Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Kindergarten

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

a. Understand that mathematics is relevant when studied in a cultural context.
b. Explain the meaning of a problem and restate it in their words.
c. Analyze given information to develop possible strategies for solving the problem.
d. Identify and execute appropriate strategies to solve the problem.
e. Evaluate progress toward the solution and make revisions if necessary.
f. Check their answers using a different method, and continually ask "Does this make sense?"

2. Reason abstractly and quantitatively.

Mathematically proficient students:

a. Make sense of quantities and their relationships in problem situations.
b. Use varied representations and approaches when solving problems.
c. Know and flexibly use different properties of operations and objects.
d. Change perspectives, generate alternatives and consider different options.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

a. Understand and use prior learning in constructing arguments.
b. Habitually ask "why" and seek an answer to that question.
c. Question and problem-pose.
d. Develop questioning strategies to generate information.
e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.
f. Justify their conclusions, communicate them to others, and respond to the arguments of others.
g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.

Mathematically proficient students:

a. **Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians.**

b. **Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.**

c. **Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.**

d. **Analyze mathematical relationships to draw conclusions.**

5. Use appropriate tools strategically.

Mathematically proficient students:

a. **Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)**

b. **Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.**

6. Attend to precision.

Mathematically proficient students:

a. **Communicate their understanding of mathematics to others.**

b. **Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.**

c. **Specify units of measure and use label parts of graphs and charts**

d. **Strive for accuracy.**

7. Look for and make use of structure.

Mathematically proficient students:

a. **Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.**

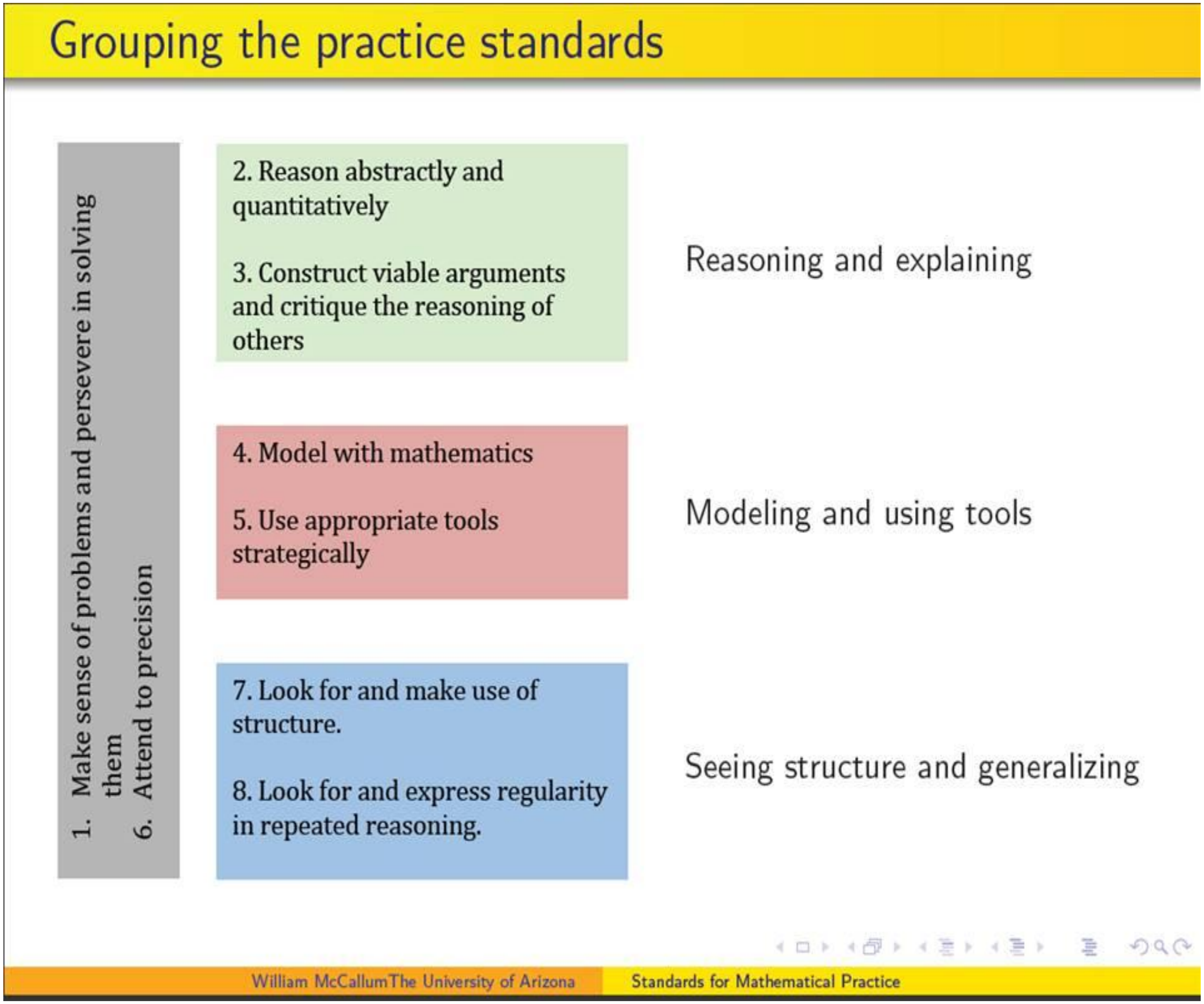
b. **Apply and discuss properties.**

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

a. **Look for mathematically sound shortcuts.**

b. **Use repeated applications to generalize properties.**


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallum The University of Arizona
Standards for Mathematical Practice

Standards for Mathematical Practice: Kindergarten Explanations and Examples

Standards	*Explanations and Examples*
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
K.MP.1. Make sense of problems and persevere in solving them.	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
K.MP.2. Reason abstractly and quantitatively.	Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
K.MP.3. Construct viable arguments and critique the reasoning of others.	Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
K.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
K.MP.5. Use appropriate tools strategically.	Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.
K.MP.6. Attend to precision.	As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.
K.MP.7. Look for and make use of structure.	Younger students begin to discern a pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that 3 + 2 = 5 and 2 + 3 = 5.
K.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). In addition, students continually check their work by asking themselves, "Does this make sense?"

KINDERGARTEN

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade K Overview

Counting and Cardinality
- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten
- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data
- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry
- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Counting and Cardinality **K.CC** Know number names and the count sequence.		
Standard / Learning Objective		***Explanations and Examples***
K.CC.1. Count to 100 by ones and by tens.		The emphasis of this standard is on the counting sequence. When counting by ones, students need to understand that the next number in the sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). Instruction on the counting sequence should be scaffolded (e.g., 1-10, then 1-20, etc.). Counting should be reinforced throughout the day, not in isolation. Examples: • Count the number of chairs of the students who are absent. • Count the number of stairs, shoes, etc. • Counting groups of ten such as "fingers in the classroom" (ten fingers per student). When counting orally, students should recognize the patterns that exist from 1 to 100. They should also recognize the patterns that exist when counting by 10s.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).		The emphasis of this standard is on the counting sequence to 100. Students should be able to count forward from any number, 1-99.
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).		Students should be given multiple opportunities to count objects and recognize that a number represents a specific quantity. Once this is established, students begin to read and write numerals (numerals are the symbols for the quantities). The emphasis should first be on quantity and then connecting quantities to the written symbols. • A sample unit sequence might include: 1. Counting up to 20 objects in many settings and situations over several weeks. 2. Beginning to recognize, identify, and read the written numerals, and match the numerals to given sets of objects. 3. Writing the numerals to represent counted objects. Since the teen numbers are not written as they are said, teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represents each teen number. For example, when focusing on the number "14," students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten and four extra ones. Students should connect the representation to the symbol "14."

Counting and Cardinality Count to tell the number of objects.		**K.CC**
Standard / Learning Objective		***Explanations and Examples***
K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians.. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.		This standard focuses on one-to-one correspondence and how cardinality connects with quantity. • For example, when counting three bears, the student should use the counting sequence, "1-2-3," to count the bears and recognize that "three" represents the group of bears, not just the third bear. A student may use an interactive whiteboard to count objects, cluster the objects, and state, "This is three". In order to understand that each successive number name refers to a quantity that is one larger, students should have experience counting objects, placing one more object in the group at a time. • For example, using cubes, the student should count the existing group, and then place another cube in the set. Some students may need to re-count from one, but the goal is that they would count on from the existing number of cubes. S/he should continue placing one more cube at a time and identify the total number in order to see that the counting sequence results in a quantity that is one larger each time one more cube is placed in the group. • A student may use a clicker (electronic response system) to communicate his/her count to the teacher.
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.		Students should develop counting strategies to help them organize the counting process to avoid re-counting or skipping objects. Examples: • If items are placed in a circle, the student may mark or identify the starting object. • If items are in a scattered configuration, the student may move the objects into an organized pattern. • Some students may choose to use grouping strategies such as placing objects in twos, fives, or tens (note: this is not a kindergarten expectation). • Counting up to 20 objects should be reinforced when collecting data to create charts and graphs. A student may use a clicker (electronic response system) to communicate his/her count to the teacher.

Counting and Cardinality Compare numbers.		**K.CC**
Standard / Learning Objective		***Explanations and Examples***
K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects)		Students should develop a strong sense of the relationship between quantities and numerals before they begin comparing numbers. Other strategies: • Matching: Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects. • Counting: Students count the objects in each set, and then identify which set has more, less, or an equal number of objects. • Observation: Students may use observation to compare two quantities (e.g., by looking at two sets of objects, they may be able to tell which set has more or less without counting). • Observations in comparing two quantities can be accomplished through daily routines of collecting and organizing data in displays. Students create object graphs and pictographs using data relevant to their lives (e.g., favorite ice cream, eye color, pets, etc.). Graphs may be constructed by groups of students as well as by individual students. • Benchmark Numbers: This would be the appropriate time to introduce the use of 0, 5 and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers. ○ Students state whether the number of objects in a set is more, less, or equal to a set that has 0, 5, or 10 objects.
K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.		Given two numerals, students should determine which is greater or less than the other.

Operations and Algebraic Thinking Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although the standards state, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required", please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7).		**K.OA**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **join, add, separate, subtract, and, same amount as, equal, less, more, total**		
Standard / Learning Objective		***Explanations and Examples***
K.0A.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problems. This applies wherever drawings are mentioned in the Standards.) ▪ Know adding is putting together parts to make the whole ▪ Know subtracting is taking apart or taking away from the whole to find the other part ▪ Know the symbols and the words for adding and subtracting ▪ Analyze an addition or subtraction problem to determine whether to put together or take apart ▪ Model an addition/subtraction problem given a real-life story		Using addition and subtraction in a word problem context allows students to develop their understanding of what it means to add and subtract. Students should use objects, fingers, mental images, drawing, sounds, acting out situations and verbal explanations in order to develop the concepts of addition and subtraction. Then, they should be introduced to writing expressions and equations using appropriate terminology and symbols which include "+," "–," and "=". • Addition terminology: add, join, put together, plus, combine, total • Subtraction terminology: minus, take away, separate, difference, compare Students may use document cameras or interactive whiteboards to represent the concept of addition or subtraction. This gives them the opportunity to communicate their thinking.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although the standards state, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required", please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7).

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Standard / Learning Objective		***Explanations and Examples***

K.0A.2. Solve addition and subtraction word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
▪ Add and subtract within 10 (maximum sum and minuend is 10)
▪ Use objects/drawings to represent an addition and subtraction word problem
		Using a word problem context allows students to develop their understanding about what it means to add and subtract. Addition is putting together and adding to. Subtraction is taking apart and taking from. Kindergarteners develop the concept of addition/subtraction by modeling the actions in word problem using objects, fingers, mental images, drawings, sounds, acting out situations, and/or verbal explanations. Students may use different representations based on their experiences, preferences, etc. They may connect their conceptual representations of the situation using symbols, expressions, and/or equations. Students should experience the following addition and subtraction problem types (see Table 1).
		• Add To word problems, such as, "Mia had 3 apples. Her friend gave her 2 more. How many does she have now?"
		  ○ A student's "think aloud" of this problem might be, "I know that Mia has some apples and she's getting some more. So she's going to end up with more apples than she started with."
		• Take From problems such as:
		  ○ José had 8 markers and he gave 2 away. How many does he have now? When modeled, a student would begin with 8 objects and remove two to get the result.
		• Put Together/Take Apart problems with Total Unknown gives students opportunities to work with addition in another context such as:
		  ○ There are 2 red apples on the counter and 3 green apples on the counter. How many apples are on the counter?
		• Solving Put Together/Take Apart problems with Both Addends Unknown provides students with experiences with finding all the decompositions of a number and investigating the patterns involved.
		  ○ There are 10 apples on the counter. Some are red and some are green. How many apples could be green? How many apples could be red?
		
		Students may use a document camera or interactive whiteboard to demonstrate addition or subtraction strategies. This gives them the opportunity to communicate and justify their thinking.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although the standards state, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required", please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7).

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Standard / Learning Objective

K.0A.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

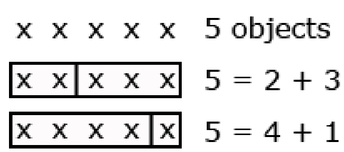
- Solve addition number sentences within 10
- Decompose numbers less than or equal to 10 into pairs in more than one way

Explanations and Examples

This standard focuses on number pairs which add to a specified total, 1-10. These number pairs may be examined either in or out of context.

Students may use objects such as cubes, two-color counters, square tiles, etc. to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, etc.

Students may also use drawings to show different number pairs for a given number. For example, students may draw 5 objects, showing how to decompose in several ways.



Sample unit sequence:

- A contextual problem (word problem) is presented to the students such as, "Mia goes to Nan's house. Nan tells her she may have 5 pieces of fruit to take home. There are lots of apples and bananas. How many of each can she take?"
- Students find related number pairs using objects (such as cubes or two-color counters), drawings, and/or equations. Students may use different representations based on their experiences, preferences, etc.
- Students may write equations that equal 5 such as:
  - 5=4+1
  - 3+2=5
  - 2+3=4+1

This is a good opportunity for students to systematically list all the possible number pairs for a given number. For example, all the number pairs for 5 could be listed as 0+5, 1+4, 2+3, 3+2, 4+1, and 5+0. Students should describe the pattern that they see in the addends, e.g., each number is one less or one than the previous addend.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although the standards state, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required", please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **join, add, separate, subtract, and, same amount as, equal, less, more, total**

Standard / Learning Objective

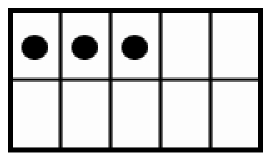
K.0A.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

- Know that two numbers can be added together to make ten
- Using materials or representations, find the number that makes 10 when added to the given number for any number from 1 to 9, and record the answer using materials representations, or equations

Explanations and Examples

The number pairs that total ten are foundational for students' ability to work fluently within base-ten numbers and operations. Different models, such as ten-frames, cubes, two-color counters, etc., assist students in visualizing these number pairs for ten.

Example 1:
Students place three objects on a ten frame and then determine how many more are needed to "make a ten."
Students may use electronic versions of ten frames to develop this skill.



Example 2:
The student snaps ten cubes together to make a "train."

- Student breaks the "train" into two parts. S/he counts how many are in each part and record the associated equation (10 = ___ + ___).
- Student breaks the "train into two parts. S/he counts how many are in one part and determines how many are in the other part without directly counting that part. Then s/he records the associated equation (if the counted part has 4 cubes, the equation would be 10 = 4 + ___).
- Student covers up part of the train, without counting the covered part. S/he counts the cubes that are showing and determines how many are covered up. Then s/he records the associated equation (if the counted part has 7 cubes, the equation would be 10 = 7 + ___).

Example 3:
The student tosses ten two-color counters on the table and records how many of each color are facing up.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

For numbers 0 – 10, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although the standards state, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required", please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7).

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **join, add, separate, subtract, and, same amount as, equal, less, more, total**

Standard / Learning Objective		***Explanations and Examples***
K.0A.5. Fluently add and subtract within 5.		This standard focuses on students being able to add and subtract numbers within 5. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Strategies students may use to attain fluency include: • Counting on (e.g., for 3+2, students will state, "3," and then count on two more, "4, 5," and state the solution is "5") • Counting back (e.g., for 4-3, students will state, "4," and then count back three, "3, 2, 1" and state the solution is "1") • Counting up to subtract (e.g., for 5-3, students will say, "3," and then count up until they get to 5, keeping track of how many they counted up, stating that the solution is "2") • Using doubles (e.g., for 2+3, students may say, "I know that 2+2 is 4, and 1 more is 5") • Using commutative property (e.g., students may say, "I know that 2+1=3, so 1+2=3") • Using fact families (e.g., students may say, "I know that 2+3=5, so 5-3=2") Students may use electronic versions of five frames to develop fluency of these facts.

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

Rather than unitizing a ten (recognizing that a set of 10 objects is a unit called a "ten"), which is a standard for First Grade (1.NBT.1a), kindergarteners keep each count as a single unit as they explore a set of 10 objects and leftovers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: number words (**one, two… thirteen, fourteen, … nineteen**), **leftovers**

Standard / Learning Objective		***Explanations and Examples***
K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. ▪ Know that the numbers 11-19 represent a quantity ▪ Represent compositions or decompositions of 11-19 by a drawing or equation		Special attention needs to be paid to this set of numbers as they do not follow a consistent pattern in the verbal counting sequence. • Eleven and twelve are special number words. • "Teen" means one "ten" plus ones. • The verbal counting sequence for teen numbers is backwards – we say the ones digit before the tens digit. For example "27" reads tens to ones (twenty-seven), but 17 reads ones to tens (seven-teen). • In order for students to interpret the meaning of written teen numbers, they should read the number as well as describe the quantity. For example, for 15, the students should read "fifteen" and state that it is one group of ten *and* five ones and record that 15 = 10 + 5. Teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represent each teen number. For example, when focusing on the number "14," students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten ones and four additional ones. Students should connect the representation to the symbol "14." Students should recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated.

Measurement and Data Describe and compare measurable attributes.		**K.MD**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **length, weight, heavy, long, more of, less of, longer, taller, shorter.**		
Standard / Learning Objective		***Explanations and Examples***
K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. ▪ Know that objects have measurable attributes and know what they are called, such as length and weight ▪ Describe an object using multiple attributes such as: width, height, length, weight, etc.		In order to describe attributes such as length and weight, students must have many opportunities to informally explore these attributes. • Students should compare objects verbally and then focus on specific attributes when making verbal comparisons for K.MD.2. They may identify measurable attributes such as length, width, height, and weight. For example, when describing a soda can, a student may talk about how tall, how wide, how heavy, or how much liquid can fit inside. These are all measurable attributes. Non-measurable attributes include: words on the object, colors, pictures, etc. An interactive whiteboard or document camera may be used to model objects with measurable attributes.
K.MD.2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* ▪ *Know the meaning of a variety of attributes* ▪ *Know that two objects can be compared using a particular attribute*		When making direct comparisons for length, students must attend to the “starting point” of each object. For example, the ends need to be lined up at the same point, or students need to compensate when the starting points are not lined up (conservation of length includes understanding that if an object is moved, its length does not change; an important concept when comparing the lengths of two objects). Language plays an important role in this standard as students describe the similarities and differences of measurable attributes of objects (e.g., shorter than, taller than, lighter than, the same as, etc.). An interactive whiteboard or document camera may be used to compare objects with measurable attributes.

Measurement and Data Classify objects and count the number of objects in each category.		K.MD
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: color words **(e.g., blue, green, red,** etc.**),** descriptive words (e.g., **small, big, rough, smooth, bumpy, round, flat,** etc.), **more, less, same amount**		
Standard / Learning Objective		***Explanations and Examples***
K.MD.3. Classify objects from a variety of cultural contexts, including those of Montana American Indians, into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10). ▪ Know what classify and sort mean ▪ Understand one to one correspondence with tem or less objects		Possible objects to sort include buttons, shells, shapes, beans, etc. After sorting and counting, it is important for students to: • explain how they sorted the objects; • label each set with a category; • answer a variety of counting questions that ask, "How many …"; and • compare sorted groups using words such as, "most", "least", "alike" and "different".

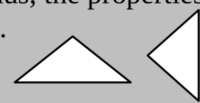
Geometry **K.G**

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

This entire cluster asks students to understand that certain attributes define what a shape is called (number of sides, number of angles, etc.) and other attributes do not (color, size, orientation). Using geometric attributes, the student identifies and describes squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Throughout the year, Kindergarten students move from informal language to describe what shapes look like (e.g., "That looks like an ice cream cone!") to more formal mathematical language (e.g., "That is a triangle. All of its sides are the same length").

In Kindergarten, students need ample experiences exploring various forms of the shapes (e.g., *size*: big and small; *types*: triangles, equilateral, isosceles, scalene; *orientation*: rotated slightly to the left, 'upside down') using geometric vocabulary to describe the different shapes.

Students in Kindergarten typically recognize figures by appearance alone, often by comparing them to a known example of a shape, such as the triangle on the left (see below). For example, students in Kindergarten typically recognize that the figure on the left as a triangle, but claim that the figure on the right is not a triangle, since it does not have a flat bottom. Thus, the properties of a figure are not recognized or known. Students make decisions on identifying and describing shapes based on perception, not reasoning.



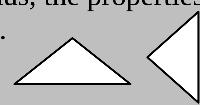
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, spheres, flat, solid, side, corner, angle, edge, face,** positional vocabulary (e.g., **above, below, beside, in front of, behind, next to, same, different,** etc.).

Standard / Learning Objective		***Explanations and Examples***
K.G.1. Describe objects, including those of Montana American Indians, in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. ▪ Construct viable arguments and critique the reasoning of others		Examples of environments in which students would be encouraged to identify shapes would include nature, buildings, and the classroom using positional words in their descriptions. Teachers should work with children and pose four mathematical questions: Which way? How far? Where? And what objects? To answer these questions, children develop a variety of important skills contributing to their spatial thinking. Examples: • Teacher holds up an object such as an ice cream cone, a number cube, ball, etc. and asks students to identify the shape. Teacher holds up a can of soup and asks," What shape is this can?" Students respond "cylinder!" • Teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions. Where is the water bottle? (water bottle is placed behind a book) Students say "The water bottle is behind the book." Students should have multiple opportunities to identify shapes; these may be displayed as photographs, or pictures using the document camera or interactive whiteboard.

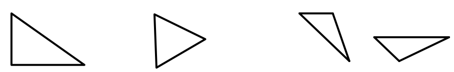
Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

This entire cluster asks students to understand that certain attributes define what a shape is called (number of sides, number of angles, etc.) and other attributes do not (color, size, orientation). Using geometric attributes, the student identifies and describes squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Throughout the year, Kindergarten students move from informal language to describe what shapes look like (e.g., “That looks like an ice cream cone!”) to more formal mathematical language (e.g., “That is a triangle. All of its sides are the same length”).
In Kindergarten, students need ample experiences exploring various forms of the shapes (e.g., *size*: big and small; *types*: triangles, equilateral, isosceles, scalene; *orientation*: rotated slightly to the left, ‘upside down’) using geometric vocabulary to describe the different shapes.
Students in Kindergarten typically recognize figures by appearance alone, often by comparing them to a known example of a shape, such as the triangle on the left (see below). For example, students in Kindergarten typically recognize that the figure on the left as a triangle, but claim that the figure on the right is not a triangle, since it does not have a flat bottom. Thus, the properties of a figure are not recognized or known. Students make decisions on identifying and describing shapes based on perception, not reasoning.



Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, spheres, flat, solid, side, corner, angle, edge, face,** positional vocabulary (e.g., **above, below, beside, in front of, behind, next to, same, different,** etc.).

Standard / Learning Objective		***Explanations and Examples***
K.G.2. Correctly name shapes regardless of their orientations or overall size. ▪ Recognize that size does not affect the name of the shape ▪ Recognize that orientation does not affect the name of the shape		Students should be exposed to many types of triangles in many different orientations in order to eliminate the misconception that a triangle is always right-side-up and equilateral.  Students should also be exposed to many shapes in many different sizes. Examples: • Teacher makes pairs of paper shapes that are different sizes. Each student is given one shape and the objective is to find the partner who has the same shape. • Teacher brings in a variety of spheres (tennis ball, basketball, globe, ping pong ball, etc) to demonstrate that size doesn’t change the name of a shape.
K.G.3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). ▪ Define the difference between tow and three-dimensional shapes		Student should be able to differentiate between two dimensional and three dimensional shapes. • Student names a picture of a shape as two dimensional because it is flat and can be measured in only two ways (length and width). • Student names an object as three dimensional because it is not flat (it is a solid object/shape) and can be measured in three different ways (length, width, height/depth).

Geometry Analyze, compare, create, and compose shapes.		**K.G**
Standard / Learning Objective		***Explanations and Examples***
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). ▪ Identify attributes of shapes ▪ Describe attributes of a variety of two-and three-dimensional shapes		Students analyze and compare two- and three-dimensional shapes by observations. Their visual thinking enables them to determine if things are alike or different based on the appearance of the shape. Students sort objects based on appearance. Even in early explorations of geometric properties, they are introduced to how categories of shapes are subsumed within other categories. For instance, they will recognize that a square is a special type of rectangle. Students should be exposed to triangles, rectangles, and hexagons whose sides are not all congruent. They first begin to describe these shapes using everyday language and then refine their vocabulary to include sides and vertices/corners. Opportunities to work with pictorial representations, concrete objects, as well as technology helps student develop their understanding and descriptive vocabulary for both two- and three- dimensional shapes.
K.G.5. Model shapes in the world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes. ▪ Identify basic shapes ▪ Recognize and identify basic shapes in the real world		Because two-dimensional shapes are flat and three-dimensional shapes are solid, students should draw two-dimensional shapes and build three-dimensional shapes. Shapes may be built using materials such as clay, toothpicks, marshmallows, gumdrops, straws, etc.
K.G.6. Compose simple shapes to form larger shapes. For example*, "Can you join these two triangles with full sides touching to make a rectangle?”*		Students use pattern blocks, tiles, or paper shapes and technology to make new two- and three-dimensional shapes. Their investigations allow them to determine what kinds of shapes they can join to create new shapes. They answer questions such as “What shapes can you use to make a square, rectangle, circle, triangle? …etc.” Students may use a document camera to display shapes they have composed from other shapes. They may also use an interactive whiteboard to copy shapes and compose new shapes. They should describe and name the new shape.

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product 3 × 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 × ? = 18, and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? × 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. **Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a – c = b – c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $–a < –b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

LEARNING PROGRESSIONS BY DOMAIN

<table>
<tr><th colspan="10">Mathematics Learning Progressions by Domain</th></tr>
<tr><td>K</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>HS</td></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">Ratios and Proportional Relationship</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td colspan="2">Functions</td></tr>
<tr><td colspan="10">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="4">Statistics and Probability</td></tr>
</table>