Billings PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

First Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.
Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.
Mathematically proficient students:
- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.
Mathematically proficient students:
- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students:
- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**
- **f. Justify their conclusions, communicate them to others, and respond to the arguments of others.**
- **g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.**

4. Model with mathematics.

Mathematically proficient students:

- **a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.**
- **b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.**
- **c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.**
- **d. Analyze mathematical relationships to draw conclusions.**

5. Use appropriate tools strategically.

Mathematically proficient students:

- **a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)**
- **b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.**

6. Attend to precision.

Mathematically proficient students:

- **a. Communicate their understanding of mathematics to others.**
- **b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.**
- **c. Specify units of measure and use label parts of graphs and charts**
- **d. Strive for accuracy.**

7. Look for and make use of structure.

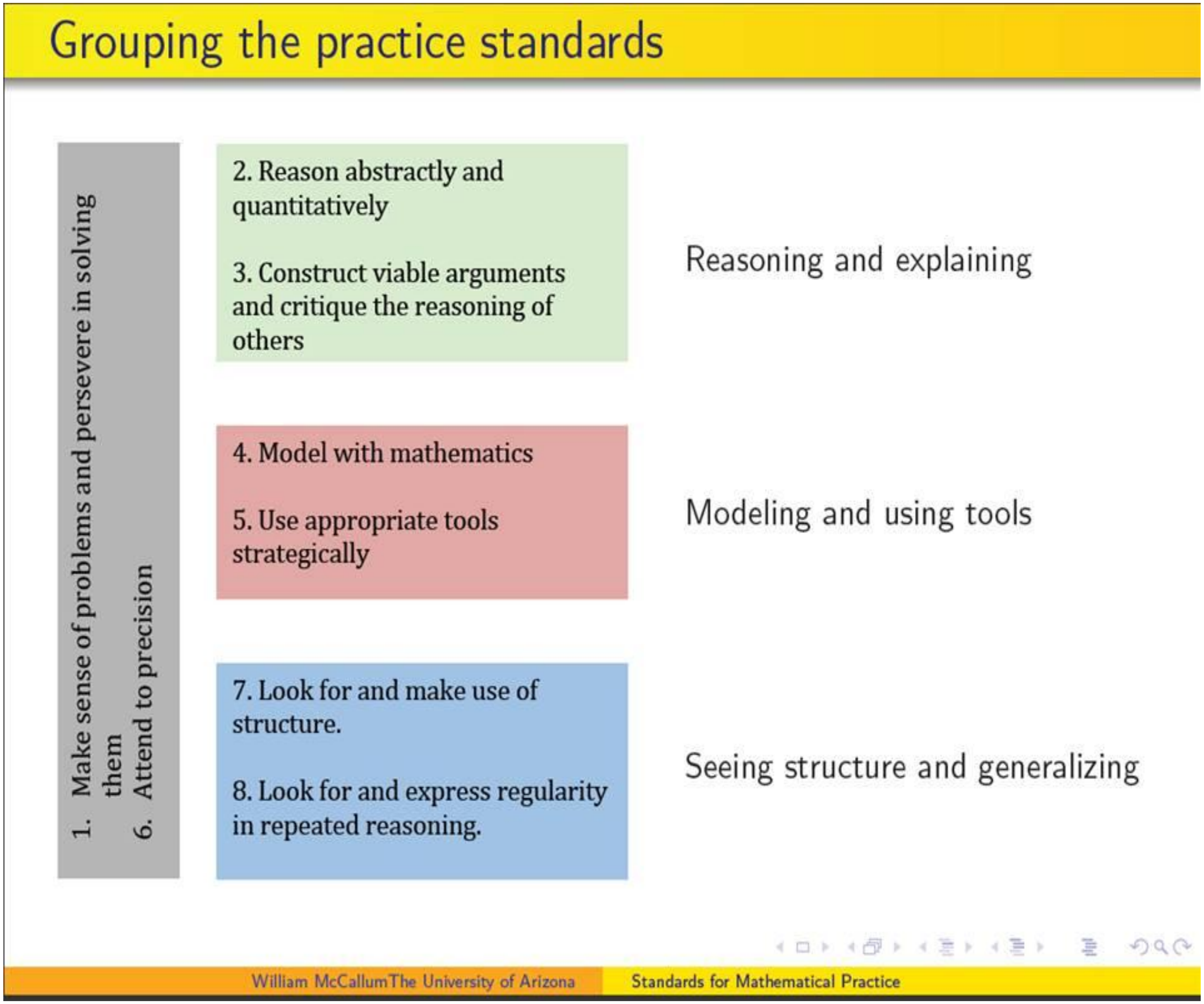
Mathematically proficient students:

- **a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.**
- **b. Apply and discuss properties.**

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- **a. Look for mathematically sound shortcuts.**
- **b. Use repeated applications to generalize properties.**


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallumThe University of Arizona
Standards for Mathematical Practice

Standards for Mathematical Practice: Grade 1 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
1.MP.1. Make sense of problems and persevere in solving them.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
1.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
1.MP.3. Construct viable arguments and critique the reasoning of others.	First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions.
1.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
1.MP.5. Use appropriate tools strategically.	In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem.
1.MP.6. Attend to precision.	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
1.MP.7. Look for and make use of structure.	First graders begin to discern a pattern or structure. For instance, if students recognize $12 + 3 = 15$, *then they also know* $3 + 12 = 15$. *(Commutative property of addition.) To add* $4 + 6 + 4$, *the first two numbers can be added to make a ten, so* $4 + 6 + 4 = 10 + 4 = 14$.
1.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, "Does this make sense?"

FIRST GRADE

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 1 Overview

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten
- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data
- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry
- Reason with shapes and their attributes.

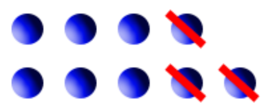
Operations and Algebraic Thinking **1.OA**

Represent and solve problems involving addition and subtraction.

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with the operations.

An important component of solving problems involving addition and subtraction is the ability to recognize that any given group of objects (up to 10) can be separated into sub groups in multiple ways and remain equivalent in amount to the original group (Ex: A set of 6 cubes can be separated into a set of 2 cubes and a set of 4 cubes and remain 6 total cubes).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **adding to, taking from, putting together, taking apart, comparing, unknown, sum, less than, equal to, minus, subtract, the same amount as, and** (to describe (+) symbol)
***NOTE:** *Subtraction names a missing part. Therefore, the minus sign should be read as "minus" or "subtract" but not as "take away". Although "take away" has been a typical way to define subtraction, it is a narrow and incorrect definition.* (*Fosnot & Dolk, 2001; Van de Walle & Lovin, 2006)

Standard / Learning Objective	***Explanations and Examples***
1.OA.1. Use addition and subtraction within 20 to solve word problems within a cultural context, including those of Montana American Indians, involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Table 1.) ▪ Use a symbol for an unknown number in an addition or subtraction problem within 20 ▪ Interpret situations to solve word problems with unknowns in all positions within 20 using addition and subtraction ▪ Solve word problems within 20 using addition and subtraction	Contextual problems that are closely connected to students' lives should be used to develop fluency with addition and subtraction. Table 1 describes the four different addition and subtraction situations and their relationship to the position of the unknown. Students use objects or drawings to represent the different situations. • Take From example: Abel has 9 balls. He gave 3 to Susan. How many balls does Abel have now?  • Compare example: Abel has 9 balls. Susan has 3 balls. How many more balls does Abel have than Susan? A student will use 9 objects to represent Abel's 9 balls and 3 objects to represent Susan's 3 balls. Then they will compare the 2 sets of objects. Note that even though the modeling of the two problems above is different, the equation, 9 - 3 = ?, can represent both situations yet the compare example can also be represented by 3 + ? = 9 (How many more do I need to make 9?) It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown. • Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students. • The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown • The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems). Students may use document cameras to display their combining or separating strategies. This gives them the opportunity to communicate and justify their thinking.

Operations and Algebraic Thinking **1.OA**

Represent and solve problems involving addition and subtraction.

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with the operations.

An important component of solving problems involving addition and subtraction is the ability to recognize that any given group of objects (up to 10) can be separated into sub groups in multiple ways and remain equivalent in amount to the original group (Ex: A set of 6 cubes can be separated into a set of 2 cubes and a set of 4 cubes and remain 6 total cubes).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **adding to, taking from, putting together, taking apart, comparing, unknown, sum, less than, equal to, minus, subtract, the same amount as, and** (to describe (+) symbol)

***NOTE:** *Subtraction names a missing part. Therefore, the minus sign should be read as "minus" or "subtract" but not as "take away". Although "take away" has been a typical way to define subtraction, it is a narrow and incorrect definition.* (*Fosnot & Dolk, 2001; Van de Walle & Lovin, 2006)

Standard / Learning Objective

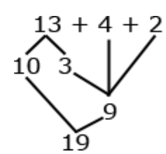
1.OA.2. Solve word problems within a cultural context, including those of Montana American Indians,that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

- Know how to add three whole numbers whose sum is less than or equal to 20
- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

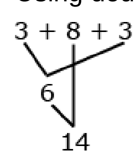
Explanations and Examples

To further students' understanding of the concept of addition, students create word problems with three addends. They can also increase their estimation skills by creating problems in which the sum is less than 5, 10 or 20. They use properties of operations and different strategies to find the sum of three whole numbers such as:

- Counting on and counting on again (e.g., to add 3 + 2 + 4 a student writes 3 + 2 + 4 = ? and thinks, "3, 4, 5, that's 2 more, 6, 7, 8, 9 that's 4 more so 3 + 2 + 4 = 9."
- Making tens (e.g., 4 + 8 + 6 = 4 + 6 + 8 = 10 + 8 = 18)
- Using "plus 10, minus 1" to add 9 (e.g., 3 + 9 + 6 A student thinks, "9 is close to 10 so I am going to add 10 plus 3 plus 6 which gives me 19. Since I added 1 too many, I need to take 1 away so the answer is 18.)
- Decomposing numbers between 10 and 20 into 1 ten plus some ones to facilitate adding the ones



- Using doubles



Students will use different strategies to add the 6 and 8.

- Using near doubles (e.g.,5 + 6 + 3 = 5 + 5 + 1 + 3 = 10 + 4 =14)

Students may use document cameras to display their combining strategies. This gives them the opportunity to communicate and justify their thinking.

Operations and Algebraic Thinking 1.OA

Understand and apply properties of operations and the relationship between addition and subtraction.

Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **order, first, second**

Standard/Learning Objective		***Explanations and Examples***
1.OA.3. Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* (Students need not use formal terms for these properties.) ▪ Define properties of operation strategies ▪ Apply properties of operation as strategies to solve addition and subtraction problems		Students should understand the important ideas of the following properties: • Identity property of addition (e.g., 6 = 6 + 0) • Identity property of subtraction (e.g., 9 – 0 = 9) • Commutative property of addition (e.g., 4 + 5 = 5 + 4) • Associative property of addition (e.g., 3 + 9 + 1 = 3 + 10 = 13) Students need several experiences investigating whether the commutative property works with subtraction. The intent is not for students to experiment with negative numbers but only to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should recognize that they will be working with numbers later on that will allow them to subtract larger numbers from smaller numbers. However, in first grade we do not work with negative numbers.
1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.* ▪ Identify the unknown in a subtraction problem ▪ Solve subtraction problems to find the missing addend ▪ Explain the relationship of addition and subtraction		Example: 10 – 2 = ☐ **Student**: "2 and what make 10? I know that 8 and 2 make 10. So, 10 – 2 = 8." Example: 15 – 9 = ☐ **Student** : "I'll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That's 1 and 5- so it's 6. 15 – 9 = 6."

Operations and Algebraic Thinking Add and subtract within 20.		**1.OA**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **addition, subtraction, counting all, counting on, counting back**		
Standard/Learning Objective		***Explanations and Examples***
1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). ▪ Know how to count on and count back ▪ Explain how counting on and counting back relate to addition and subtraction		Students' multiple experiences with counting may hinder their understanding of counting on and counting back as connected to addition and subtraction. To help them make these connections when students count on 3 from 4, they should write this as 4 + 3 = 7. When students count back (3) from 7, they should connect this to 7 – 3 = 4. Students often have difficulty knowing where to begin their count when counting backward.
1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). ▪ Apply strategies to add and subtract within 20 ▪ Add fluently within 20 ▪ Subtract fluently within 20		This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 10 and having experiences adding and subtracting within 20. By studying patterns and relationships in addition facts and relating addition and subtraction, students build a foundation for fluency with addition and subtraction facts. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. The use of objects, diagrams, or interactive whiteboards and various strategies will help students develop fluency.

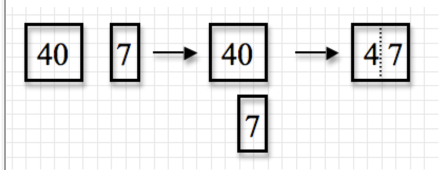
Operations and Algebraic Thinking Work with addition and subtraction equations.		**1.OA**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **addition, subtraction, counting all, counting on, counting back**		
Standard/Learning Objectives		***Explanations and Examples***
1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* ▪ Compare the values on each side of an equal sign ▪ Determine if an equation is true or false		Interchanging the language of "equal to" and "the same as" as well as "not equal to" and "not the same as" will help students grasp the meaning of the equal sign. Students should understand that *"*equality*"* means "the same quantity as". In order for students to avoid the common pitfall that the equal sign means "to do something" or that the equal sign means "the answer is," they need to be able to: • Express their understanding of the meaning of the equal sign • Accept sentences other than a + b = c as true (a = a, c = a + b, a = a + 0, a + b = b + a) • Know that the equal sign represents a relationship between two equal quantities These key skills are hierarchical in nature and need to be developed over time. Experiences determining if equations are true or false help student develop these skills. Initially, students develop an understanding of the meaning of equality using models. However, the goal is for students to reason at a more abstract level. At all times students should justify their answers, make conjectures (e.g., if you add a number and then subtract that same number, you always get zero), and make estimations. Once students have a solid foundation of the key skills listed above, they can begin to rewrite true/false statements using the symbols, < and >. Examples of true and false statements: • 7 = 8 – 1 • 8 = 8 • 1 + 1 + 3 =7 • 4 + 3 = 3 + 4 • 6 – 1 = 1 – 6 • 12 + 2 – 2 = 12 • 9 + 3 = 10 • 5 + 3 = 10 – 2 • 3 + 4 + 5 = 3 + 5 + 4 • 3 + 4 + 5 = 7 + 5 • 13 = 10 + 4 • 10 + 9 + 1 = 19 Students can use a clicker (electronic response system) or interactive whiteboard to display their responses to the equations. This gives them the opportunity to communicate and justify their thinking.

Operations and Algebraic Thinking Work with addition and subtraction equations.		**1.OA**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **addition, subtraction, counting all, counting on, counting back**		
Standard/Learning Objectives		***Explanations and Examples***
1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations: 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*		Students need to understand the meaning of the equal sign and know that the quantity on one side of the equal sign must be the same quantity on the other side of the equal sign. They should be exposed to problems with the unknown in different positions. Having students create word problems for given equations will help them make sense of the equation and develop strategic thinking. Examples of possible student "think-throughs": • *8 + ? = 11*: "8 and some number is the same as 11. 8 and 2 is 10 and 1 more makes 11. So the answer is 3." • *5 = □ – 3*: "This equation means I had some cookies and I ate 3 of them. Now I have 5. How many cookies did I have to start with? Since I have 5 left and I ate 3, I know I started with 8 because I count on from 5. . . 6, 7, 8." Students may use a document camera or interactive whiteboard to display their combining or separating strategies for solving the equations. This gives them the opportunity to communicate and justify their thinking.

Number and Operations in Base Ten — 1.NBT

Extend the counting sequence.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: *number words* **0-120**

Standard/Learning Objectives	***Explanations and Examples***
1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. ▪ Represent a number of objects up to 120 with a written numeral ▪ Count to 120, starting at any number less than 120 ▪ Read and write numerals up to 120	Students use objects, words, and/or symbols to express their understanding of numbers. They extend their counting beyond 100 to count up to 120 by counting by 1s. Some students may begin to count in groups of 10 (while other students may use groups of 2s or 5s to count). Counting in groups of 10 as well as grouping objects into 10 groups of 10 will develop students understanding of place value concepts. Students extend reading and writing numerals beyond 20 to 120. After counting objects, students write the numeral or use numeral cards to represent the number. Given a numeral, students read the numeral, identify the quantity that each digit represents using numeral cards, and count out the given number of objects.  Students should experience counting from different starting points (e.g., start at 83; count to 120). To extend students' understanding of counting, they should be given opportunities to count backwards by ones and tens. They should also investigate patterns in the base 10 system.

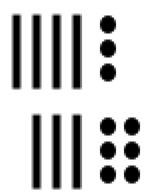
Number and Operations in Base Ten **1.NBT**

Understand place value.

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **tens, ones, bundle, left-overs, singles, groups, greater/less than, equal to**

Standard/Learning Objectives		***Explanations and Examples***
1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).		Understanding the concept of 10 is fundamental to children's mathematical development. Students need multiple opportunities counting 10 objects and "bundling" them into one group of ten. They count between 10 and 20 objects and make a bundle of 10 with or without some left over (this will help students who find it difficult to write teen numbers). Finally, students count any number of objects up to 99, making bundles of 10s with or without leftovers. As students are representing the various amounts, it is important that an emphasis is placed on the language associated with the quantity. For example, 53 should be expressed in multiple ways such as 53 ones or 5 groups of ten with 3 ones leftover. When students read numbers, they read them in standard form as well as using place value concepts. For example, 53 should be read as "fifty-three" as well as five tens, 3 ones. Reading 10, 20, 30, 40, 50 as "one ten, 2 tens, 3 tens, etc." helps students see the patterns in the number system. Students may use the document camera or interactive whiteboard to demonstrate their "bundling" of objects. This gives them the opportunity to communicate their thinking.
1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. ▪ Know what each symbol represents >, <, and = ▪ Compare two two-digit numbers based on meanings of the tens and ones digits		Students use models that represent two sets of numbers. To compare, students first attend to the number of tens, then, if necessary, to the number of ones. Students may also use pictures, number lines, and spoken or written words to compare two numbers. Comparative language includes but is not limited to more than, less than, greater than, most, greatest, least, same as, equal to and not equal to.

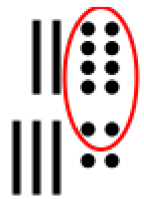
Standard/Learning Objectives		***Explanations and Examples***
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. ▪ Decompose any number within one hundred into ten(s) and one(s) ▪ Choose an appropriate strategy for solving an addition problem within 100 ▪ Relate the chosen strategy (using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction) to a written method (equation) and explain the reasoning used		Students extend their number fact and place value strategies to add within 100. They represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. It is important for students to understand if they are adding a number that has 10s to a number with 10s, they will have more tens than they started with; the same applies to the ones. Also, students should be able to apply their place value skills to decompose numbers. For example, 17 + 12 can be thought of 1 ten and 7 ones plus 1 ten and 2 ones. Numeral cards may help students decompose the numbers into 10s and 1s. Students should be exposed to problems both in and out of context and presented in horizontal and vertical forms. As students are solving problems, it is important that they use language associated with proper place value (see example). They should always explain and justify their mathematical thinking both verbally and in a written format. Estimating the solution prior to finding the answer focuses students on the meaning of the operation and helps them attend to the actual quantities. This standard focuses on developing addition - the intent is not to introduce traditional algorithms or rules. Examples: • 43 + 36 Student counts the 10s (10, 20, 30…70 or 1, 2, 3…7 tens) and then the 1s.  Continued on next page

Number and Operations in Base Ten **1.NBT**
Use place value understanding and properties of operations to add and subtract.

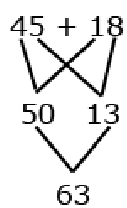
Standard/Learning Objectives	***Explanations and Examples***

- 28
  +34

  Student thinks: 2 tens plus 3 tens is 5 tens or 50. S/he counts the ones and notices there is another 10 plus 2 more. 50 and 10 is 60 plus 2 more or 62.



- 45 + 18
  Student thinks: Four 10s and one 10 are 5 tens or 50. Then 5 and 8 is 5 + 5 + 3 (or 8 + 2 + 3) or 13. 50 and 13 is 6 tens plus 3 more or 63.



- 29
  +14

  Student thinks: "29 is almost 30. I added one to 29 to get to 30. 30 and 14 is 44. Since I added one to 29, I have to subtract one so the answer is 43."

Number and Operations in Base Ten — 1.NBT

Use place value understanding and properties of operations to add and subtract.

Standard/Learning Objectives	***Explanations and Examples***
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. ▪ Explain how to mentally find 10 more or 10 less than a given two-digit number ▪ Apply knowledge of place value to mentally add or subtract 10 to/from a given two digit number	This standard requires students to understand and apply the concept of 10 which leads to future place value concepts. It is critical for students to do this without counting. Prior use of models such as base ten blocks, number lines, and 100s charts helps facilitate this understanding. It also helps students see the pattern involved when adding or subtracting 10. Examples: • 10 more than 43 is 53 because 53 is one more 10 than 43 • 10 less than 43 is 33 because 33 is one 10 less than 43 Students may use interactive versions of models (base ten blocks,100s charts, number lines, etc) to develop prior understanding.
1.NBT.6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. ▪ Subtract multiples of 10 in the range of 10-90 from multiples of 10 in the range of 10-90 (positive or zero difference) ▪ Choose appropriate strategy (concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction) for solving subtraction problems with multiples of 10	This standard is foundational for future work in subtraction with more complex numbers. Students should have multiple experiences representing numbers that are multiples of 10 (e.g. 90) with models or drawings. Then they subtract multiples of 10 (e.g. 20) using these representations or strategies based on place value. These opportunities develop fluency of addition and subtraction facts and reinforce counting up and back by 10s. Examples: • 70 - 30: Seven 10s take away three 10s is four 10s • 80 - 50: 80, 70 (one 10), 60 (two 10s), 50 (three 10s), 40 (four 10s), 30 (five 10s) • 60 - 40: I know that 4 + 2 is 6 so four 10s + two 10s is six 10s so 60 - 40 is 20 Students may use interactive versions of models (base ten blocks,100s charts, number lines, etc.) to demonstrate and justify their thinking.

Measurement and Data — 1.MD

Measure lengths indirectly and by iterating length units.

Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, order, length, height, more, less, longer than, shorter than, first, second, third, gap, overlap, about, a little less than, a little more than**

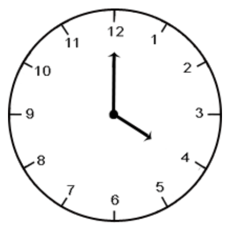
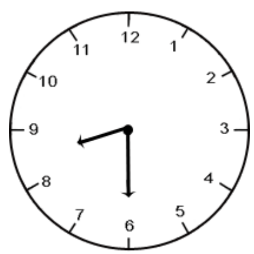
Standards	***Explanations and Examples***
1.MD.1. Order three objects from a variety of cultural contexts, including those of Montana American Indians, by length; compare the lengths of two objects indirectly by using a third object. ▪ Identify the measurement known as the length of an object ▪ Directly compare the length of three objects ▪ Order three objects by length ▪ Compare the lengths of two objects indirectly by using a third object (e.g., if the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C)	In order for students to be able to compare objects, students need to understand that length is measured from one end point to another end point. They determine which of two objects is longer, by physically aligning the objects. Typical language of length includes taller, shorter, longer, and higher. When students use bigger or smaller as a comparison, they should explain what they mean by the word. Some objects may have more than one measurement of length, so students identify the length they are measuring. Both the length and the width of an object are measurements of length. Examples for ordering: • Order three students by their height • Order pencils, crayons, and/or markers by length • Build three towers (with cubes) and order them from shortest to tallest • Three students each draw one line, then order the lines from longest to shortest Example for comparing indirectly: • Two students each make a dough "snake." Given a tower of cubes, each student compares his/her snake to the tower. Then students make statements such as, "My snake is longer than the cube tower and your snake is shorter than the cube tower. So, my snake is longer than your snake." Students may use interactive whiteboard or document camera to demonstrate and justify comparisons.

Measurement and Data Measure lengths indirectly and by iterating length units. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.		**1.MD**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, order, length, height, more, less, longer than, shorter than, first, second, third, gap, overlap, about, a little less than, a little more than**		
Standards		***Explanations and Examples***
1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* ▪ Know to use the same size non-standard objects as repeating units ▪ Know that length can be measured with various units ▪ Compare a smaller unit of measurement to a larger object ▪ Determine the length of a measured object to be the number of smaller iterated or repeated objects that equal its length		Students use their counting skills while measuring with non-standard units. While this standard limits measurement to whole numbers of length, in a natural environment, not all objects will measure to an exact whole unit. When students determine that the length of a pencil is six to seven paperclips long, they can state that it is about six paperclips long. Example: • Ask students to use multiple units of the same object to measure the length of a pencil. (How many paper clips will it take to measure how long the pencil is?)  Students may use the document camera or interactive whiteboard to demonstrate their counting and measuring skills.

Measurement and Data 1.MD

Tell and write time.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **time, hour, half-hour, about, o’clock, past, “six”-thirty**

Standard/Learning Objectives	***Explanations and Examples***
1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks. ▪ Recognize that analog and digital clocks are objects that measure time ▪ Know hour hand and minute hand and distinguish between the two ▪ Determine where the minute hand must be when the time is to the hour (o’clock) ▪ Determine where the minute hand must be when the time is to the half-hour (thirty) ▪ Tell and write the time to the hour and half-hour correctly using analog and digital clocks	Ideas to support telling time: • within a day, the hour hand goes around a clock twice (the hand moves only in one direction) • when the hour hand points exactly to a number, the time is exactly on the hour • time on the hour is written in the same manner as it appears on a digital clock • the hour hand moves as time passes, so when it is half way between two numbers it is at the half hour • there are 60 minutes in one hour; so halfway between an hour, 30 minutes have passed • half hour is written with “30” after the colon “It is 4 o’clock”  “It is halfway between 8 o’clock and 9 o’clock. It is 8:30.”  The idea of 30 being “halfway” is difficult for students to grasp. Students can write the numbers from 0 - 60 counting by tens on a sentence strip. Fold the paper in half and determine that halfway between 0 and 60 is 30. A number line on an interactive whiteboard may also be used to demonstrate this.

Measurement and Data Represent and interpret data.		1.MD
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **data, more, most, less, least, same, different, category, question, collect**		
Standard/Learning Objectives		***Explanations and Examples***
1.MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. ▪ Recognize different methods to represent data ▪ Organize data with up to three categories ▪ Represent data with up to three categories		Students create object graphs and tally charts using data relevant to their lives (e.g., favorite ice cream, eye color, pets, etc.). Graphs may be constructed by groups of students as well as by individual students. Counting objects should be reinforced when collecting, representing, and interpreting data. Students describe the object graphs and tally charts they create. They should also ask and answer questions based on these charts or graphs that reinforce other mathematics concepts such as sorting and comparing. The data chosen or questions asked give students opportunities to reinforce their understanding of place value, identifying ten more and ten less, relating counting to addition and subtraction and using comparative language and symbols. Students may use an interactive whiteboard to place objects onto a graph. This gives them the opportunity to communicate and justify their thinking.

Geometry **1.G** Reason with shapes and their attributes. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.	
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **shape, closed, side, attribute, two-dimensional, rectangle, square, trapezoid, triangle, half-circle, and quarter-circle**, **three-dimensional, cube, cone, prism, cylinder, equal shares, halves, fourths, quarters, half of, fourth of, quarter of**	
Standards	***Explanations and Examples***
1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. ▪ Identify defining and non-defining attributes of shapes ▪ Draw shapes to show defining attributes	Attributes refer to any characteristic of a shape. Students use attribute language to describe a given two-dimensional shape: number of sides, number of vertices/points, straight sides, closed. A child might describe a triangle as "right side up" or "red." These attributes are not defining because they are not relevant to whether a shape is a triangle or not. Students should articulate ideas such as, "A triangle is a triangle because it has three straight sides and is closed." It is important that students are exposed to both regular and irregular shapes so that they can communicate defining attributes. Students should use attribute language to describe why these shapes are not triangles.  Students should also use appropriate language to describe a given three-dimensional shape: number of faces, number of vertices/points, number of edges. Example: A cylinder may be described as a solid that has two circular faces connected by a curved surface (which is not considered a face). Students may say, "It looks like a can." Students should compare and contrast two-and three-dimensional figures using defining attributes. Examples: • List two things that are the same and two things that are different between a triangle and a cube. • Given a circle and a sphere, students identify the sphere as being three-dimensional but both are round. • Given a trapezoid, find another two-dimensional shape that has two things that are the same. Students may use interactive whiteboards or computer environments to move shapes into different orientations and to enlarge or decrease the size of a shape still keeping the same shape. They can also move a point/vertex of a triangle and identify that the new shape is still a triangle. When they move one point/vertex of a rectangle they should recognize that the resulting shape is no longer a rectangle.

Geometry 1.G

Reason with shapes and their attributes.

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **shape, closed, side, attribute, two-dimensional, rectangle, square, trapezoid, triangle, half-circle, and quarter-circle**, **three-dimensional, cube, cone, prism, cylinder, equal shares, halves, fourths, quarters, half of, fourth of, quarter of**

Standards

1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")

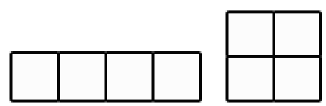
- Know that shapes can be decomposed to create composite shapes
- Describe properties of original, decomposed and composite shapes
- Determine how the original and created composite shapes are alike and different
- Create composite shapes

Explanations and Examples

The ability to describe, use and visualize the effect of composing and decomposing shapes is an important mathematical skill. It is not only relevant to geometry, but is related to children's ability to compose and decompose numbers. Students may use pattern blocks, plastic shapes, tangrams, or computer environments to make new shapes. The teacher can provide students with cutouts of shapes and ask them to combine them to make a particular shape.

Example:

- What shapes can be made from four squares?



Students can make three-dimensional shapes with clay or dough, slice into two pieces (not necessarily congruent) and describe the two resulting shapes. For example, slicing a cylinder will result in two smaller cylinders.

Geometry 1.G

Reason with shapes and their attributes.

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **shape, closed, side, attribute, two-dimensional, rectangle, square, trapezoid, triangle, half-circle, and quarter-circle**, **three-dimensional, cube, cone, prism, cylinder, equal shares, halves, fourths, quarters, half of, fourth of, quarter of**

Standards

1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

- Partition circles and squares into two and four equal shares
- Identify when shares are equal
- Describe equal shares using vocabulary: *halves, fourths and quarters, half of, fourth of, and quarter of*
- Analyze that dividing a circle or rectangle into more equal pieces creates smaller shares

Explanations and Examples

Students need experiences with different sized circles and rectangles to recognize that when they cut something into two equal pieces, each piece will equal one half of its original whole. Children should recognize that halves of two different wholes are not necessarily the same size. Also they should reason that decomposing equal shares into more equal shares results in smaller equal shares.

Examples:

- Student partitions a rectangular candy bar to share equally with one friend and thinks “I cut the rectangle into two equal parts. When I put the two parts back together, they equal the whole candy bar. One half of the candy bar is smaller than the whole candy bar.”

- Student partitions an identical rectangular candy bar to share equally with 3 friends and thinks “I cut the rectangle into four equal parts. Each piece is one fourth of or one quarter of the whole candy bar. When I put the four parts back together, they equal the whole candy bar. I can compare the pieces (one half and one fourth) by placing them side-by-side. One fourth of the candy bar is smaller than one half of the candy bar.

- Students partition a pizza to share equally with three friends. They recognize that they now have four equal pieces and each will receive a fourth or quarter of the whole pizza.

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another.Example: 3/4 and – 3/4 are additive inverses of one another because3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product 3 × 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 × ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? × 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. **Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a – c = b – c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $–a < –b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

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