



Mathematical Practice and Content

Common Core Standards

Second Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit our efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- a. Understand that mathematics is relevant when studied in a cultural context.
- b. Explain the meaning of a problem and restate it in their words.
- c. Analyze given information to develop possible strategies for solving the problem.
- d. Identify and execute appropriate strategies to solve the problem.
- e. Evaluate progress toward the solution and make revisions if necessary.
- f. Check their answers using a different method, and continually ask “Does this make sense?”

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- a. Make sense of quantities and their relationships in problem situations.
- b. Use varied representations and approaches when solving problems.
- c. Know and flexibly use different properties of operations and objects.
- d. Change perspectives, generate alternatives and consider different options.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- a. Understand and use prior learning in constructing arguments.
- b. Habitually ask “why” and seek an answer to that question.
- c. Question and problem-pose.
- d. Develop questioning strategies to generate information.
- e. Seek to understand alternative approaches suggested by others and, as a result, to adopt better approaches.
- f. Justify their conclusions, communicate them to others, and respond to the arguments of others.
- g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.

Mathematically proficient students:

- a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.
- b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- d. Analyze mathematical relationships to draw conclusions.

5. Use appropriate tools strategically.

Mathematically proficient students:

- a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)
- b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.

6. Attend to precision.

Mathematically proficient students:

- a. Communicate their understanding of mathematics to others.
- b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- c. Specify units of measure and use label parts of graphs and charts
- d. Strive for accuracy.

7. Look for and make use of structure.

Mathematically proficient students:

- a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.
- b. Apply and discuss properties.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- a. Look for mathematically sound shortcuts.
- b. Use repeated applications to generalize properties.

Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

Reasoning and explaining

4. Model with mathematics
5. Use appropriate tools strategically

Modeling and using tools

7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Seeing structure and generalizing



Standards for Mathematical Practice: Grade 2 Explanations and Examples

<u>Standards</u>	<u>Explanations and Examples</u>
<i>Students are expected to:</i>	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
2.MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.
2.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length.
2.MP.3. Construct viable arguments and critique the reasoning of others.	Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions.
2.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
2.MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.
2.MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
2.MP.7. Look for and make use of structure.	Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).
2.MP.8. Look for and express regularity in repeated reasoning.	Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, “Does this make sense?”

Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Operations and Algebraic Thinking

2.OA

Represent and solve problems involving addition and subtraction.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **add, subtract, more, less, equal, equation, putting together, taking from, taking apart**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations within a cultural context, including those of Montana American Indians, of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 1.)</p> <ul style="list-style-type: none"> Identify the unknown in an addition or subtraction word problem Determine the appropriate operations needed to solve addition and subtraction problems in situations including add to, take from, put together, take apart, and compare Use drawings or equations to represent one- and two-step word problems Add and subtract within 100 to solve one-step word problems with unknowns in any positions Write an addition and subtraction equation with a symbol for the unknown 	<p>Word problems that are connected to students' lives can be used to develop fluency with addition and subtraction. Table 1 describes the four different addition and subtraction situations and their relationship to the position of the unknown.</p> <p>Examples:</p> <ul style="list-style-type: none"> Take From example: David had 63 stickers. He gave 37 to Susan. How many stickers does David have now? $63 - 37 = \square$ Add To example: David had \$37. His grandpa gave him some money for his birthday. Now he has \$63. How much money did David's grandpa give him? $\\$37 + \square = \\63 Compare example: David has 63 stickers. Susan has 37 stickers. How many more stickers does David have than Susan? $63 - 37 = \square$ <ul style="list-style-type: none"> Even though the modeling of the two problems above is different, the equation, $63 - 37 = ?$, can represent both situations (How many more do I need to make 63?) Take From (Start Unknown) David had some stickers. He gave 37 to Susan. Now he has 26 stickers. How many stickers did David have before? $\square - 37 = 26$ <p>It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.</p> <ul style="list-style-type: none"> Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students. The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems). <p>Second graders should work on ALL problem types regardless of the level of difficulty. Mastery is expected in second grade. Students can use interactive whiteboard or document camera to demonstrate and justify their thinking.</p> <p>This standard focuses on developing an algebraic representation of a word problem through addition and subtraction -- the intent is not to introduce traditional algorithms or rules.</p>

Operations and Algebraic Thinking

2.OA

Add and subtract within 20.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **add, subtract, sum, more, less, equal, equation, putting together, taking from, taking apart**

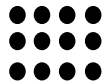
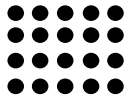
<u>Standard/ Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.OA.2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. (See standard 1.OA.6 for a list of mental strategies.)</p> <ul style="list-style-type: none">▪ Know mental strategies for addition and subtraction▪ Know from memory all sums of two one-digit numbers▪ Apply mental strategies to add and subtract fluently within 20▪ Fluently add and subtract within 20	<p>This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 20. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.</p> <p>Mental strategies help students make sense of number relationships as they are adding and subtracting within 20. The ability to calculate mentally with efficiency is very important for all students. Mental strategies may include the following:</p> <ul style="list-style-type: none">• Counting on• Making tens ($9 + 7 = 10 + 6$)• Decomposing a number leading to a ten ($14 - 6 = 14 - 4 - 2 = 10 - 2 = 8$)• Fact families ($8 + 5 = 13$ is the same as $13 - 8 = 5$)• Doubles• Doubles plus one ($7 + 8 = 7 + 7 + 1$) <p>However, the use of objects, diagrams, or interactive whiteboards, and various strategies will help students develop fluency.</p>

Operations and Algebraic Thinking

2.OA

Work with equal groups of objects to gain foundations for multiplication.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **odd, even, row, column, rectangular array, equal**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</p> <ul style="list-style-type: none">Recognize that in groups of even numbers objects will pair up evenlyRecognize that in groups of odd numbers objects will not pair up evenlyDetermine whether a group of objects is odd or even, using a variety of strategies	<p>Students explore odd and even numbers in a variety of ways including the following: students may investigate if a number is odd or even by determining if the number of objects can be divided into two equal sets, arranged into pairs or counted by twos. After the above experiences, students may derive that they only need to look at the digit in the ones place to determine if a number is odd or even since any number of tens will always split into two even groups.</p> <p>Example:</p> <p>Students need opportunities writing equations representing sums of two equal addends, such as: $2 + 2 = 4$, $3 + 3 = 6$, $5 + 5 = 10$, $6 + 6 = 12$, or $8 + 8 = 16$. This understanding will lay the foundation for multiplication and is closely connected to 2.OA.4.</p> <p>The use of objects and/or interactive whiteboards will help students develop and demonstrate various strategies to determine even and odd numbers.</p>
<p>2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</p> <ul style="list-style-type: none">Generalize that fact that arrays can be written as repeated addition problemsSolve repeated addition problems to find the number of objects using rectangular arraysWrite an equation with repeated equal addends from an array	<p>Students may arrange any set of objects into a rectangular array. Objects can be cubes, buttons, counters, etc. Objects do not have to be square to make an array. Geoboards can also be used to demonstrate rectangular arrays. Students then write equations that represent the total as the sum of equal addends as shown below.</p> <div style="display: flex; justify-content: space-around; align-items: center;"><div style="text-align: center;"> $4 + 4 + 4 = 12$</div><div style="text-align: center;"> $5 + 5 + 5 + 5 = 20$</div></div> <p>Interactive whiteboards and document cameras may be used to help students visualize and create arrays.</p>

Number and Operations in Base Ten

2.NBT

Understand place value.

Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **hundreds, tens, ones, skip count, base-ten, number names to 1,000 (e.g., one, two, thirty, etc.), expanded form, greater than ($>$), less than ($<$), equal to ($=$), digit, compare**

Standard/Learning Objectives

2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- a. 100 can be thought of as a bundle of ten tens—called a “hundred.”
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- Explain the value of each digit in a three-digit number
 - Identify a bundle of 10 tens a “hundred”
 - Represent a three digit number with hundreds, tens, and ones
 - Represent 100, 200, 300, 400, 500, 600, 700, 800, 900 with one, two, three, four, five, six, seven, eight, or nine hundreds and 0 tens and 0 ones

Explanations and Examples

Understanding that 10 ones make one ten and that 10 tens make one hundred is fundamental to students' mathematical development. Students need multiple opportunities counting and “bundling” groups of tens in first grade. In second grade, students build on their understanding by making bundles of 100s with or without leftovers using base ten blocks, cubes in towers of 10, ten frames, etc. This emphasis on bundling hundreds will support students' discovery of place value patterns.

As students are representing the various amounts, it is important that emphasis is placed on the language associated with the quantity. For example, 243 can be expressed in multiple ways such as 2 groups of hundred, 4 groups of ten and 3 ones, as well as 24 tens and 3 ones. When students read numbers, they should read in standard form as well as using place value concepts. For example, 243 should be read as “two hundred forty-three” as well as two hundreds, 4 tens, 3 ones.

A document camera or interactive whiteboard can also be used to demonstrate “bundling” of objects. This gives students the opportunity to communicate their thinking.

Number and Operations in Base Ten

2.NBT

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<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>									
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s. <ul style="list-style-type: none">Count within 1000Skip-count by 5s to 1000Skip-count by 10s to 1000Skip-count by 100s to 1000	<p>Students need many opportunities counting, up to 1000, from different starting points. They should also have many experiences skip counting by 5s, 10s, and 100s to develop the concept of place value.</p> <p>Examples:</p> <ul style="list-style-type: none">The use of the 100s chart may be helpful for students to identify the counting patterns.The use of money (nickels, dimes, dollars) or base ten blocks may be helpful visual cues.The use of an interactive whiteboard may also be used to develop counting skills. <p>The ultimate goal for second graders is to be able to count in multiple ways with no visual support.</p>									
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. <ul style="list-style-type: none">Know what expanded form meansRecognize that the digits in each place represent amounts of thousands, hundreds, tens, or onesRead numbers to 1000 using base ten numeralsRead numbers to 1000 using number namesRead numbers to 1000 using expanded formWrite numbers to 1000 using base ten numerals	<p>Students need many opportunities reading and writing numerals in multiple ways.</p> <p>Examples:</p> <table><tr><td>Base-ten numerals</td><td>637</td><td>(standard form)</td></tr><tr><td>Number names</td><td>six hundred thirty seven</td><td>(written form)</td></tr><tr><td>Expanded form</td><td>600 + 30 + 7</td><td>(expanded notation)</td></tr></table> <p>When students say the expanded form, it may sound like this: “6 hundreds plus 3 tens plus 7 ones” OR 600 plus 30 plus 7.”</p>	Base-ten numerals	637	(standard form)	Number names	six hundred thirty seven	(written form)	Expanded form	600 + 30 + 7	(expanded notation)
Base-ten numerals	637	(standard form)								
Number names	six hundred thirty seven	(written form)								
Expanded form	600 + 30 + 7	(expanded notation)								

Number and Operations in Base Ten

2.NBT

Understand place value.

Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **hundreds, tens, ones, skip count, base-ten, number names to 1,000 (e.g., one, two, thirty, etc.), expanded form, greater than (>), less than (<), equal to (=), digit, compare**

Standard/Learning Objectives

2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

- Know the value of each digit represented in a three-digit number
- Know what “ $>$ ”, “ $<$ ”, and “ $=$ ” represent
- Compare two three-digit numbers based on place value of each digit
- Use $>$, $=$ and $<$ symbols to record the results of comparisons

Explanations and Examples

Students may use models, number lines, base ten blocks, interactive whiteboards, document cameras, written words, and/or spoken words that represent two three-digit numbers. To compare, students apply their understanding of place value. They first attend to the numeral in the hundreds place, then the numeral in tens place, then, if necessary, to the numeral in the ones place.

Comparative language includes but is not limited to: more than, less than, greater than, most, greatest, least, same as, equal to and not equal to. Students use the appropriate symbols to record the comparisons.

Number and Operations in Base Ten

2.NBT

Use place value understanding and properties of operations to add and subtract.

Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fluent, compose, decompose, place value, digit, ten more, ten less, one hundred more, one hundred less, add, subtract, sum, equal, addition, subtraction**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <ul style="list-style-type: none">Know strategies for adding and subtracting based on place valueKnow strategies for adding and subtracting based on properties of operationsKnow strategies for adding and subtracting based on the relationship between addition and subtractionChoose a strategy (place value, properties of operations, and/or the relationship between addition and subtraction) to fluently add and subtract within 100Fluently add and subtract within 100	<p>Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students should have experiences solving problems written both horizontally and vertically. They need to communicate their thinking and be able to justify their strategies both verbally and with paper and pencil.</p> <p>Addition strategies based on place value for $48 + 37$ may include:</p> <ul style="list-style-type: none">Adding by place value: $40 + 30 = 70$ and $8 + 7 = 15$ and $70 + 15 = 85$.Incremental adding (breaking one number into tens and ones): $48 + 10 = 58$, $58 + 10 = 68$, $68 + 10 = 78$, $78 + 7 = 85$Compensation (making a friendly number): $48 + 2 = 50$, $37 - 2 = 35$, $50 + 35 = 85$ <p>Subtraction strategies based on place value for $81 - 37$ may include:</p> <ul style="list-style-type: none">Adding up (from smaller number to larger number): $37 + 3 = 40$, $40 + 40 = 80$, $80 + 1 = 81$, and $3 + 40 + 1 = 44$.Incremental subtracting: $81 - 10 = 71$, $71 - 10 = 61$, $61 - 10 = 51$, $51 - 7 = 44$Subtracting by place value: $81 - 30 = 51$, $51 - 7 = 44$ <p>Properties that students should know and use are:</p> <ul style="list-style-type: none">Commutative property of addition (Example: $3 + 5 = 5 + 3$)Associative property of addition (Example: $(2 + 7) + 3 = 2 + (7+3)$)Identity property of 0 (Example: $8 + 0 = 8$) <p>Students in second grade need to communicate their understanding of why some properties work for some operations and not for others.</p> <ul style="list-style-type: none">Commutative Property: In first grade, students investigated whether the commutative property works with subtraction. The intent was for students to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should also understand that they will be working with numbers in later grades that will allow them to subtract larger numbers from smaller numbers. This exploration of the commutative property continues in second grade.Associative Property: Recognizing that the associative property does not work for subtraction is difficult for students to consider at this grade level as it is challenging to determine all the possibilities.

Number and Operations in Base Ten

2.NBT

Use place value understanding and properties of operations to add and subtract.

Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fluent, compose, decompose, place value, digit, ten more, ten less, one hundred more, one hundred less, add, subtract, sum, equal, addition, subtraction**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. <ul style="list-style-type: none">Know strategies for adding two digit numbers based on place value and properties of operationsUse strategies to add up to four two-digit numbers	Students demonstrate addition strategies with up to four two-digit numbers either with or without regrouping. Problems may be written in a story problem format to help develop a stronger understanding of larger numbers and their values. Interactive whiteboards and document cameras may also be used to model and justify student thinking.
2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	There is a strong connection between this standard and place value understanding with addition and subtraction of smaller numbers. Students may use concrete models or drawings to support their addition or subtraction of larger numbers. Strategies are similar to those stated in 2.NBT.5, as students extend their learning to include greater place values moving from tens to hundreds to thousands. Interactive whiteboards and document cameras may also be used to model and justify student thinking.

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Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fluent, compose, decompose, place value, digit, ten more, ten less, one hundred more, one hundred less, add, subtract, sum, equal, addition, subtraction**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p> <ul style="list-style-type: none">▪ Know place value within 1000▪ Apply knowledge of place value to mentally add or subtract 10 to 100 to/from a given number 100-900	<p>Students need many opportunities to practice mental math by adding and subtracting multiples of 10 and 100 up to 900 using different starting points. They can practice this by counting and thinking aloud, finding missing numbers in a sequence, and finding missing numbers on a number line or hundreds chart. Explorations should include looking for relevant patterns.</p> <p>Mental math strategies may include:</p> <ul style="list-style-type: none">• counting on; 300, 400, 500, etc.• counting back; 550, 450, 350, etc. <p>Examples:</p> <ul style="list-style-type: none">• 100 more than 653 is ____ (753)• 10 less than 87 is ____ (77)• “Start at 248. Count up by 10s until I tell you to stop.” <p>An interactive whiteboard or document camera may be used to help students develop these mental math skills.</p>
<p>2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)</p> <ul style="list-style-type: none">▪ Know addition and subtraction strategies using place value and properties of operations related to addition and subtraction▪ Explain why addition and subtraction strategies based on place value and properties of operations work	<p>Students need multiple opportunities explaining their addition and subtraction thinking. Operations embedded within a meaningful context promote development of reasoning and justification.</p> <p>Example:</p> <p>Mason read 473 pages in June. He read 227 pages in July. How many pages did Mason read altogether?</p> <ul style="list-style-type: none">• Karla’s explanation: $473 + 227 = \underline{\hspace{2cm}}$. I added the ones together ($3 + 7$) and got 10. Then I added the tens together ($70 + 20$) and got 90. I knew that $400 + 200$ was 600. So I added $10 + 90$ for 100 and added $100 + 600$ and found out that Mason had read 700 pages altogether.• Debbie’s explanation: $473 + 227 = \underline{\hspace{2cm}}$. I started by adding 200 to 473 and got 673. Then I added 20 to 673 and I got 693 and finally I added 7 to 693 and I knew that Mason had read 700 pages altogether. <p>Continued on next page</p>

Number and Operations in Base Ten

2.NBT

Use place value understanding and properties of operations to add and subtract.

Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fluent, compose, decompose, place value, digit, ten more, ten less, one hundred more, one hundred less, add, subtract, sum, equal, addition, subtraction**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
	<ul style="list-style-type: none">Becky's explanation: I used base ten blocks on a base ten mat to help me solve this problem. I added 3 ones (units) plus 7 ones and got 10 ones which made one ten. I moved the 1 ten to the tens place. I then added 7 tens rods plus 2 tens rods plus 1 tens rod and got 10 tens or 100. I moved the 1 hundred to the hundreds place. Then I added 4 hundreds plus 2 hundreds plus 1 hundred and got 7 hundreds or 700. So Mason read 700 books. <p>Students should be able to connect different representations and explain the connections. Representations can include numbers, words (including mathematical language), pictures, number lines, and/or physical objects. Students should be able to use any/all of these representations as needed.</p> <p>An interactive whiteboard or document camera can be used to help students develop and explain their thinking.</p>

Measurement and Data

2.MD

Measure and estimate lengths in standard units.

Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **about, a little less than, a little more than, longer, shorter, inch, foot, centimeter, meter, ruler, yardstick, meter stick, measuring tape, estimate**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p> <ul style="list-style-type: none">▪ Identify tools that can be used to measure length▪ Identify the unit of length for the tool used (inches, centimeters, feet, meters)▪ Determine which tool is more appropriate to use to measure the length of an object▪ Measure the length of objects, using appropriate tools	<p>Students in second grade will build upon what they learned in first grade from measuring length with non-standard units to the new skill of measuring length in metric and U.S. Customary with standard units of measure. They should have many experiences measuring the length of objects with rulers, yardsticks, meter sticks, and tape measures. They will need to be taught how to actually use a ruler appropriately to measure the length of an object especially as to where to begin the measuring. Do you start at the end of the ruler or at the zero?</p>
<p>2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.</p> <ul style="list-style-type: none">▪ Know how to measure the length of objects with different units▪ Explain the length of an object in relation to the size of the units used to measure it	<p>Students need multiple opportunities to measure using different units of measure. They should not be limited to measuring within the same standard unit. Students should have access to tools, both U.S. Customary and metric. The more students work with a specific unit of measure, the better they become at choosing the appropriate tool when measuring.</p> <p>Students measure the length of the same object using different tools (ruler with inches, ruler with centimeters, a yardstick, or meter stick). This will help students learn which tool is more appropriate for measuring a given object. They describe the relationship between the size of the measurement unit and the number of units needed to measure something. For instance, a student might say, "The longer the unit, the fewer I need." Multiple opportunities to explore provide the foundation for relating metric units to customary units, as well as relating within customary (inches to feet to yards) and within metric (centimeters to meters).</p>


Measurement and Data

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Measure and estimate lengths in standard units.

Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **about, a little less than, a little more than, longer, shorter, inch, foot, centimeter, meter, ruler, yardstick, meter stick, measuring tape, estimate**

<u>Standard/Learning Objectives</u>	<u>Explanations and Examples</u>
<p>2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.</p> <ul style="list-style-type: none">▪ Know strategies for estimating length▪ Recognize the size of inches, feet, centimeters, and meters▪ Determine if an estimate is reasonable▪ Estimate lengths in units of inches, feet, centimeters, and meters	<p>Estimation helps develop familiarity with the specific unit of measure being used. To measure the length of a shoe, knowledge of an inch or a centimeter is important so that one can approximate the length in inches or centimeters. Students should begin practicing estimation with items which are familiar to them (length of desk, pencil, favorite book, etc.).</p> <p>Some useful benchmarks for measurement are:</p> <ul style="list-style-type: none">• First joint to the tip of a thumb is about an inch• Length from your elbow to your wrist is about a foot• If your arm is held out perpendicular to your body, the length from your nose to the tip of your fingers is about a yard  <p>The illustration shows a person from the chest up, wearing a green shirt. They are holding their right arm out straight to the side, perpendicular to their body. A horizontal line with arrows at both ends is drawn above the arm, starting from the person's nose and ending at the tip of their fingers. Above this line, the text "1 yard" is written.</p>
<p>2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.</p> <ul style="list-style-type: none">▪ Name standard length units▪ Compare lengths of two objects▪ Determine how much longer one object is than another in standard length units	<p>Second graders should be familiar enough with inches, feet, yards, centimeters, and meters to be able to compare the differences in lengths of two objects. They can make direct comparisons by measuring the difference in length between two objects by laying them side by side and selecting an appropriate standard length unit of measure. Students should use comparative phrases such as "It is longer by 2 inches" or "It is shorter by 5 centimeters" to describe the difference between two objects. An interactive whiteboard or document camera may be used to help students develop and demonstrate their thinking.</p>

Measurement and Data

2.MD

Relate addition and subtraction to length.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inch, foot, yard, centimeter, meter, ruler, yardstick, meter stick, measuring tape, estimate, length, equation, number line, equally spaced, point**

Standard/Learning Objectives

2.MD.5. Use addition and subtraction within 100 to solve word problems within a cultural context, including those of Montana American Indians, involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

- Add and subtract lengths within 100
- Solve word problems involving lengths that are given in the same units
- Solve word problems involving length that have equations with a symbol for the unknown number

Explanations and Examples

Students need experience working with addition and subtraction to solve word problems which include measures of length. It is important that word problems stay within the same unit of measure. Counting on and/or counting back on a number line will help tie this concept to previous knowledge. Some representations students can use include drawings, rulers, pictures, and/or physical objects. An interactive whiteboard or document camera may be used to help students develop and demonstrate their thinking.

Equations include:

- $20 + 35 = c$
- $c - 20 = 35$
- $c - 35 = 20$
- $20 + b = 55$
- $35 + a = 55$
- $55 = a + 35$
- $55 = 20 + b$

Example:

- A word problem for $5 - n = 2$ could be: Mary is making a dress. She has 5 yards of fabric. She uses some of the fabric and has 2 yards left. How many yards did Mary use?

There is a strong connection between this standard and demonstrating fluency of addition and subtraction facts. Addition facts through $10 + 10$ and the related subtraction facts should be included.

2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Students represent their thinking when adding and subtracting within 100 by using a number line. An interactive whiteboard or document camera can be used to help students demonstrate their thinking.

Example: $10 - 6 = 4$



Measurement and Data

2.MD

Work with time and money.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **clocks, hand, hour hand, minute hand, hour, minute, a.m., p.m., o'clock, multiples of 5** (e.g., **five, ten, fifteen**, etc.), **analog clock, digital clock, quarter 'til, quarter after, half past, quarter hour, half hour, thirty minutes before, 30 minutes after, 30 minutes until, 30 minutes past, quarter, dime, nickel, dollar, cent(s), \$, ¢, heads, tails**

Standard/Learning Objectives

2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

- Look for and make use of structure
- Determine what time is represented by the combination of the number on the clock face and the position of the hands
- Tell time using analog clocks to the nearest 5 minutes
- Tell time using digital clocks to the nearest 5 minutes
- Write time using analog clocks and digital clocks
- Identify the hour and minute hand on an analog clock
- Identify and label when a.m. and p.m. occur

Explanations and Examples

In first grade, students learned to tell time to the nearest hour and half-hour. Students build on this understanding in second grade by skip-counting by 5 to recognize 5-minute intervals on the clock. They need exposure to both digital and analog clocks. It is important that they can recognize time in both formats and communicate their understanding of time using both numbers and language. Common time phrases include the following: quarter till ____, quarter after ____, ten till ____, ten after ____, and half past ____.

Students should understand that there are 2 cycles of 12 hours in a day - a.m. and p.m. Recording their daily actions in a journal would be helpful for making real-world connections and understanding the difference between these two cycles. An interactive whiteboard or document camera may be used to help students demonstrate their thinking.

Measurement and Data

2.MD

Relate addition and subtraction to length.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inch, foot, yard, centimeter, meter, ruler, yardstick, meter stick, measuring tape, estimate, length, equation, number line, equally spaced, point**

Standard/Learning Objectives

2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

- Identify and recognize the value of dollar bills, quarters, dimes, nickels, and pennies
- Identify the \$ and ¢ symbol
- Solve word problems involving dollar bills, quarters, dimes, nickels

Explanations and Examples

Since money is not specifically addressed in kindergarten, first grade, or third grade, students should have multiple opportunities to identify, count, recognize, and use coins and bills in and out of context. They should also experience making equivalent amounts using both coins and bills. “Dollar bills” should include denominations up to one hundred (\$1.00, \$5.00, \$10.00, \$20.00, \$100.00).

Students should solve story problems connecting the different representations. These representations may include objects, pictures, charts, tables, words, and/or numbers. Students should communicate their mathematical thinking and justify their answers. An interactive whiteboard or document camera may be used to help students demonstrate and justify their thinking.

Example:

- Sandra went to the store and received \$ 0.76 in change. What are three different sets of coins she could have received?

Measurement and Data

2.MD

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **collect, organize, display, show, data, attribute, sort, line plot, picture graph, bar graph**

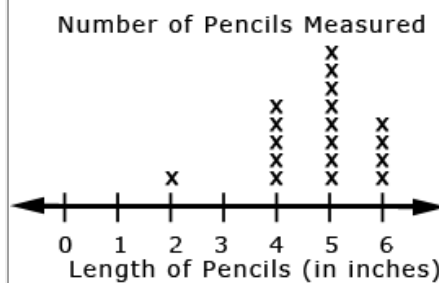
Standard/Learning Objectives

2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

- Read tools of measurement to the nearest unit
- Represent measurement data on a line plot
- Measure lengths of several objects to the nearest whole unit
- Measure lengths of objects by making repeated measurements of the same object
- Create a line plot with a horizontal scale marked in whole numbers using measurements

Explanations and Examples

This standard emphasizes representing data using a line plot. Students will use the measurement skills learned in earlier standards to measure objects. Line plots are first introduced in this grade level. A line plot can be thought of as plotting data on a number line. An interactive whiteboard may be used to create and/or model line plots.



Measurement and Data

2.MD

Represent and interpret data.

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Standard/Learning Objectives

2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (See Table 1.)

- Recognize and identify picture graphs and bar graphs
- Identify and label the components of a picture graph and bar graph
- Make comparisons between categories in the graph using more than, less than, etc.
- Solve problems relating to data in graphs by using addition and subtraction
- Draw a single-unit scale picture graph to represent a given set of data with up to four categories
- Draw a single-unit scale bar graph to represent a given set of data with up to four categories

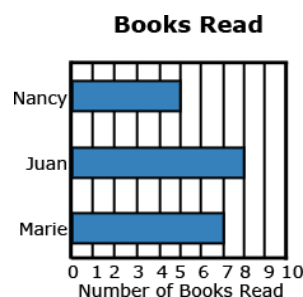
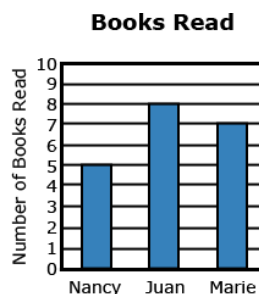
Explanations and Examples

Students should draw both picture and bar graphs representing data that can be sorted up to four categories using single unit scales (e.g., scales should count by ones). The data should be used to solve put together, take-apart, and compare problems as listed in Table 1.

In second grade, picture graphs (pictographs) include symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.

Number of Books Read	
Nancy	✧ ✧ ✧ ✧ ✧
Juan	✧ ✧ ✧ ✧ ✧ ✧ ✧ ✧
✧ = 1 Book	

Second graders should draw both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Geometry

2.G

Reason with shapes and their attributes.

Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, angle, side, triangle, quadrilateral, square, rectangle, trapezoid, pentagon, hexagon, cube, face, edge, vertex, surface, figure, shape, closed, open, partition, equal size, equal shares, half, halves, thirds, half of, a third of, whole, two halves, three thirds, four fourths, partition**

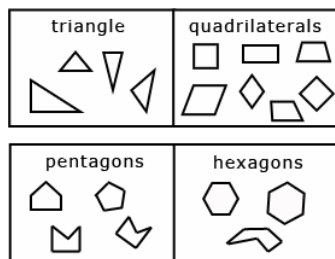
Standard/Learning Objectives

2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)

- Identify the attributes of triangles, quadrilaterals, pentagons, hexagons, and cubes (e.g., faces, angles, sides, vertices, etc.)
- Identify triangles, quadrilaterals, pentagons, hexagons, and cubes based on the given attributes
- Describe and analyze shapes by examining their sides and angles, not by measuring
- Compare shapes by their attributes (e.g., faces, angles)
- Draw shapes with specified attributes

Explanations and Examples

Students identify, describe, and draw triangles, quadrilaterals, pentagons, and hexagons. Pentagons, triangles, and hexagons should appear as both regular (equal sides and equal angles) and irregular. Students recognize all four sided shapes as quadrilaterals. Students use the vocabulary word “angle” in place of “corner” but they do not need to name angle types. Interactive whiteboards and document cameras may be used to help identify shapes and their attributes. Shapes should be presented in a variety of orientations and configurations.



Geometry

2.G

Reason with shapes and their attributes.

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Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, angle, side, triangle, quadrilateral, square, rectangle, trapezoid, pentagon, hexagon, cube, face, edge, vertex, surface, figure, shape, closed, open, partition, equal size, equal shares, half, halves, thirds, half of, a third of, whole, two halves, three thirds, four fourths, partition**

Standard/Learning Objectives

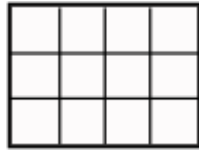
2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

- Define partition
- Identify a row
- Identify a column
- Determine how to partition a rectangle into same-size squares
- Count to find the total number of same-size squares

Explanations and Examples

This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication. An interactive whiteboard or manipulatives such as square tiles, cubes, or other square shaped objects can be used to help students partition rectangles.

Rows are horizontal and columns are vertical.



Geometry

2.G

Reason with shapes and their attributes.

Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, angle, side, triangle, quadrilateral, square, rectangle, trapezoid, pentagon, hexagon, cube, face, edge, vertex, surface, figure, shape, closed, open, partition, equal size, equal shares, half, halves, thirds, half of, a third of, whole, two halves, three thirds, four fourths, partition**

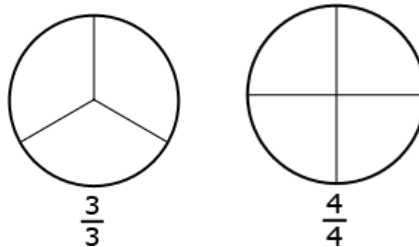
Standard/Learning Objectives

2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

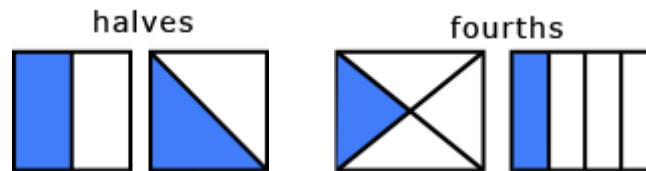
- Identify two, three, and four equal shares of a whole
- Describe equal shares using vocabulary: *halves*, *thirds*, *fourths*, *half of*, *third of*, etc.
- Describe the whole as *two halves*, *three thirds*, or *four fourths*
- Justify why equal shares of identical wholes need not have the same shape

Explanations and Examples

This standard introduces fractions in an area model. Students need experiences with different sizes, circles, and rectangles. For example, students should recognize that when they cut a circle into three equal pieces, each piece will equal one third of its original whole. In this case, students should describe the whole as three thirds. If a circle is cut into four equal pieces, each piece will equal one fourth of its original whole and the whole is described as four fourths.



Students should see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole. An interactive whiteboard may be used to show partitions of shapes.



GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another. Example: $3/4$ and $-3/4$ are additive inverses of one another because $3/4 + (-3/4) = (-3/4) + 3/4 = 0$.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.¹

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

¹ Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/mathglos.html>, accessed March 2, 2010.

² Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median M , the first quartile is the median of the data values less than M . Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or $-a$ for some whole number a .

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is $15 - 6 = 9$. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.³

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.⁴ Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $3/4$ and $4/3$ are multiplicative inverses of one another because $3/4 \times 4/3 = 4/3 \times 3/4 = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

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³Adapted from Wisconsin Department of Public Instruction, *op. cit.*

⁴To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $5/50 = 10\%$ per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b . The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.⁵

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

⁵Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Third quartile. For a data set with median M , the third quartile is the median of the data values greater than M . Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

⁵Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Tables

Table 1. Common addition and subtraction situations.¹

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
Put Together/ Take Apart¹	Total Unknown	Addend Unknown	Both Addends Unknown²
	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
Compare²	Difference Unknown	Bigger Unknown	Smaller Unknown
	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

¹These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

²Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

³For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

¹Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.¹

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,⁴ Area⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

⁴The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

⁵Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. Here a , b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a , b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a .

Table 5. The properties of inequality. Here a , b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

Mathematics Learning Progressions by Domain									
K	1	2	3	4	5	6	7	8	HS
Counting and Cardinality									Number and Quantity
Number and Operations in Base Ten						Ratios and Proportional Relationship			
			Number and Operations – Fractions			The Number System			
Operations and Algebraic Thinking						Expressions and Equations		Algebra	
							Functions		
Geometry									
Measurement and Data						Statistics and Probability			