shapes and find the area.

__Explanations and Examples__

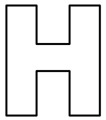
Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on NCTM's Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D (http://illuminations.nctm.org/ActivityDetail.aspx?ID=125)

Examples:

- Find the area of a triangle with a base length of three units and a height of four units.
- Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



- A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?
- The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?
- The sixth grade class at Hernandez School is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.
    - How large will the H be if measured in square feet?
    - The truck that will be used to bring the wood from the lumber yard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many pieces and what dimensions) are needed to complete the project?



Geometry **6.G**

Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism**

Standards/Learning Objectives

6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

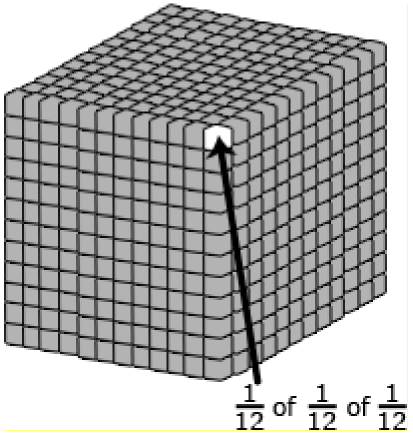
Explanations and Examples

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.

Examples:

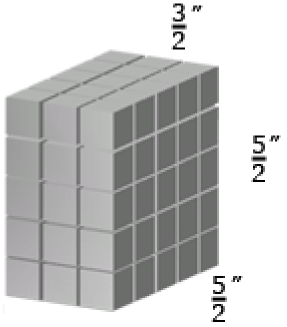
- The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft^3.



Continued on next page

Geometry **6.G**
Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism**

Standards/Learning Objectives		***Explanations and Examples***
		• The models show a rectangular prism with dimensions 3/2 inches, 5/2 inches, and 5/2 inches. Each of the cubic units in the model is $\frac{1}{8}$ in^3**.** Students work with the model to illustrate 3/2 x 5/2 x 5/2 = (3 x 5 x 5) x 1/8. Students reason that a small cube has volume 1/8 because 8 of them fit in a unit cube. 
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	.	Example: • On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile. ○ What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know? ○ What shape is formed by connecting the three locations? The city council is planning to place a city park in this area. How large is the area of the planned park?

Geometry **6.G**

Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism**

Standards/Learning Objectives

6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians.

Explanations and Examples

Students construct models and nets of three dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

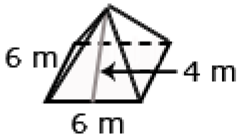
Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205).

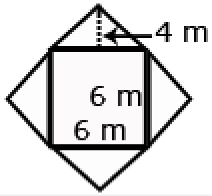
Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Examples:

- Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

- Create the net for a given prism or pyramid, and then use the net to calculate the surface area.





Statistics and Probability **6.SP**

Develop understanding of statistical variability.

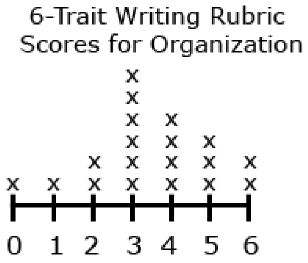
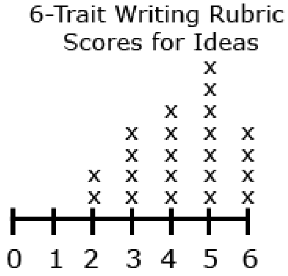
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean**

Standards/Learning Objectives		***Explanations and Examples***
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* ▪ Recognize that data can have variability ▪ Recognize a statistical question (examples versus non-examples)		Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Questions can result in a narrow or wide range of numerical values. For example, asking classmates "How old are the students in my class in years?" will result in less variability than asking "How old are the students in my class in months?" Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be:"How many hours per week on average do students at Jefferson Middle School exercise?" To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.

Statistics and Probability **6.SP**

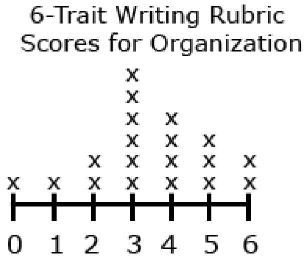
Develop understanding of statistical variability.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean**

Standards/Learning Objectives		***Explanations and Examples***
6.SP.2. Understand that a set of data collected (including Montana American Indian demographic data) to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.		The two dot plots show the 6-trait writing scores for a group of students on two different traits, organization and ideas. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.  

Statistics and Probability **6.SP**
Develop understanding of statistical variability.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean**

Standards/Learning Objectives		***Explanations and Examples***
6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	.	When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values. Example: • Consider the data shown in the dot plot of the six trait scores for organization for a group of students. ○ How many students are represented in the data set? ○ What are the mean, median, and mode of the data set? What do these values mean? How do they compare? ○ What is the range of the data? What does this value mean?  6-Trait Writing Rubric Scores for Organization

Statistics and Probability **6.SP**
Summarize and describe distributions.

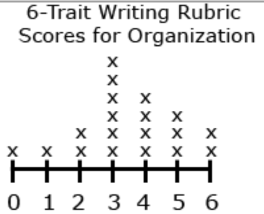
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives		***Explanations and Examples***
6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. ▪ Identify the components of dot plots, histograms, and box plots ▪ Find the median, quartile and interquartile range of a set of data ▪ Analyze a set of data to determine its variance ▪ Create a dot plot to display a set of numerical data ▪ Create a histogram to display a set of numerical data ▪ Create a box plot to display a set of numerical data		In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations. Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77 Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78 Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it. Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summary of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data. Examples: • Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Statistics and Probability **6.SP**
Summarize and describe distributions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

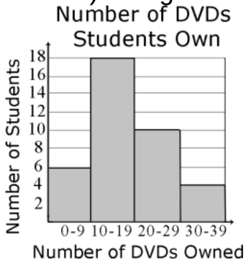
<u>Standards/Learning Objectives</u> | ***<u>Explanations and Examples</u>***


6-Trait Writing Rubric
Scores for Organization

- Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

A histogram using 5 ranges (0-9, 10-19, …30-39) to organize the data is displayed below.


Number of DVDs
Students Own

- Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

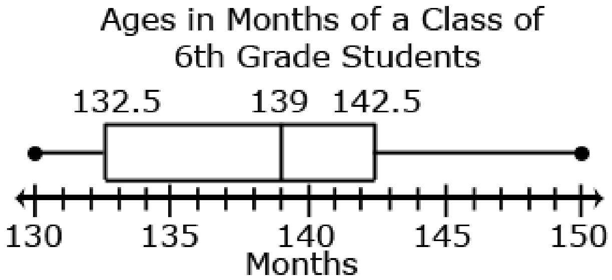
Statistics and Probability **6.SP**
Summarize and describe distributions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives		***Explanations and Examples***

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months



This box plot shows that

- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- the median class age is 139 months.

Statistics and Probability 6.SP

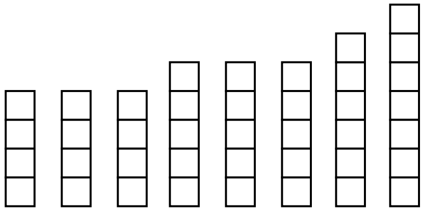
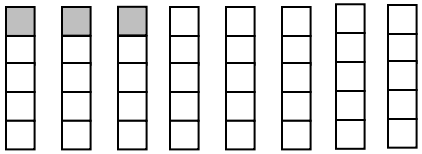
Summarize and describe distributions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives		***Explanations and Examples***
6.SP.5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.		Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set. <u>Understanding the Mean</u> The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean. For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes. Continued on next page

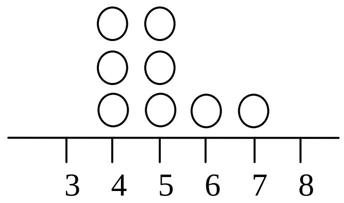
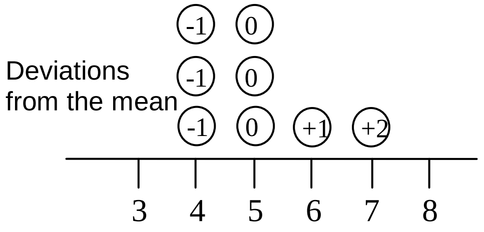
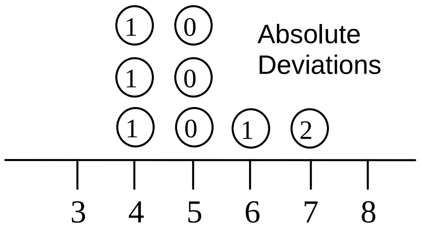
Statistics and Probability **6.SP**
Summarize and describe distributions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives		***Explanations and Examples***
		Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data set could be represented with stacking cubes.  Students can model the mean by “leveling” the stacks or distributing the blocks so the stacks are “fair”. Students are seeking to answer the question “If all of the students had the same number of letters in their name, how many letters would each person have?” One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.  If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have. Continued on next page

Statistics and Probability **6.SP**
Summarize and describe distributions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives	***Explanations and Examples***
	Understanding Mean Absolute Deviation The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets. In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.  To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.   Continued on next page

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives

Explanations and Examples

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
John	4	-1	1
Luis	4	-1	1
Mike	4	-1	1
Carol	5	0	0
Maria	5	0	0
Karen	5	0	0
Sierra	6	+1	1
Monique	7	+2	2
Total	40	0	6

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be 6 ÷ 8 or ¾ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?

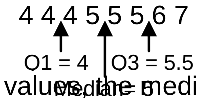
The mean of this data set is still 5. $\frac{(3+3+3+3+7+7+7)}{8} = \frac{40}{8} = 5$

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Sue	3	-2	2
Joe	3	-2	2
Jim	3	-2	2
Amy	3	-2	2
Sabrina	7	+2	2
Timothy	7	+2	2
Adelita	7	+2	2
Monique	7	+2	2
Total	40	0	16

The mean deviation of this data set is 16 ÷ 8 or 2. Although the mean is the same, there is much more variability in this data set.

Statistics and Probability **6.SP**
Summarize and describe distributions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier**

Standards/Learning Objectives		***Explanations and Examples***
		Understanding Medians and Quartiles Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed. Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest. 5 4 5 4 7 6 4 5 → 4 4 4 5 5 5 6 7  The middle value in the ordered data set is the median. If there are an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values which are both 5. Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2nd and 3rd value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data.

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another.Example: 3/4 and – 3/4 are additive inverses of one another because3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

---

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

---

[3] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

[4] To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

---

[5] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

---

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product **3 × 6 = ?**	**Group Size Unknown (“How many in each group?” Division)** **3 × ? = 18, and 18 ÷ 3 = ?**	**Number of Groups Unknown (“How many groups?” Division)** **? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a – c = b – c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $–a < –b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

<table>
<tr><th colspan="10">Mathematics Learning Progressions by Domain</th></tr>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">Ratios and Proportional Relationship</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td colspan="2">Functions</td></tr>
<tr><td colspan="10">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="4">Statistics and Probability</td></tr>
</table>