Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Fifth Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**
- **f. Justify their conclusions, communicate them to others, and respond to the arguments of others.**

g. **Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.**

4. Model with mathematics.

Mathematically proficient students:

a. **Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.**
b. **Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.**
c. **Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.**
d. **Analyze mathematical relationships to draw conclusions.**

5. Use appropriate tools strategically.

Mathematically proficient students:

a. **Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)**
b. **Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.**

6. Attend to precision.

Mathematically proficient students:

a. **Communicate their understanding of mathematics to others.**
b. **Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.**
c. **Specify units of measure and use label parts of graphs and charts**
d. **Strive for accuracy.**

7. Look for and make use of structure.

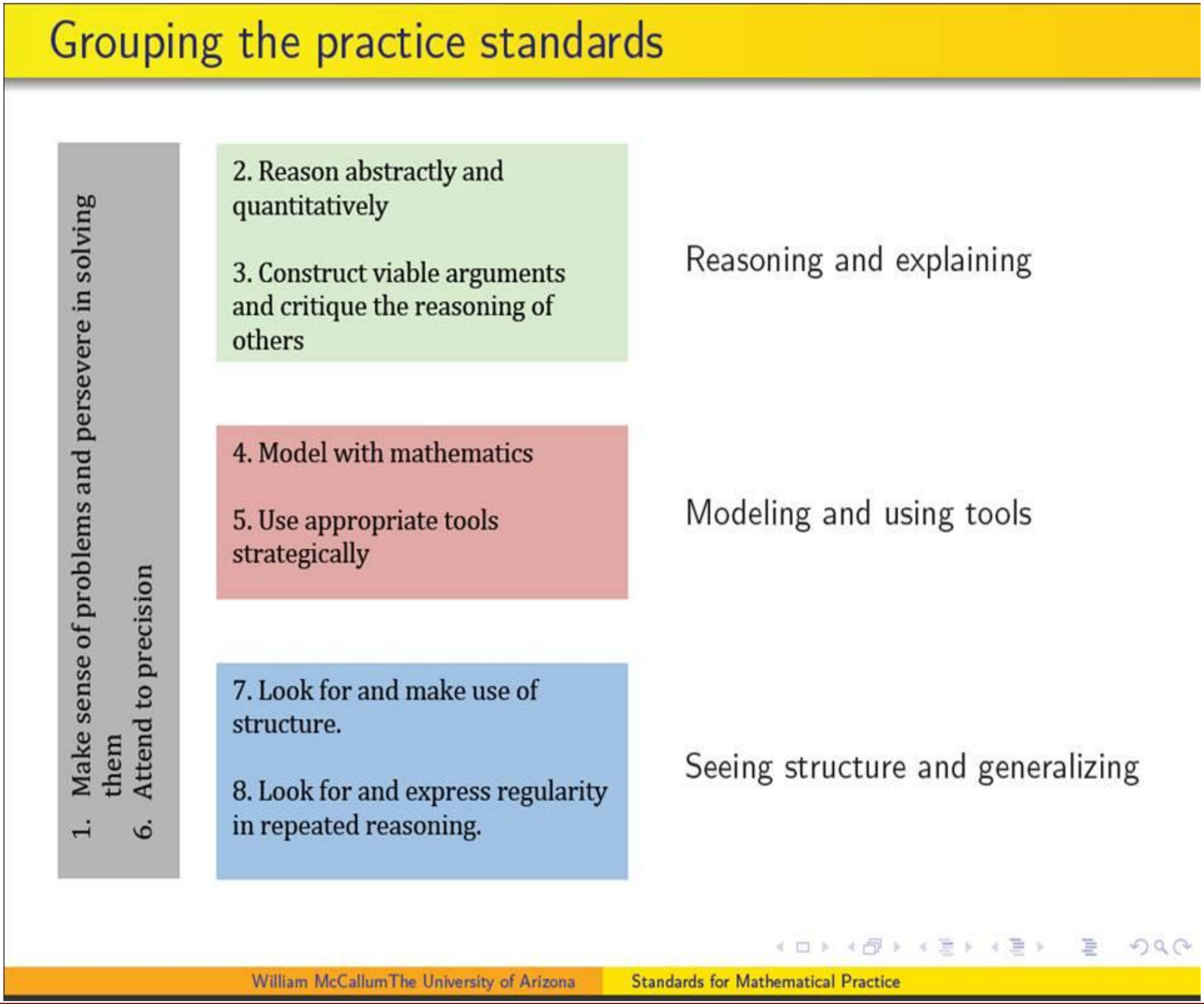
Mathematically proficient students:

a. **Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.**
b. **Apply and discuss properties.**

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

a. **Look for mathematically sound shortcuts.**
b. **Use repeated applications to generalize properties.**


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallumThe University of Arizona
Standards for Mathematical Practice

Standards for Mathematical Practice: Grade 5 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
5.MP.1. Make sense of problems and persevere in solving them.	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
5.MP.2. Reason abstractly and quantitatively.	Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
5.MP.3. Construct viable arguments and critique the reasoning of others.	In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
5.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5.MP.5. Use appropriate tools strategically.	Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
5.MP.6. Attend to precision.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
5.MP.7. Look for and make use of structure.	In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
5.MP.8. Look for and express regularity in repeated reasoning.	Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 5 Overview

Operations and Algebraic Thinking
- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data
- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Operations and Algebraic Thinking **5.OA**

Write and interpret numerical expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **parentheses, brackets, braces, numerical expressions**

Standards/Learning Objectives	***Explanations and Examples***
5.OA.1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. ▪ Use order of operations including parenthesis, brackets, or braces ▪ Evaluate expressions using the order of operations (including using parenthesis, brackets, or braces)	This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions. Examples: • (26 + 18) ÷ 4 Answer: 11 • {[2 x (3+5)] – 9} + [5 x (23-18)] Answer: 32 • 12 – (0.4 x 2) Answer: 11.2 • (2 + 3) x (1.5 – 0.5) Answer: 5 • $6-\left(\frac{1}{2}+\frac{1}{3}\right)$ Answer: 5 1/6 • { 80 ÷ [2 x (3 ½ + 1 ½)] }+ 100 Answer: 108 To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently. Examples: • 15 - 7 – 2 = 10 → 15 - (7 – 2) = 10 • 3 x 125 ÷ 25 + 7 = 22 → [3 x (125 ÷ 25)] + 7 = 22 • 24 ÷ 12 ÷ 6 ÷ 2 = 2 x 9 + 3 ÷ ½ → 24 ÷ [(12 ÷ 6) ÷ 2] = (2 x 9) + (3 ÷ ½) • Compare 3 x 2 + 5 and 3 x (2 + 5) • Compare 15 – 6 + 7 and 15 – (6 + 7)
5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression. Examples: • Students write an expression for calculations given in words such as "divide 144 by 12, and then subtract 7/8." They write (144 ÷ 12) – 7/8. • Students recognize that 0.5 x (300 ÷ 15) is ½ of (300 ÷ 15) without calculating the quotient.

Operations and Algebraic Thinking **5.OA**
Analyze patterns and relationships.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **numerical patterns, rules, ordered pairs, coordinate plane**

Standards/Learning Objectives

5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

- Generate two numerical patterns using two given rules
- Form ordered pairs consisting of corresponding terms for the two patterns
- Analyze and explain the relationships between corresponding terms in the two numerical patterns
- Graph generated ordered pairs on a coordinate plane

Explanations and Examples

Example:

Use the rule "add 3" to write a sequence of numbers. Starting with a 0, students write 0, 3, 6, 9, 12, . . .

Use the rule "add 6" to write a sequence of numbers. Starting with 0, students write 0, 6, 12, 18, 24, . . .

After comparing these two sequences, the students notice that each term in the second sequence is twice the corresponding terms of the first sequence. One way they justify this is by describing the patterns of the terms. Their justification may include some mathematical notation (See example below). A student may explain that both sequences start with zero and to generate each term of the second sequence he/she added 6, which is twice as much as was added to produce the terms in the first sequence. Students may also use the distributive property to describe the relationship between the two numerical patterns by reasoning that 6 + 6 + 6 = 2 (3 + 3 + 3).

$0, \quad ^{+3}3, \quad ^{+3}6, \quad ^{+3}9, \quad ^{+3}12, \ldots$

$0, \quad ^{+6}6, \quad ^{+6}12, \quad ^{+6}18, \quad ^{+6}24, \ldots$

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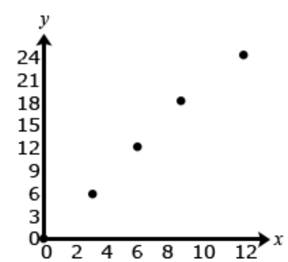
Once students can describe that the second sequence of numbers is twice the corresponding terms of the first sequence, the terms can be written in ordered pairs and then graphed on a coordinate grid. They should recognize that each point on the graph represents two quantities in which the second quantity is twice the first quantity.

<u>Ordered pairs</u>

(0, 0)
(3, 6)
(6, 12)
(9, 18)
(12, 24)



Number and Operations in Base Ten **5.NBT**

Understand the place value system.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round**

Standards/Learning Objectives	***Explanations and Examples***
5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $^{1}/_{10}$ of what it represents in the place to its left.	In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons.

Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left.

A student thinks, "I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is 1/10 of the value of a 5 in the hundreds place.

To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe 1/10 of that model using fractional language ("This is 1 out of 10 equal parts. So it is 1/10". I can write this using 1/10 or 0.1"). They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning, "0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit."

In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.

5	5	.	5	5

The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.

5	5	.	5	5

The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.

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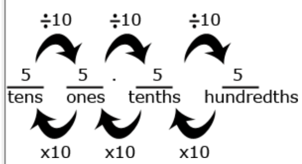
Number and Operations in Base Ten Understand the place value system.	5.NBT
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round**	
Standards/Learning Objectives	***Explanations and Examples***
	
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. ▪ Translate between powers of 10 written as 10 raised to a whole number exponent, in expanded form, and standard notation ▪ Represent powers of 10 using whole number exponents	Examples: Students might write: • 36 x 10 = 36 x 10^1 = 360 • 36 x 10 x 10 = 36 x 10^2 = 3600 • 36 x 10 x 10 x 10 = 36 x 10^3 = 36,000 • 36 x 10 x 10 x 10 x 10 = 36 x 10^4 = 360,000 Students might think and/or say: • I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left. • When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones). Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense. • $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places. • $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places. • $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

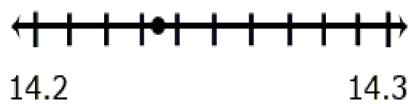
Number and Operations in Base Ten Understand the place value system.	**5.NBT**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round**	

Standards/Learning Objectives	***Explanations and Examples***
5.NBT.3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = $3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (^1/_{10}) + 9 \times (^1/_{100}) + 2 \times (^1/_{1000})$. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800). Example: Some equivalent forms of 0.72 are: 72/100 70/100 + 2/100 7/10 + 2/100 0.720 7 x (1/10) + 2 x (1/100) 7 x (1/10) + 2 x (1/100) + 0 x (1/1000) 0.70 + 0.02 720/1000 Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals. Example: Comparing 0.25 and 0.17, a student might think, “25 hundredths is more than 17 hundredths”. They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison. Comparing 0.207 to 0.26, a student might think, “Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, “I know that 0.207 is 207 thousandths (and may write 207/1000). 0.26 is 26 hundredths (and may write 26/100) but I can also think of it as 260 thousandths (260/1000). So, 260 thousandths is more than 207 thousandths.

Number and Operations in Base Ten **5.NBT**

Understand the place value system.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round**

Standards/Learning Objectives		***Explanations and Examples***
5.NBT.4. Use place value understanding to round decimals to any place.		When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers. Example: Round 14.235 to the nearest tenth. • Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30). 

Number and Operations in Base Ten **5.NBT**
Perform operations with multi-digit whole numbers and with decimals to hundredths.

Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/division, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning**

Standards/Learning Objectives	***Explanations and Examples***
5.NBT.5. Fluently multiply multi-digit whole numbers using the standard algorithm.	In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value. Example: • 123 x 34. When students apply the standard algorithm, they, decompose 34 into 30 + 4. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products.

5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.
	
	Example:
	• Using expanded notation ~ 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25
	• Using his or her understanding of the relationship between 100 and 25, a student might think ~
	  ○ I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
	  ○ 600 divided by 25 has to be 24.
	  ○ Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80)
	  ○ I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
	  ○ 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.

Number and Operations in Base Ten **5.NBT**

Perform operations with multi-digit whole numbers and with decimals to hundredths.

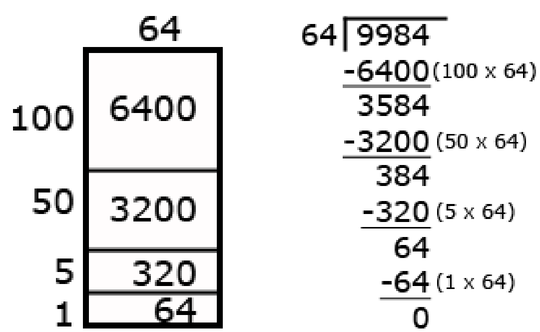
Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/division, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning**

Standards/Learning Objectives	***Explanations and Examples***
	Using an equation that relates division to multiplication, 25 x *n* = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 x 100 = 2500.

Example: 9984 ÷ 64

- An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.



Number and Operations in Base Ten Perform operations with multi-digit whole numbers and with decimals to hundredths. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately		**5.NBT**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/division, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning**		
Standards/Learning Objectives		***Explanations and Examples***

5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.		This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.
		
		Examples:
		• 3.6 + 1.7
		  ○ A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½.
		• 5.4 – 0.8
		  ○ A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.
		• 6 x 2.4
		  ○ A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 x 2 ½ and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 (½ of a group of 6).
		
		Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.
		
		Continued on next page

Number and Operations in Base Ten **5.NBT**

Perform operations with multi-digit whole numbers and with decimals to hundredths.

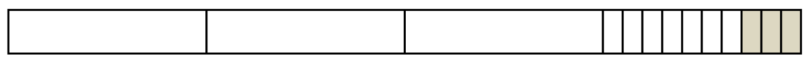
Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/division, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning**

Standards/Learning Objectives		***Explanations and Examples***

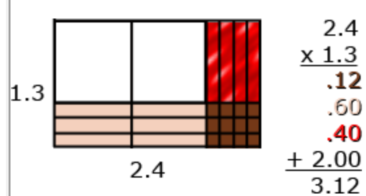
Example: 4 - 0.3

- 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.



The answer is 3 and 7/10 or 3.7.

Example: An area model can be useful for illustrating products.



Students should be able to describe the partial products displayed by the area model. For example,

“3/10 times 4/10 is 12/100.
3/10 times 2 is 6/10 or 60/100.
1 group of 4/10 is 4/10 or 40/100.
1 group of 2 is 2.”

Continued on next page

Number and Operations in Base Ten		**5.NBT**
Perform operations with multi-digit whole numbers and with decimals to hundredths. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately		
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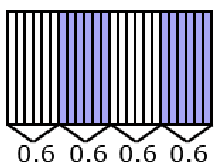
Standards/Learning Objectives		***Explanations and Examples***
		Example of division: finding the number in each group or share • Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as *2.4 ÷ 4 = 0.6*  Example of division: find the number of groups • Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut? • To divide to find the number of groups, a student might ○ draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths. Continued on next page

Number and Operations in Base Ten **5.NBT**

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately

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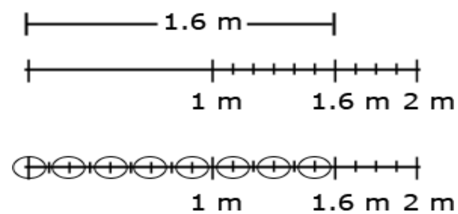
Standards/Learning Objectives		***Explanations and Examples***
		 ○ count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as 10/10, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . .16 tenths, a student can count 8 groups of 2 tenths. ○ Use their understanding of multiplication and think, “8 groups of 2 is 16, so 8 groups of 2/10 is 16/10 or 1 6/10.” Technology Connections: Create models using Interactive Whiteboard software

Number and Operations—Fractions Use equivalent fractions as a strategy to add and subtract fractions. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.		**5.NF**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers**		
Standards/Learning Objectives		***Explanations and Examples***
5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $^2/_3 + {^5/_4} = {^8/_{12}} + {^{15}/_{12}} = {^{23}/_{12}}$. *(In general,* $^a/_b + {^c/_d} = {^{(ad + bc)}/_{bd}}$.*)* ▪ Generate equivalent fractions to find like denominator ▪ Solve addition and subtraction problems involving fractions (including mixed numbers) with like and unlike denominators using an equivalent fraction strategy		Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator. Examples: • $\frac{2}{5}+\frac{7}{8}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$ • $3\frac{1}{4}-\frac{1}{6}=3\frac{3}{12}-\frac{2}{12}=3\frac{1}{12}$

Number and Operations—Fractions **5.NF**

Use equivalent fractions as a strategy to add and subtract fractions.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers**

Standards/Learning Objectives

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.*

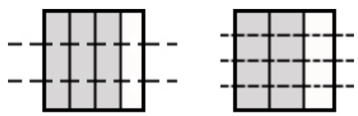
- Generate equivalent fractions to find like denominators
- Evaluate the reasonableness of an answer, using fractional number sense, by comparing it to a benchmark fraction
- Solve word problems involving addition and subtraction of fractions with unlike denominators referring to the same whole

Explanations and Examples

Examples:

Jerry was making two different types of cookies. One recipe needed ¾ cup of sugar and the other needed $^2/_3$ cup of sugar. How much sugar did he need to make both recipes?

- Mental estimation:
  - A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.
- Area model



$\frac{3}{4}$ cup of sugar $\quad$ $\frac{2}{3}$ cup of sugar

$\frac{3}{4}=\frac{9}{12}$ $\qquad$ $\frac{2}{3}=\frac{8}{12}$ $\qquad$ $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

Continued on next page

Number and Operations—Fractions **5.NF**

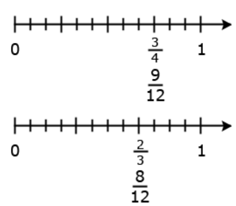
Use equivalent fractions as a strategy to add and subtract fractions.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.

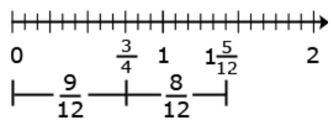
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers**

Standards/Learning Objectives	***Explanations and Examples***

- Linear model



Solution:



Example: Using a bar diagram

- Sonia had 2 1/3 candy bars. She promised her brother that she would give him ½ of a candy bar. How much will she have left after she gives her brother the amount she promised?
- If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran 1 ¾ miles. How many miles does she still need to run the first week?
  - Using addition to find the answer:1 ¾ + n = 3
  - A student might add 1 ¼ to 1 ¾ to get to 3 miles. Then he or she would add 1/6 more. Thus 1 ¼ miles + 1/6 of a mile is what Mary needs to run during that week.

Continued on next page

Number and Operations—Fractions **5.NF**

Use equivalent fractions as a strategy to add and subtract fractions.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.

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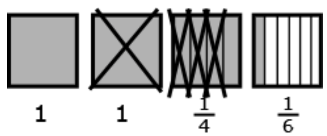
Standards/Learning Objectives	***Explanations and Examples***

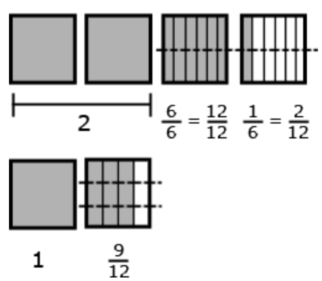
Example: Using an area model to subtract

- This model shows 1 ¾ subtracted from 3 1/6 leaving 1 + ¼ + 1/6 which a student can then change to 1 + 3/12 + 2/12 = 1 5/12.



3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this is done a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.

- This diagram models a way to show how 3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.



Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.

Number and Operations—Fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)		**5.NF**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**		
Standards/Learning Objectives		***Explanations and Examples***
5.NF.3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*		Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as "three fifths" and after many experiences with sharing problems, learn that 3/5 can also be interpreted as "3 divided by 5." Examples: • Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? ○ When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting 3/10 of a box. • Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend? • The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive? Students may recognize this as a whole number division problem but should also express this equal sharing problem as $^{27}/_{6}$. They explain that each classroom gets $^{27}/_{6}$ boxes of pencils and can further determine that each classroom get 4 $^{3}/_{6}$ or 4 $^{1}/_{2}$ boxes of pencils.

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**

Standards/Learning Objectives

5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$*, and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$*. (In general,* $(a/b) \times (c/d) = ac/bd$*.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Explanations and Examples

Students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.

As they multiply fractions such as 3/5 x 6, they can think of the operation in more than one way.

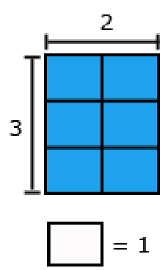
- ○ 3 x (6 ÷ 5) or (3 x 6/5)
- ○ (3 x 6) ÷ 5 or 18 ÷ 5 (18/5)

Students create a story problem for 3/5 x 6 such as,

- Isabel had 6 feet of wrapping paper. She used 3/5 of the paper to wrap some presents. How much does she have left?
- Every day Tim ran 3/5 of mile. How far did he run after 6 days? (Interpreting this as 6 x 3/5)

Examples: Building on previous understandings of multiplication

- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.



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Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

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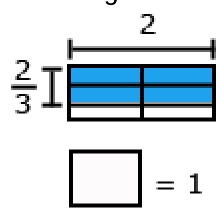
Standards/Learning Objectives

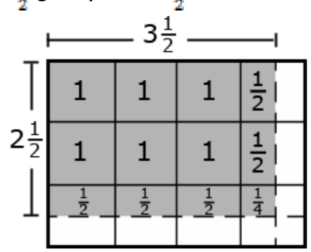
- Multiply fractions by whole numbers
- Multiply fractions by fractions
- Interpret the product of a fraction times a whole number as total number of parts of the whole
- Determine the sequence of operations that results in the total number of parts of the whole
- Interpret the product of a fraction times a fraction as the total number of parts of the whole
- Represent fraction products as rectangular areas
- Justify multiplying fractional side lengths to find the area is the same as tiling a rectangle with unit squares of the appropriate unit fraction side lengths

Explanations and Examples

- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that 2 x 2/3 = 4/3



- $2\frac{1}{2}$ groups of $3\frac{1}{2}$:



- In solving the problem $\frac{2}{3}$ x $\frac{4}{5}$, students use an area model to visualize it as a 2 by 4 array of small rectangles each of which has side lengths 1/3 and 1/5. They reason that 1/3 x 1/5 = 1/(3 x 5) by counting squares in the entire rectangle, so the area of the shaded area is (2 x 4) x 1/(3 x 5) = $\frac{2 \times 4}{3 \times 5}$. They can explain that the product is less than $\frac{4}{5}$ because they are finding $\frac{2}{3}$ of $\frac{4}{5}$. They can further estimate that the answer must be between $\frac{2}{5}$ and $\frac{4}{5}$ because $\frac{2}{3}$ of $\frac{4}{5}$ is more than $\frac{1}{2}$ of $\frac{4}{5}$ and less than one group of $\frac{4}{5}$.

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Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

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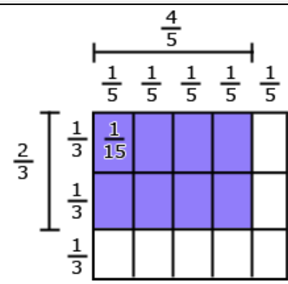
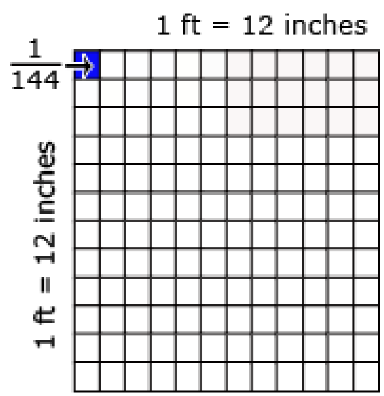
Standards/Learning Objectives	***Explanations and Examples***



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

- Larry knows that $\frac{1}{12}$ x $\frac{1}{12}$ is $\frac{1}{144}$. To prove this he makes the following array.



Technology Connections:

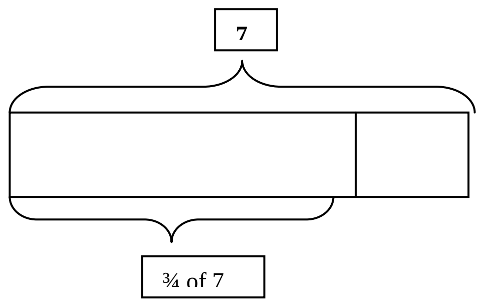
- Create story problems for peers to solve using digital tools.
- Use a tool such as Jing to digitally communicate story problems.

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**

Standards/Learning Objectives	***Explanations and Examples***
5.NF.5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Examples: • $\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.  • $2\frac{2}{3}$ x 8 must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24. • $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$ because multiplying $\frac{3}{4}$ by $\underline{5}$ is the same as multiplying by 1.

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**

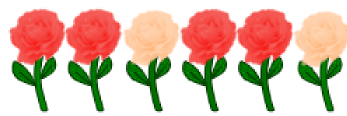
Standards/Learning Objectives

5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem within cultural contexts, including those of Montana American Indians.

Explanations and Examples

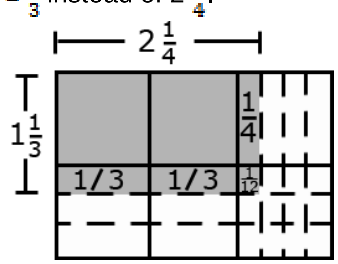
Examples:

- Evan bought 6 roses for his mother. $\frac{2}{3}$ of them were red. How many red roses were there?
  - Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.



  - A student can use an equation to solve.
    $\frac{2}{3} \times 6 = \frac{12}{3} = 4$ red roses

- Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?
  - A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by $1\frac{1}{3}$ instead of $2\frac{1}{4}$.



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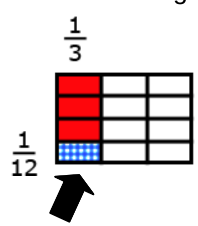
Number and Operations—Fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)		**5.NF**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**		
<u>Standards/Learning Objectives</u>		***<u>Explanations and Examples</u>***
		The explanation may include the following: • First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$. • When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$. • Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$. • $\frac{1}{3}$ times 2 is $\frac{2}{3}$. • $\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$. • So the answer is $2\frac{1}{4} + \frac{2}{3} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**

Standards/Learning Objectives		***Explanations and Examples***
5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for ($^1/_3$) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ($^1/_3$) ÷ 4 = $^1/_{12}$ because ($^1/_{12}$) × 4 = $^1/_3$.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for 4 ÷ ($^1/_5$), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4÷($^1/_5$) = 20 because 20 × ($^1/_5$) = 4.*		In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions. Division Example: Knowing the number of groups/shares and finding how many/much in each group/share • Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally? ○ The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.  Continued on next page

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**

Standards/Learning Objectives

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $^1/_2$ lb of chocolate equally? How many $^1/_3$-cup servings are in 2 cups of raisins?*

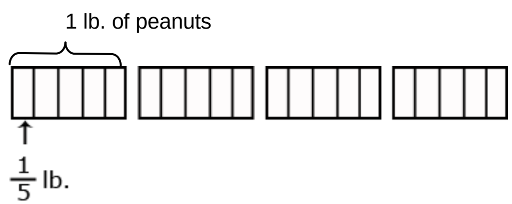
Explanations and Examples

Examples:

Knowing how many in each group/share and finding how many groups/shares

- Angelo has 4 lbs of peanuts. He wants to give each of his friends 1/5 lb. How many friends can receive 1/5 lb of peanuts?

A diagram for 4 ÷ 1/5 is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs.



- How much rice will each person get if 3 people share 1/2 lb of rice equally?

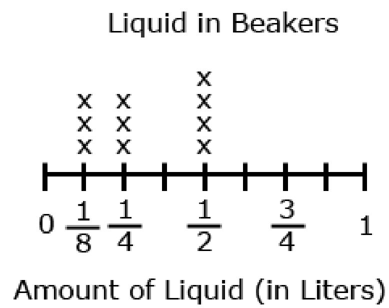
$$\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$$

  - A student may think or draw ½ and cut it into 3 equal groups then determine that each of those part is 1/6.
  - A student may think of ½ as equivalent to 3/6. 3/6 divided by 3 is 1/6.

Measurement and Data	**5.MD**
Convert like measurement units within a given measurement system.	

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **conversion/convert, metric and customary measurement**. **From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second**

Standards/Learning Objectives	***Explanations and Examples***
5.MD.1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems within a cultural context, including those of Montana American Indians. ▪ Recognize units of measurement within the same system ▪ Divide and multiply to change units ▪ Convert units of measurement within the same system ▪ Solve multi-step, real world problems that involve converting units	In fifth grade, students build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less units than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.

Measurement and Data Represent and interpret data.		**5.MD**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **line plot, length, mass, liquid volume**		
Standards/Learning Objectives		***Explanations and Examples***
5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (${}^1/_2$, ${}^1/_4$, ${}^1/_8$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* ▪ Identify benchmark fractions ▪ Make a line plot to display a data set of measurements in fractions of a unit ▪ Solve problems involving information presented in line plots which use fractions of a unit by adding, subtracting, multiplying, and dividing fraction		Ten beakers, measured in liters, are filled with a liquid.  The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.) Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

Measurement and Data **5.MD** Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.	
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in. cubic ft. nonstandard cubic units), multiplication, addition, edge lengths, height, area of base**	

Standards/Learning Objectives	***Explanations and Examples***
5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Students’ prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Students estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.
5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Students understand that same sized cubic units are used to measure volume. They select appropriate units to measure volume. For example, they make a distinction between which units are more appropriate for measuring the volume of a gym and the volume of a box of books. They can also improvise a cubic unit using any unit as a length (e.g., the length of their pencil). Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume. They may also use drawings or interactive computer software to simulate the same filling process. Technology Connections: http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

Measurement and Data **5.MD**

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in. cubic ft. nonstandard cubic units), multiplication, addition, edge lengths, height, area of base**

<u>Standards/Learning Objectives</u>

5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume within cultural contexts, including those of Montana American Indians.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

<u>Explanations and Examples</u>

Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.

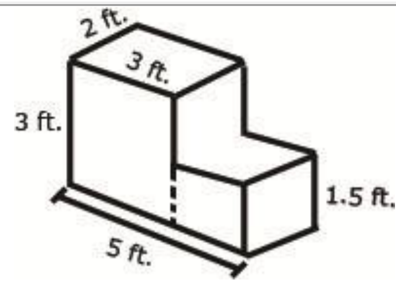
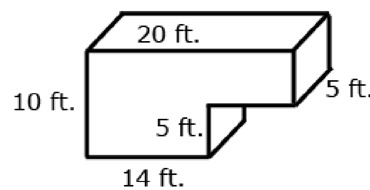
Examples:

- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

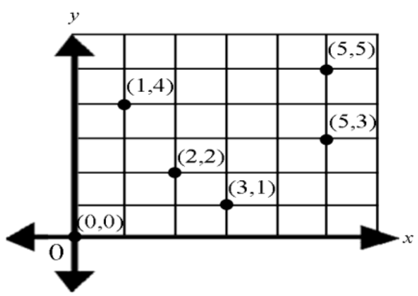
- Students determine the volume of concrete needed to build the steps in the diagram below.

Continued on next page

Measurement and Data Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.		**5.MD**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in. cubic ft. nonstandard cubic units), multiplication, addition, edge lengths, height, area of base**		
Standards/Learning Objectives		***Explanations and Examples***
c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.		 • A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below. 

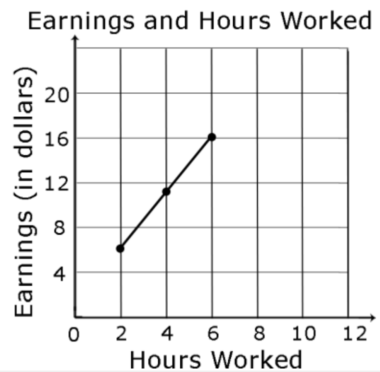
Geometry Graph points on the coordinate plane to solve real-world and mathematical problems.	**5.G**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, x-axis, y-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, x-coordinate, y-coordinate**

Standards/Learning Objectives	***Explanations and Examples***
5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	Examples: • Students can use a classroom size coordinate system to physically locate the coordinate point (5, 3) by starting at the origin point (0,0), walking 5 units along the x axis to find the first number in the pair (5), and then walking up 3 units for the second number in the pair (3). The ordered pair names a point in the plane.  • Graph and label the points below in a coordinate system. ○ A (0, 0) ○ B (5, 1) ○ C (0, 6) ○ D (2.5, 6) ○ E (6, 2) ○ F (4, 1) ○ G (3, 0)

Geometry Graph points on the coordinate plane to solve real-world and mathematical problems.	**5.G**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, x-axis, y-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, x-coordinate, y-coordinate**	

Standards/Learning Objectives	***Explanations and Examples***

5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation including those found in Montana American Indian designs.	Examples:
	• Sara has saved $20. She earns $8 for each hour she works.
	  ○ If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?
	  ○ Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved.
	  ○ What other information do you know from analyzing the graph?
	
	• Use the graph below to determine how much money Jack makes after working exactly 9 hours.
	

Geometry Classify two-dimensional figures into categories based on their properties.	**5.G**

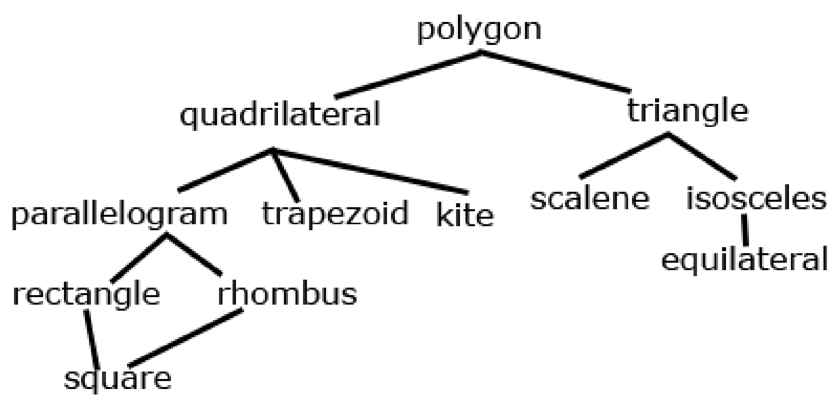
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, category, subcategory, hierarchy, (properties)-rules about how numbers work, two dimensional. From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle**

Standards/Learning Objectives	***Explanations and Examples***
5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). Example: • If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms A sample of questions that might be posed to students include: • A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms? • Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons. • All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False? • A trapezoid has 2 sides parallel so it must be a parallelogram. True or False? Technology Connections: http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

Geometry Classify two-dimensional figures into categories based on their properties.	**5.G**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, category, subcategory, hierarchy, (properties)-rules about how numbers work, two dimensional. From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle**

Standards/Learning Objectives	***Explanations and Examples***

5.G.4. Classify two-dimensional figures in a hierarchy based on properties.	Properties of figure may include:
	• Properties of sides—parallel, perpendicular, congruent, number of sides
	• Properties of angles—types of angles, congruent
	
	Examples:
	• A right triangle can be both scalene and isosceles, but not equilateral.
	• A scalene triangle can be right, acute and obtuse.
	
	Triangles can be classified by:
	• Angles
	  ○ Right: The triangle has one angle that measures 90º.
	  ○ Acute: The triangle has exactly three angles that measure between 0º and 90º.
	  ○ Obtuse: The triangle has exactly one angle that measures greater than 90º and less than 180º.
	• Sides
	  ○ Equilateral: All sides of the triangle are the same length.
	  ○ Isosceles: At least two sides of the triangle are the same length.
	  ○ Scalene: No sides of the triangle are the same length.
	

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another.Example: 3/4 and – 3/4 are additive inverses of one another because3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

[3] Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4] To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. **Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

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