Mathematical Practice and Content

Common Core Standards

Third Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.
Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**

f. Justify their conclusions, communicate them to others, and respond to the arguments of others.
g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

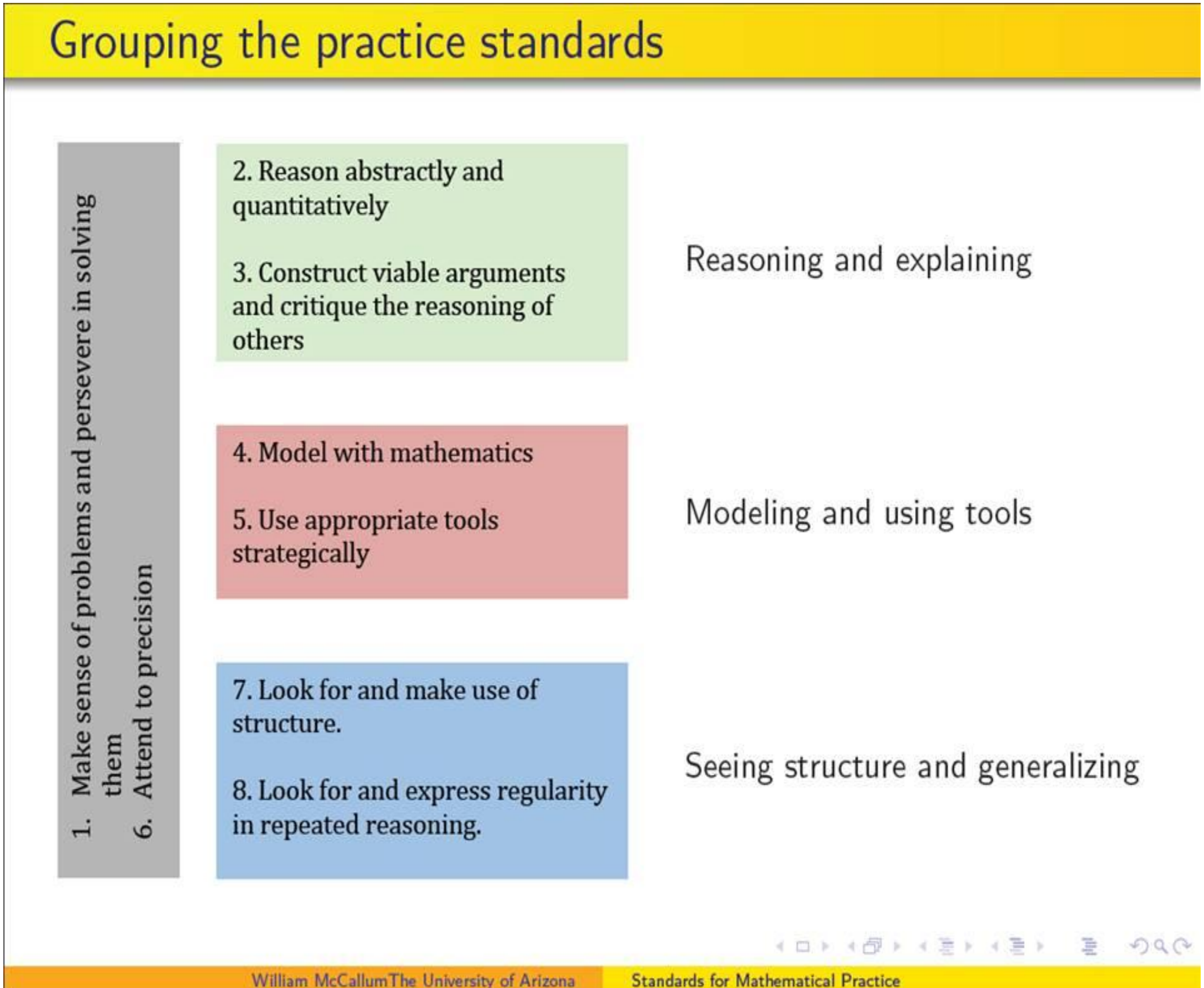
4. Model with mathematics.
Mathematically proficient students:
a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.
b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
d. Analyze mathematical relationships to draw conclusions.

5. Use appropriate tools strategically.
Mathematically proficient students:
a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)
b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.

6. Attend to precision.
Mathematically proficient students:
a. Communicate their understanding of mathematics to others.
b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
c. Specify units of measure and use label parts of graphs and charts
d. Strive for accuracy.

7. Look for and make use of structure.
Mathematically proficient students:
a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.
b. Apply and discuss properties.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students:
a. Look for mathematically sound shortcuts.
b. Use repeated applications to generalize properties.


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallumThe University of Arizona
Standards for Mathematical Practice

Standards for Mathematical Practice: Grade 3 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
3.MP.1. Make sense of problems and persevere in solving them.	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
3.MP.2. Reason abstractly and quantitatively.	Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3.MP.3. Construct viable arguments and critique the reasoning of others.	In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
3.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
3.MP.5. Use appropriate tools strategically.	Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
3.MP.6. Attend to precision.	As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
3.MP.7. Look for and make use of structure.	In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
3.MP.8. Look for and express regularity in repeated reasoning.	Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Third Grade

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 3 Overview

Operations and Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Geometry
- Reason with shapes and their attributes.

Number and Operations—Fractions
- Develop understanding of fractions as numbers.

Measurement and Data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Operations and Algebraic Thinking Represent and solve problems involving multiplication and division. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.		**3.OA**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **products, groups of, quotients, partitioned equally, multiplication**, **division, equal groups, arrays, equations**, **unknown**		
Standard/Learning Objective		***Explanations and Examples***
3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* ▪ Find the product of multiple groups of objects ▪ Interpret products of whole number as a total number of objects in a number of groups		Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7. To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication expression (e.g., 5 x 6) students interpret the expression using a multiplication context. (See Table 2) They should begin to use the terms, *factor* and *product,* as they describe multiplication. Students may use interactive whiteboards to create digital models.

Operations and Algebraic Thinking Represent and solve problems involving multiplication and division. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.		**3.OA**
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Standard/Learning Objective		***Explanations and Examples***
3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* ▪ Know what numbers in a division problem represent ▪ Explain what division means and how it relates to equal shares ▪ Interpret quotients as the number of shares or the number of groups when a set of objects is divided equally		Students recognize the operation of division in two different types of situations. One situation requires determining hov many groups and the other situation requires sharing (determining how many in each group). Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor). To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., 24 ÷ 6) students interpret the expression in contexts that require both interpretations of division. (See Table 2) Students may use interactive whiteboards to create digital models.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.

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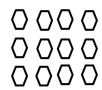
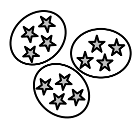
Standard/Learning Objective	***Explanations and Examples***
3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 2.) ▪ Multiply and divide within 100 ▪ Solve word problems in situations involving equal groups, arrays, and measurement quantities ▪ Represent a word problem using a picture, an equation with a symbol for the unknown number, or in other ways	Students use a variety of representations for creating and solving one-step word problems, i.e., numbers, words, pictures, physical objects, or equations. They use multiplication and division of whole numbers up to 10 x10. Students explain their thinking, show their work by using at least one representation, and verify that their answer is reasonable. Word problems may be represented in multiple ways: • Equations: 3 x 4 = ?, 4 x 3 = ?, 12 ÷ 4 = ? and 12 ÷ 3 = ? • Array:  • Equal groups  • Repeated addition: 4 + 4 + 4 or repeated subtraction • Three equal jumps forward from 0 on the number line to 12 or three equal jumps backwards from 12 to 0  Continued on next page

Operations and Algebraic Thinking 3.OA Represent and solve problems involving multiplication and division. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.	
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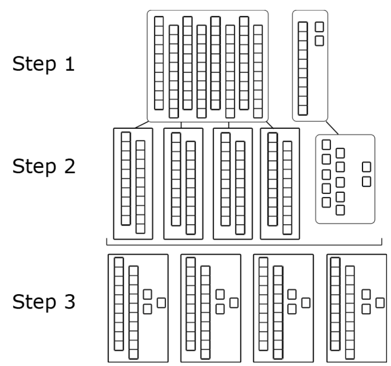
Standard/Learning Objective	***Explanations and Examples***
	Examples of division problems: • Determining the number of objects in each share (partitive division, where the size of the groups is unknown): ○ The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?  Continued on next page

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.

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Standard/Learning Objective	***Explanations and Examples***
	• Determining the number of shares (measurement division, where the number of groups is unknown) Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Startin g	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24-4= 20	20-4= 16	16-4= 12	12-4= 8	8-4= 4	4-4= 0

Solution: The bananas will last for 6 days.

Students may use interactive whiteboards to show work and justify their thinking.

Operations and Algebraic Thinking 3.OA Represent and solve problems involving multiplication and division. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.	
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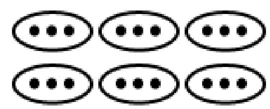
Standard/Learning Objective	***Explanations and Examples***
3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.* ▪ Multiply and divide within 100 ▪ Determine which operation (multiplication or division)is needed to determine the unknown whole number ▪ Solve to find the unknown whole number in a multiplication or division equation	This standard is strongly connected to 3.AO.3 when students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation. Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 x ? = 40, they might think: • 4 groups of some number is the same as 40 • 4 times some number is the same as 40 • I know that 4 groups of 10 is 40 so the unknown number is 10 • The missing factor is 10 because 4 times 10 equals 40. Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions. Examples: • Solve the equations below: 24 = ? x 6 $72 \div \Delta = 9$ • Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m* Students may use interactive whiteboards to create digital models to explain and justify their thinking.

Operations and Algebraic Thinking 3.OA

Understand properties of multiplication and the relationship between multiplication and division.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, strategies,(properties)-rules about how numbers work**

Standard/ Learning Objectives	***Explanations and Examples***
3.OA.5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* ▪ Multiply and divide within 100 ▪ Explain how the properties of operations work ▪ Apply properties of operations as strategies to multiply and divide	Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. Models help build understanding of the commutative property: Example: 3 x 6 = 6 x 3 In the following diagram it may not be obvious that 3 groups of 6 is the same as 6 groups of 3. A student may need to count to verify this.  is the same quantity as  Example: 4 x 3 = 3 x 4 An array explicitly demonstrates the concept of the commutative property.  4 rows of 3 or 4 x 3  3 rows of 4 or 3 x 4 Continued on next page

Operations and Algebraic Thinking 3.OA

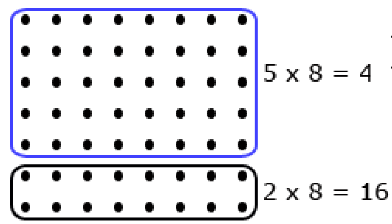
Understand properties of multiplication and the relationship between multiplication and division.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, strategies,(properties)-rules about how numbers work**

Standard/ Learning Objectives	***Explanations and Examples***
	Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. Students should learn that they can decompose either of the factors. It is important to note that the students may record their thinking in different ways.

5 x 8 = 40
2 x 8 = <u>16</u>
56



7 x 4 = 28
7 x 4 = <u>28</u>
56

To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication)
- 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1)
- 3 x 6 = 6 x 3 (Commutative Property)
- 8 ÷ 2 = 2 ÷ 8 (Students are only to determine that these are not equal)
- 2 x 3 x 5 = 6 x 5
- 10 x 2 < 5 x 2 x 2
- 2 x 3 x 5 = 10 x 3
- 0 x 6 > 3 x 0 x 2
-

Operations and Algebraic Thinking 3.OA

Understand properties of multiplication and the relationship between multiplication and division.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

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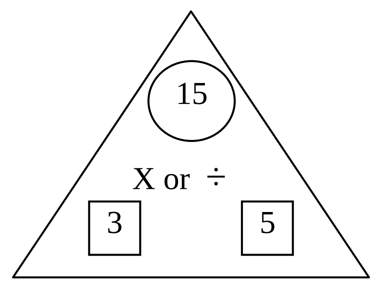
Standard/ Learning Objectives	***Explanations and Examples***
3.OA.6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* ▪ Identify the multiplication problem as related to the division problem ▪ Identify the unknown factor in the related to the division problem ▪ Recognize multiplication and division as related operations and explain how they are related ▪ Use multiplication to solve division problems	Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Examples: • 3 x 5 = 15 5 x 3 = 15 • 15 ÷ 3 = 5 15 ÷ 5 = 3  Continued on next page Students use their understanding of the meaning of the equal sign as “the same as” to interpret an equation with an unknown. When given 32 ÷ ☐ = 4, students may think: • 4 groups of some number is the same as 32 • 4 times some number is the same as 32 • I know that 4 groups of 8 is 32 so the unknown number is 8 • The missing factor is 8 because 4 times 8 is 32. Equations in the form of a ÷ b = c and c = a ÷ b need to be used interchangeably, with the unknown in different positions.

Operations and Algebraic Thinking Multiply and divide within 100	3.OA
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property**	
Standard/Learning Objectives	***Explanations and Examples***
3.OA.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. ▪ Know from memory all products of two one-digit numbers ▪ Fluently multiply and divide within 100 ▪ Analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 100	By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Strategies students may use to attain fluency include: • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • Tens facts (relating to place value, 5 x 10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of __ and knowing how many groups have been counted) • Square numbers (ex: 3 x 3) • Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3) • Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6) • Turn-around facts (Commutative Property) • Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24) • Missing factors General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

Operations and Algebraic Thinking 3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, (properties)-rules about how numbers work**

Standard/Learning Objectives

3.OA.8. Solve two-step word problems using the four operations within cultural contexts, including those of Montana American Indians. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

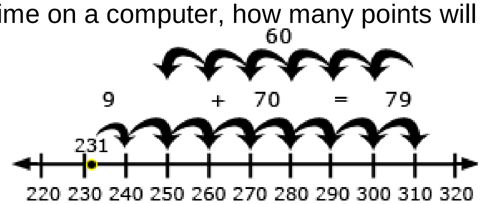
- Know the order of operations
- Know strategies for estimating
- Construct an equation with a letter standing for the unknown quantity
- Solve two-step word problems using the four operations
- Justify answers to problems using various estimation strategies

Explanations and Examples

Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use.

Examples:

- Jerry earned 231 points at school last week. This week he earned 79 points. If he uses 60 points to earn free time on a computer, how many points will he have left?



A student may use the number line above to describe his/her thinking, "231 + 9 = 240 so now I need to add 70 more. 240, 250 (10 more), 260 (20 more), 270, 280, 290, 300, 310 (70 more). Now I need to count back 60. 310, 300 (back 10), 290 (back 20), 280, 270, 260, 250 (back 60)."

A student writes the equation, 231 + 79 – 60 = m and uses rounding (230 + 80 – 60) to estimate.

A student writes the equation, 231 + 79 – 60 = m and calculates 79-60 = 19 and then calculates 231 + 19 = m.

- The soccer club is going on a trip to the water park. The cost of attending the trip is $63. Included in that price is $13 for lunch and the cost of 2 wristbands, one for the morning and one for the afternoon. Write an equation representing the cost of the field trip and determine the price of one wristband.

Continued on next page

Operations and Algebraic Thinking Solve problems involving the four operations, and identify and explain patterns in arithmetic.		**3.OA**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, (properties)-rules about how numbers work**		
Standard/Learning Objectives		***Explanations and Examples***
		<table><tr><td>w</td><td>w</td><td>13</td></tr><tr><td colspan="3">63</td></tr></table> The above diagram helps the student write the equation, w + w + 13 = 63. Using the diagram, a student might think, “I know that the two wristbands cost $50 ($63-$13) so one wristband costs $25.” To check for reasonableness, a student might use front end estimation and say 60-10 = 50 and 50 ÷ 2 = 25. When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions. Estimation strategies include, but are not limited to: • using benchmark numbers that are easy to compute • front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts) • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding changed the original values)

Operations and Algebraic Thinking 3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

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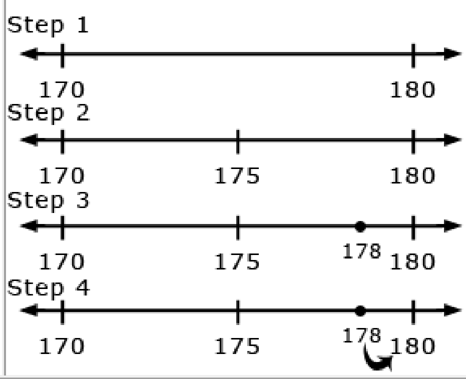
Standard/Learning Objectives	***Explanations and Examples***
3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* ▪ Identify arithmetic patterns such as even and odd numbers, patterns in an addition table, patterns in a multiplication table, and patterns regarding multiples and sums ▪ Explain rules for a pattern using properties of operations ▪ Explain relationships between numbers in a pattern	Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. For example: • Any sum of two even numbers is even. • Any sum of two odd numbers is even. • Any sum of an even number and an odd number is odd. • The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups. • The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines. • The multiples of any number fall on a horizontal and a vertical line due to the commutative property. • All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10. Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
•	•	•
•	•	•
•	•	•
20	0	20

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties)-rules about how numbers work**

Standard/Learning Objectives	***Explanations and Examples***
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100. ▪ Define "round or rounding" in relation to place value ▪ Round a whole number to the nearest 10 ▪ Round a whole number to the nearest 100	Students learn when and why to round numbers. They identify possible answers and halfway points. Then they narrow where the given number falls between the possible answers and halfway points. They also understand that by convention if a number is exactly at the halfway point of the two possible answers, the number is rounded up. Example: Round 178 to the nearest 10.  Step 1: The answer is either 170 or 180. Step 2: The halfway point is 175. Step 3: 178 is between 175 and 180. Step 4: Therefore, the rounded number is 180.
3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. ▪ Know strategies and algorithms for adding and subtracting within 1000 ▪ Fluently add and subtract within 1000	Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable. An interactive whiteboard or document camera may be used to show and share student thinking. Example: • Mary read 573 pages during her summer reading challenge. She was only required to read 399 pages. How many extra pages did Mary read beyond the challenge requirements? Continued on next page

Number and Operations in Base Ten **3.NBT**
Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

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Standard/Learning Objectives	***Explanations and Examples***
	Students may use several approaches to solve the problem including the traditional algorithm. Examples of other methods students may use are listed below: • 399 + 1 = 400, 400 + 100 = 500, 500 + 73 = 573, therefore 1+ 100 + 73 = 174 pages (Adding up strategy) • 400 + 100 is 500; 500 + 73 is 573; 100 + 73 is 173 plus 1 (for 399, to 400) is 174 (Compensating strategy) • Take away 73 from 573 to get to 500, take away 100 to get to 400, and take away 1 to get to 399. Then 73 +100 + 1 = 174 (Subtracting to count down strategy) • 399 + 1 is 400, 500 (that's 100 more). 510, 520, 530, 540, 550, 560, 570, (that's 70 more), 571, 572, 573 (that's 3 more) so the total is 1 + 100 + 70 + 3 = 174 (Adding by tens or hundreds strategy)
3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. ▪ Know strategies to multiply one-digit numbers by multiples of 10 (up to 90) ▪ Apply knowledge of place value to multiply one-digit whole numbers by multiples of 10 in the range 10-90	Students use base ten blocks, diagrams, or hundreds charts to multiply one-digit numbers by multiples of 10 from 10-90. They apply their understanding of multiplication and the meaning of the multiples of 10. For example, 30 is 3 tens and 70 is 7 tens. They can interpret 2 x 40 as 2 groups of 4 tens or 8 groups of ten. They understand that 5 x 60 is 5 groups of 6 tens or 30 tens and know that 30 tens is 300. After developing this understanding they begin to recognize the patterns in multiplying by multiples of 10. Students may use manipulatives, drawings, document camera, or interactive whiteboard to demonstrate their understanding.

Number and Operations—Fractions 3.NF

Develop understanding of fractions as numbers.

[1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, . of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), equal parts, fraction, equal distance (intervals), equivalent, equivalence, reasonable, denominator, numerator, comparison, compare, ‹, ›, = , justify**

Standards

3.NF.1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

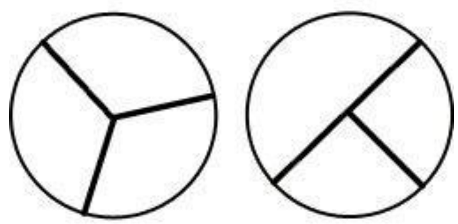
- Recognize a unit fraction such as ¼ as the quantity formed when the whole is partitioned into 4 equal parts
- Identify a fraction such as 2/3 and explain that the quantity formed is 2 equal parts of the whole partitioned into 3 parts (1/3 and 1/3 of the whole 3/3)
- Express a fraction as the number of unit fractions

Explanations and Examples

Some important concepts related to developing understanding of fractions include:

- Understand fractional parts must be equal-sized

Example Non-example



These are thirds These are NOT thirds

- The number of equal parts tell how many make a whole
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases
- The size of the fractional part is relative to the whole
    - The number of children in one-half of a classroom is different than the number of children in one-half of a school. (the whole in each set is different therefore the half in each set will be different)
- When a whole is cut into equal parts, the denominator represents the number of equal parts

Continued on next page

Number and Operations—Fractions 3.NF

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Standards	***Explanations and Examples***

	• The numerator of a fraction is the count of the number of equal parts
	  ○ ¾ means that there are 3 one-fourths
	  ○ Students can count *one fourth, two fourths, three fourths*
	
	Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.
	
	To develop understanding of fair shares, students first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.
	
	Examples:
	• Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
	• Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?
	• What fraction of the rectangle is shaded? How might you draw the rectangle in another way but with the same fraction shaded?
	
	Continued on next page

Number and Operations—Fractions 3.NF

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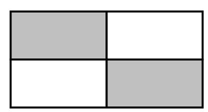
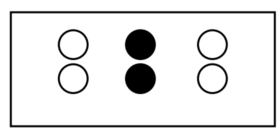
Standards		***Explanations and Examples***

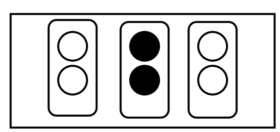


Solution: $\frac{2}{4}$ or $\frac{1}{2}$

What fraction of the set is black?



Solution: $\frac{2}{6}$



Solution: $\frac{1}{3}$

Number and Operations—Fractions 3.NF

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Standards

3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

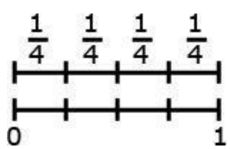
b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

- Divide a whole on a number line into equal parts

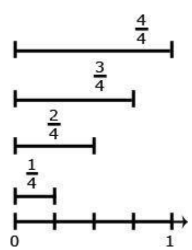
Explanations and Examples

Students transfer their understanding of parts of a whole to partition a number line into equal parts. There are two new concepts addressed in this standard which students should have time to develop.

1. On a number line from 0 to 1, students can partition (divide) it into equal parts and recognize that each segmented part represents the same length.



2. Students label each fractional part based on how far it is from zero to the endpoint.



An interactive whiteboard may be used to help students develop these concepts.

Number and Operations—Fractions **3.NF**

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Standards	***Explanations and Examples***
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. Students recognize when examining fractions with common denominators, the wholes have been divided into the same number of equal parts. So the fraction with the larger numerator has the larger number of equal parts. $\frac{2}{6} < \frac{5}{6}$ To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts are different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces. $\frac{3}{8} < \frac{3}{4}$

Measurement and Data — 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **estimate, time, time intervals, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L)**

Standard/Learning Objectives		***Explanations and Examples***
3.MD.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. ▪ Recognize minute marks on an analog clock face and minute position on a digital clock face ▪ Know how to write time to the minute ▪ Compare an analog clock face with a number line diagram ▪ Use a number line diagram to add and subtract time intervals in minutes ▪ Tell time to the minute ▪ Solve word problems involving addition and subtraction of time intervals in minutes		Students in second grade learned to tell time to the nearest five minutes. In third grade, they extend telling time and measure elapsed time both in and out of context using clocks and number lines. Students may use an interactive whiteboard to demonstrate understanding and justify their thinking.

Measurement and Data Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		**3.MD**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **estimate, time, time intervals, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L)**		
Standard/Learning Objectives		***Explanations and Examples***
3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Table 2). ▪ Explain how to measure liquid volume in liters ▪ Explain how to measure mass in grams and kilograms ▪ Know various strategies to represent a word problem involving liquid volume or mass ▪ Solve one-step word problems involving masses given in the same units ▪ Solve one-step word problems involving liquid volume given in the same units		Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Example: Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram.

Measurement and Data **3.MD**

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale, scaled picture graph, scaled bar graph, line plot, data**

Standard/Learning Objectives

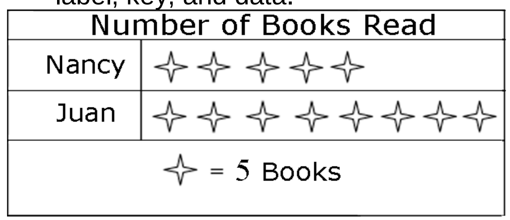
3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories, within cultural contexts, including those of Montana American Indians.. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

- Explain the scale of a graph with a scale greater than one
- Identify the scale of a graph with a scale greater than one
- Analyze a graph with a scale greater than one
- Choose a proper scale for a bar graph or picture graph
- Create a scaled picture graph to show data
- Create a scaled bar graph to show data

Explanations and Examples

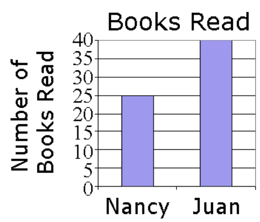
Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts.

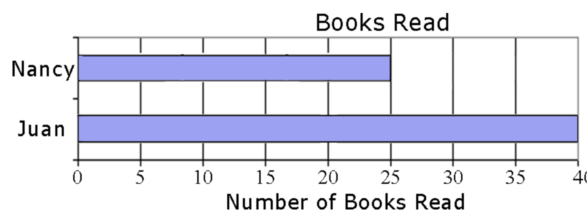
- Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data.



How many more books did Juan read than Nancy?

- Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.

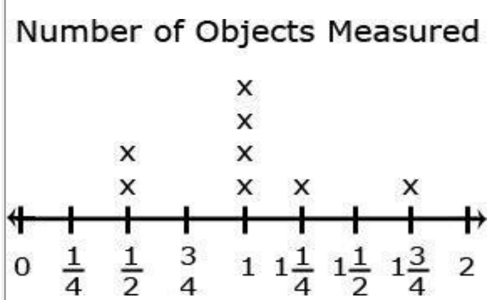


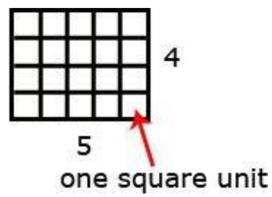


Measurement and Data — 3.MD

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale, scaled picture graph, scaled bar graph, line plot, data**

Standard/Learning Objectives	***Explanations and Examples***
3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. ▪ Define horizontal axis ▪ Identify each plot on the line as data or a number of objects ▪ Analyze data from a line plot ▪ Determine appropriate unit of measurement ▪ Determine appropriate scale for a line plot ▪ Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch ▪ Create a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters	Students in second grade measured length in whole units using both metric and U.S. customary systems. It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment. Some important ideas related to measuring with a ruler are: • The starting point of where one places a ruler to begin measuring • Measuring is approximate. Items that students measure will not always measure exactly ¼, ½ or one whole inch. Students will need to decide on an appropriate estimate length. • Making paper rulers and folding to find the half and quarter marks will help students develop a stronger understanding of measuring length Students generate data by measuring and create a line plot to display their findings. An example of a line plot is shown below: 

Measurement and Data Geometric measurement: understand concepts of area and relate area to multiplication and to addition. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.	**3.MD**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, area, square unit, plane figure, gap, overlap, square cm, square m , square in., square ft, nonstandard units, tiling, side length, decomposing**	
Standard/Learning Objectives	***Explanations and Examples***
3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. ▪ Define "unit square" ▪ Define area ▪ Relate the number (n) of unit squares to the area of a plane figure	Students develop understanding of using square units to measure area by: • Using different sized square units • Filling in an area with the same sized square units and counting the number of square units • An interactive whiteboard would allow students to see that square units can be used to cover a plane figure. 
3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	Using different sized graph paper, students can explore the areas measured in square centimeters and square inches. An interactive whiteboard may also be used to display and count the unit squares (area) of a figure.

Measurement and Data 3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, area, square unit, plane figure, gap, overlap, square cm, square m , square in., square ft, nonstandard units, tiling, side length, decomposing**

Standard/Learning Objectives

3.MD.7. Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems, including those of Montana American Indians.

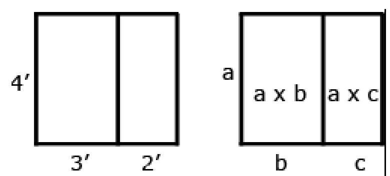
Explanations and Examples

Students tile areas of rectangles, determine the area, record the length and width of the rectangle, investigate the patterns in the numbers, and discover that the area is the length times the width.

Example:

Joe and John made a poster that was 4' by 3'. Mary and Amir made a poster that was 4' by 2'. They placed their posters on the wall side-by-side so that that there was no space between them. How much area will the two posters cover?

Students use pictures, words, and numbers to explain their understanding of the distributive property in this context.



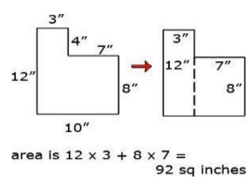
$4 \times 3 + 4 \times 2 = 20$

$4\ (3 + 2) = 20$

$4 \times 5 = 20$

Example:

Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



Measurement and Data 3.MD

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, perimeter, plane figure, linear, area, polygon, side length**

Standard/Learning Objectives	***Explanations and Examples***
3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard. Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

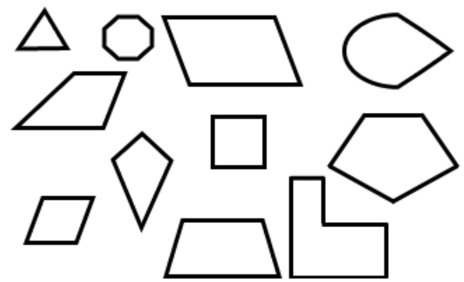
The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

Geometry **3.G**

Reason with shapes and their attributes.

Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: attributes, properties, quadrilateral, open figure, closed figure , threesided, 2-dimensional, 3-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, cubes, cones, cylinders, and rectangular prisms are subcategories of 3-dimensional figures shapes: polygon, rhombus/rhombi, rectangle, square, partition, unit fraction**
From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere

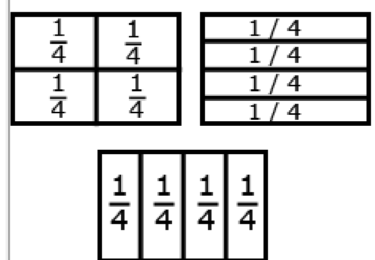
Standard/Learning Objectives		***Explanations and Examples***
3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ▪ Identify and define rhombuses, rectangles, and squares as examples of quadrilaterals based on their attributes ▪ Describe, analyze, and compare properties of two-dimensional shapes ▪ Compare and classify shapes by attributes, sides and angles ▪ Group shapes with shared attributes to define a larger category (e.g., quadrilaterals ▪ Draw examples of quadrilaterals that do and do not belong to any of the subcategories		In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals. 

Geometry 3.G

Reason with shapes and their attributes.

Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: attributes, properties, quadrilateral, open figure, closed figure , threesided, 2-dimensional, 3-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, cubes, cones, cylinders, and rectangular prisms are subcategories of 3-dimensional figures shapes: polygon, rhombus/rhombi, rectangle, square, partition, unit fraction**
From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere

Standard/Learning Objectives		***Explanations and Examples***
3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.* ▪ Know that shapes can be partitioned into equal areas ▪ Describe the area of each part as a fractional part of the whole ▪ Relate fractions to geometry by expressing the area of part of a shape as a unit fraction of the whole		Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways. 

[1] See Glossary, Table 2.
[2] Students need not use formal terms for these properties.
[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.
[4] A range of algorithms may be used.
[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.
[6] Excludes compound units such as cm3 and finding the geometric volume of a container.
[7] Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another.Example: 3/4 and – 3/4 are additive inverses of one another because3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

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[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product 3 × 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 × ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? × 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

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