Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Algebra II

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**

f. Justify their conclusions, communicate them to others, and respond to the arguments of others.
g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.
Mathematically proficient students:
a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians.
b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
d. Analyze mathematical relationships to draw conclusions.

5. Use appropriate tools strategically.
Mathematically proficient students:
a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)
b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.

6. Attend to precision.
Mathematically proficient students:
a. Communicate their understanding of mathematics to others.
b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
c. Specify units of measure and use label parts of graphs and charts
d. Strive for accuracy.

7. Look for and make use of structure.
Mathematically proficient students:
a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.
b. Apply and discuss properties.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students:
a. Look for mathematically sound shortcuts.
b. Use repeated applications to generalize properties.

Traditional Pathway: Algebra II

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radial functions are limited to square roots or cube roots of at most quadratic polynomials. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course, organized into four units, are as follows.

Critical Area/Unit 1: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area/Unit 2: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Critical Area/Unit 3: In this unit students synthesize and generate what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as *"the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions"* is the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Critical Area/Unit 4: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data-including sample surveys, experiments, and simulations-and the role that randomness and careful design play in the conclusions that can be drawn.

Algebra II		
Unit Overviews		
Units	Standards Clusters	Mathematical Practices
Unit 1: Polynomial, Rational, and Radical Relationships	~Perform arithmetic operations with complex numbers ~Use complex numbers in polynomial identities and equations ~Interpret the structure of expressions ~Write expressions in equivalent forms to solve problems ~Perform arithmetic operations on polynomials ~Understand the relationship between zeros and factors of polynomials ~Use polynomial identities to solve problems ~Rewrite rational expressions ~Understand solving equations as a process of reasoning and explain the reasoning ~Represent and solve equations and inequalities graphically ~Analyze functions using different representations	***1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.***
Unit 2: Trigonometric Functions	~Extend the domain of trigonometric functions using the unit circle ~Model periodic phenomena with trigonometric function ~Prove and apply trigonometric identities	
Unit 3: Modeling with Functions	~Create equations that describe numbers or relationships ~Interpret functions that arise in applications in terms of a context ~Analyze functions using different representations ~Build a function that models a relationship between two quantities ~Build new functions from existing functions ~Construct and compare linear, quadratic, and exponential models and solve problems	
Unit 4: Inferences and Conclusions from Data	~Summarize, represent, and interpret data on single count or measurement variable ~Understand and evaluate random processes underlying statistical experiments ~Make inferences and justify conclusions from sample surveys, experiments and observational studies ~Use probability to evaluate outcomes of decisions	

Algebra II

Unit 1: Polynomial, Rational, and Radical Relationships

Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
The Complex Number System	**Perform arithmetic operations with complex numbers.**	**N.CN.1 – Know there is a complex number i such that i2 = -1, and every complex number has the form a + bi with a and b real numbers.** • Define i as $\sqrt{-1}$ or i2 = -1 • Define complex numbers • Write complex numbers in the form a + bi with a and b being real numbers	6. Attend to precision. 7. Look for and make use of structure.
The Complex Number System	**Perform arithmetic operations with complex numbers.**	**N.CN.2 – Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.** • Know that the commutative, associative, and distributive properties extend to the set of complex numbers over the operations of addition and multiplication • Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers	2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use
The Complex Number System	**Use complex numbers in polynomial identities and equations.** *Limit to polynomial and rational expressions.*	**N.CN.7 – Solve quadratic equations with real coefficients that have complex solutions.** • Solve quadratic equations with real coefficients that have complex solutions	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically. 7. Look for and make use of structure.
The Complex Number System	**Use complex numbers in polynomial identities and equations.** *Limit to polynomial and rational expressions.*	**N.CN.8 – (+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).** • Explain that an identity shows a relationship between two quantities, or expressions, that is true for all values of the variables, over a specified set • Give examples of polynomial identities • Extend polynomial identities to the complex numbers	2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
The Complex Number System	**Use complex numbers in polynomial identities and equations.** *Limit to polynomial and rational expressions.*	**N.CN.9 – (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.** • State, in written or verbal form, the Fundamental Theorem of Algebra • Verify that the Fundamental Theorem of Algebra is true for second degree quadratic polynomials	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reason of others. 6. Attend to precision. 7. Look for and make use of structure.

Algebra II

Unit 1: Polynomial, Rational, and Radical Relationships

Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Seeing Structure in Expressions	**Interpret the structure of expressions.** *Extend to polynomial and rational expressions.*	**A.SSE.1 – Interpret expressions that represent a quantity in terms of its context.** **a. Interpret parts of an expression, such as terms, factors, and coefficients.** **b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example interpret P(1 + r)n as the product of P and a factor not depending on P.** • Define and recognize parts of an expression, such as terms, factors, and coefficients • Interpret parts of an expression, such as terms, factors, and coefficients in terms of the context • Interpret complicated expressions, in terms of the context, by viewing one or more of their parts as a single entity	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure.
Seeing Structure in Expressions	**Interpret the structure of expressions.** *Extend to polynomial and rational expressions.*	**A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x2)^2$ – $(y2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.** • Identify ways to rewrite expressions, such as difference of squares, factoring out a common monomial, and regrouping • Identify various structures of expressions • Use the structure of an expression to identify ways to rewrite it • Classify expressions by structure and develop strategies to assist in classification	1. Make sense of problems and persevere in solving them. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Seeing Structure in Expressions	**Write expressions in equivalent forms to solve problems.** *Consider extending A.SSE.4 to infinite geometric series in curricular implementations of this course description.*	**A.SSE.4 – Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.** • Find the first term in a geometric sequence given at least two other terms • Define a geometric series as a series with a constant ratio between successive terms • Use the formula S + a (1-rn)/(1-r) to solve problems • Derive a formula [i.e., equivalent to the formula S + a (1-rn)/(1–r)] for the sum of a finite geometric series (when the common ratio is not 1)	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 6. Attend to precision.
Arithmetic with Polynomials and Rational Expressions	**Perform arithmetic operations on polynomials.** *Extend beyond the quadratic polynomials found in Algebra I.*	**A.APR.1 – Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.** • Define closure • Identify that the sum, difference, or product of two polynomials will always be a polynomial, which means that polynomials are closed under the operations of addition, subtraction, and multiplication • Apply arithmetic operations of addition, subtraction, and multiplication to polynomials	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra II			
Unit 1: Polynomial, Rational, and Radical Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Arithmetic with Polynomials and Rational Expressions	**Understand the relationship between polynomials.**	**A.APR.2 – Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder of division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).** • Define the remainder theorem for polynomial division and divide polynomials • Given a polynomial p(x) and a number a, divide p(x) by (x-a) to find p(a), then apply the remainder theorem and conclude that p(x) is divisible by x - a, if and only if p(a) = 0	3. Construct viable arguments and critique the reasoning of others. 8. Look for and express regularity in repeated reasoning.
Arithmetic with Polynomials and Rational Expressions	**Understand the relationship between polynomials.**	**A.APR.3 – Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.** • Factor polynomials using any available method • Create a sign chart for a polynomial f(x) using the polynomial's x-intercepts and testing the domain intervals for which f(x) greater than and less than zero • Use the x-intercepts of a polynomial function and the sign chart to construct a rough graph of the function	5. Use appropriate tools strategically. 7. Look for and make use of structure.
Arithmetic with Polynomials and Rational Expressions	**Use polynomial identities to solve problems.** *This cluster has many possibilities for optional enrichment such as relating the example in A.APR.4 to the solution of the system $u^2+v^2=1$, $v=t(u=1)$, relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1}=(x+y)(x+y)^n$, deriving explicit formulas for the coefficients or proving the binomial theorem by induction.*	**A.APR.4 – Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.** • Explain that an identity shows a relationship between two quantities or expressions, that is true for all values of the variables, over a specified set • Prove polynomial identities • Use polynomial identities to describe numerical relationships	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 7. Look for and make use of structure.
Arithmetic with Polynomials and Rational Expressions	**Use polynomial identities to solve problems.** *This cluster has many possibilities for optional enrichment such as relating the example in A.APR.4 to the solution of the system $u^2+v^2=1$, $v=t(u=1)$, relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1}=(x+y)(x+y)^n$, deriving explicit formulas for the coefficients or proving the binomial theorem by induction.*	**A.APR.5 – (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.** • Define the Binomial Theorem and compute combinations • Apply the Binomial Theorem to expand (x+y)n, when n is a positive integer and x and y are any number, rather than expanding by multiplying • Explain the connection between Pascal's Triangle and the determination of the coefficients in the expansion of (x+y)n, when n is a positive integer and x and y are any number	1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra II			
Unit 1: Polynomial, Rational, and Radical Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Arithmetic with Polynomials and Rational Expressions	**Rewrite rational expressions.** *The limitations on rational functions apply to the rational expressions in A.APR.6. A.APR.7 requires the general division algorithm for polynomials.*	**A.APR.6 – (+) Rewrite simple rational expressions in different forma; writ a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.** • Use inspection to rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less that the degree of b(x) • Use long division to rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x) • Use a computer algebra system to rewrite complicated rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x) and r(x) are polynomials with the degree of r(x) less than the degree of b(x)	5. Use appropriate tools strategically. 7. Look for and make use of structure.
Arithmetic with Polynomials and Rational Expressions	**Rewrite rational expressions.** The limitations on rational functions apply to the rational expressions in A.APR.6. A.APR.7 requires the general division algorithm for polynomials.	**A.APR.7 – (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.** • Add, subtract, multiply, and divide rational expressions • Informally verify that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression	1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Reasoning with Equations and Inequalities	**Understand solving equations as a process of reasoning and explain the reasoning.** *Extend to simple rational and radical equations.*	**A.REI.2 – Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.** • Determine the domain of a rational function • Determine the domain of a radical function • Solve radical equations in one variable • Solve rational equations in one variable • Give examples showing how extraneous solutions may arise when solving rational and radical equations	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically.

Algebra II			
Unit 1: Polynomial, Rational, and Radical Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Reasoning with Equations and Inequalities	**Represent and solve equations and inequalities graphically.** *Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.*	**A.REI.11 – Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.** • Recognize and use function notation to represent linear and exponential equations • Recognize that if (x1, y1) and (x2, y2) share the same location in the coordinate plane that x1 = x2 and y1 = y2 • Recognize that f(x) = g(x) means that there may be particular inputs of f and g for which the outputs of f and g are equal • Recognize and use function notation to represent linear, polynomial, rational, absolute value, exponential, and radical equations • Explain why the x-coordinates of the points where the graph of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x) • Approximate/find the solution(s) using an appropriate method. For example, using technology to graph the functions, make tables of values or find successive approximations	3. Construct variable arguments and critique the reasoning of other. 4. Model with Mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.
Interpreting Functions	**Analyze functions using different representations.** *Relate F.FIF.7c to the relationship between zeros of quadratic functions and their factored forms.*	**F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.** **c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.** • Determine the difference between simple and complicated polynomial functions • Determine the difference between simple and complicated linear, quadratic, square root, cube root, and piecewise-defined functions • Determine the differences between simple and complicated linear and exponential functions and know when the use of technology is appropriate • Compare and contrast absolute value, step- and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of absolute value, step- and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of exponential, logarithmic, and trigonometric functions with linear, quadratic, absolute value, step-and piecewise-defined functions • Analyze the difference between simple and complicated linear, quadratic, square root, cube root, piecewise-defined, exponential, logarithmic, and trigonometric functions, including step and absolute value functions • Select the appropriate type of function, taking into considerations the key features, domain, and range, to model a real-world situation • Relate the relationship between zeros of quadratic functions and their factored forms to the relationship between polynomial functions of degrees greater than two • Graph exponential functions, by hand in simple cases or using technology for more complicated cases, and show intercepts and end behavior • Graph polynomial functions, by hand in simple cases or using technology for more complicated cases, and show/label maxima and minima of the graph, identify zeros when suitable factorizations are available, and show end behavior • Graph exponential, logarithmic, and trigonometric functions, by hand in simple cases or using technology for more complicated cases. For exponential and logarithmic functions, show: intercepts and end behavior, for trigonometric functions, show: period, midline, and amplitude • Graph linear functions by hand in simple cases or using technology for complicated cases and show/label intercepts of the graph	1. Make sense of problems and preserver in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra II			
Unit 2: Trigonometric Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Trigonometric Functions	**Extend the domain of trigonometric functions using the unit circle.**	**F.TF.1 – Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.** • Define a radian measure of an angle as the length of the arc on the unit circle subtended by the angle • Define terminal and initial side of an angle on the unit circle	6. Attend to precision.
Trigonometric Functions	**Extend the domain of trigonometric functions using the unit circle.**	**F.TF.2 – Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.** • Explain the relationship between a counterclockwise radian measure of an angle along the unit circle, terminal coordinate on the unit circle of that angle, and the associated real number • Explain how radian measures of angles of the unit circle in the coordinate plane enable the extension of trigonometric functions to all real numbers	6. Attend to precision. 7. Look for and make use of structure.
Trigonometric Functions	**Model periodic phenomena with trigonometric functions.**	**F.TF.5 – Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g., science, history, culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.** • Define and recognize amplitude, frequency, and midline parameters in a symbolic trigonometric function • Interpret the parameters of a trigonometric function (amplitude, frequency, midline) in the context of real-world situations • Choose trigonometric functions to model periodic phenomena for which amplitude, frequency, and midline are already specified • Explain why real-world or mathematical phenomena exhibit characteristics of periodicity	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure.
Trigonometric Functions	**Prove and apply trigonometric identities.** *An Algebra II course with an additional focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could be limited to acute angles in Algebra II*	**F.TF.8 – Prove the Pythagorean identity sin2(θ) +cos2 (θ) = 1 and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.** • Define trigonometric ratios as related to the unit circle • Prove the Pythagorean identity sin2(θ) + cos 2(θ) = 1 • Use the Pythagorean identify, sin2(θ) + cos2(θ) = 1, to find sin(θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.	3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 7. Look for and make use of structure.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Creating Equations	**Create equations that describe numbers or relationships.** *For A.CED.1, use all available types of functions to create such equations, including root functions, but constrain to simple cases.*	**A.CED.1 – Create equations and inequalities in one variable and use them to solve problems from a variety of contexts(e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.** **• Describe the relationships between the quantities in the problem (for example, how the quantities are changing or growing with respect to each other); express these relationships using mathematical operations to create an appropriate equation or inequality to solve.** • Use all available types of functions to create such equations, including root functions, but constrain to simple cases • Compare and contrast problems that can be solved by different types of equations • Compare and contrast problems that can be solved by different types of equations (linear, exponential) • Solve linear and exponential equations in one variable • Solve inequalities in one variable • Solve all available types of equations and inequalities including root equations and inequalities, in one variable • Create equations and inequalities in one variable and use them to solve problems • Create equations and inequalities in one variable to model real-world situations • Create equations (linear, exponential) and inequalities in one variable and use them to solve problems	1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 8. Look for and express regularity in repeated reasoning.
Creating Equations	**Create equations that describe numbers or relationships.** *While functions used in A.CED.2,3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Note that the example give for A.CED.4 applies to earlier instances of this standard, not to the current course.*	**A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.** • Identify the quantities in a mathematical problem or real-world situation that should be represented by distinct variables and describe what quantities the variable represent • Graph one or more created equation on coordinate axes with appropriate labels and scales • Justify which quantities in a mathematical problem or real-world situation are dependent and independent of one another and which operations represent those relationships • Determine appropriate units for the labels and scale of graph depicting the relationship between equations created in two or more variables • Create at least two equations in two or more variables to represent relationships between quantities	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Creating Equations	**Create equations that describe numbers or relationships.** *While functions used in A.CED.2,3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Note that the example give for A.CED.4 applies to earlier instances of this standard, not to the current course.*	**A.CED.3 – Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.** • Recognize when a modeling context involves constraints • Interpret solutions as viable or nonviable options in a modeling context • Determine when a problem should be represented by equations, inequalities, systems of equations and/or inequalities • Represent constraints by equations or inequalities, and by systems of equations and/or inequalities	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically.
Creating Equations	**Create equations that describe numbers or relationships.** *While functions used in A.CED.2,3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Note that the example give for A.CED.4 applies to earlier instances of this standard, not to the current course.*	**A.CED.4 – Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.** • Define a quantity of interest to mean any numerical or algebraic quantity (e.g., 2(a/b)=d in which 2 is the quantity of interest showing that d must be even; (ϖr2h/3)=Vcone and ϖr2h=Vcylinder showing that Vcylinder=3*Vcone) • Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. (e.g., ϖr2 can be re-written as (ϖr)r which makes the form of this expression resemble Bh. The quantity of interest could also be (a + b)n = an b0 + a(n-1)b1 +...+ a0b n)	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Interpret functions that arise in applications in terms of a context.** *Emphasize the selections of a model function based on behavior of data and context.*	**F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** • Define and recognize key features in tables and graphs of linear and exponential functions; intercepts; intervals where the function is increasing, decreasing, positive, or negative, and end behavior • Define and recognize key features in tables and graphs of linear, exponential, and quadratic functions: intercepts; intervals where the function is increasing, decreasing, positive, or negative, relative maximums, symmetries, end behavior and periodicity • Identify the type of function, given a table or graph • Identify whether a function is linear or exponential, given its table or graph • Interpret key features of graphs and tables of functions in terms of the contextual quantities each function represents • Sketch graphs showing the key features of a function, modeling a relationship between two quantities, given a verbal description of the relationship	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics.
Interpreting Functions	**Interpret functions that arise in applications in terms of a context.** *Emphasize the selections of a model function based on behavior of data and context.*	**F.IF.5 – Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** • Identify and describe the domain of a function, given the graph or a verbal/written description of a function • Identify an appropriate domain based on the unit, quantity, and type of function it describes • Relate the domain of a function to its graph and to the quantitative relationship it describes, where applicable • Explain why a domain is appropriate for a given situation	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.
Interpreting Functions	**Interpret functions that arise in applications in terms of a context.** *Emphasize the selections of a model function based on behavior of data and context.*	**F.IF.6 – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.** • Recognize slope as an average rate of change • Estimate the rate of change from a linear or exponential graph • Interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval • Calculate the average rate of change of a function (presented symbolically or as a table) over a specified interval	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Analyze functions using different representations.** *Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.*	**F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases** **b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.** **e. Graph exponential and log**arithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. • Determine the difference between simple and complicated polynomial functions • Determine the difference between simple and complicated linear, quadratic, square root, cube root, and piecewise-defined functions • Determine the differences between simple and complicated linear and exponential functions and know when the use of technology is appropriate • Compare and contrast absolute value, step-and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of absolute value, step-and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of exponential, logarithmic, and trigonometric functions with linear, quadratic, absolute value, step-and piecewise-defined functions • Analyze the difference between simple and complicated linear, quadratic, square root, cube root, piecewise-defined, exponential, logarithmic, and trigonometric functions, including step and absolute value functions • Select the appropriate type of function, taking into consideration the key features, domain, and range, to model a real-world situation • Relate the relationship between zeros of quadratic functions and their factored forms to the relationship between polynomial functions of degrees greater than two • Graph exponential functions, by hand in simple cases or using technology for more complicated cases, and show intercepts and end behavior • Graph polynomial functions, by hand in simple cases or using technology for more complicated cases, and show/label maxima and minima of the graph, identify zeros when suitable factorizations are available, and show end behavior • Graph exponential, logarithmic, and trigonometric functions, by hand in simple cases or using technology for more complicated cases. For exponential and logarithmic functions, show: period, midline, and amplitude • Graph linear functions by hand in simple cases or using technology for more complicated cases and show/label intercepts of the graph • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions, by hand in simple case or using technology for more complicated cases, and show/label key features of the graph	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
	Analyze functions using different representations. *Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.*	**F.IF.8 – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.** **a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.** **b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t y = (1.01)12t, y = (1.2)/10, and classify them as representing exponential growth or decay.** • Identify different forms of a quadratic expression • Identify zeros, extreme values, and symmetry of the graph of a quadratic function • Identify how key features of a quadratic function relate to characteristics of a real-world context • Classify the exponential function as exponential growth or decay by examining the base • Identify how key features of an exponential function relate to tis characteristics in a real-world context • Interpret different yet equivalent forms of a function, as defined by an expression in terms of • Use the properties of exponents to interpret expressions for exponential functions in a real-world context • Given the expression of an exponential function, interpret the expression in terms of a real-world context, using the properties of exponents • Given the expression of a quadratic function, interpret zeros, extreme values, and symmetry of the graph in terms of a real-world context • Write a quadratic function defined by an expression in different but equivalent forms to reveal and explain various properties of the function and determine which form of the quadratic is the most appropriate for showing zeros and symmetry of a graph in terms of a real-world context • Write an exponential function defined by an expression in different but equivalent forms to reveal and explain different properties of the function, and determine which form of the function is the most appropriate for interpretation in a real-world context • Write functions in equivalent forms using the process of factoring • Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure.
Interpreting Functions	**Analyze functions using different representations.** *Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.*	**F.IF.9 – Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.** • Identify types of functions based on verbal, numerical, algebraic, and graphical descriptions and state key properties • Differentiate between exponential and linear functions using a variety of descriptors (graphical, verbal, numerical, algebraic) • Differentiate between different types of functions using a variety of descriptors (graphical, verbal, numerical, algebraic) • Use a variety of function representations (algebraic, graphical, numerical in tables, or by verbal descriptions) to compare and contrast properties of two functions	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically. 7. Look for and make use of structure.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Building Functions	**Build a function that models a relationship between two quantities.** *Develop models for more complex or sophisticated situations than in previous courses.*	**F.BF.1 – Write a function that describes a relationship between two quantities.** **b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.** • Define explicit function and recursive process • Combine two functions using the operations of addition, subtraction, multiplication, and division • Evaluate the domain of the combined function • Given a real-world situation or mathematical problem, build standard functions to represent relevant relationships/quantities • Given a real-world situation or mathematical problem, determine which arithmetic operation should be performed to build the appropriate combined function • Given a real-world situation or mathematical problem, relate the combined function to the context of the problem • Write a function that describes a relationship between two quantities by determining an explicit expression, a recursive process, or steps for calculation from a context	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Building Functions	**Build new functions from existing functions.** *Use transformations of functions to find models as students consider increasingly more complex situations. For F.BF.3, note the effect of multiple transformations on a single graph and the common effect of each transformations across function types.*	**F.BF.3– Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.** • Given a single transformation on a function symbolic or graphic identify the effect on the graph • Using technology, identify effects of single transformations on graphs of functions • Recognize even and odd functions from their graphs and equations • Describe the differences and similarities between a parent function and the transformed function • Find the value of k, given the graphs of a parent function, f(x), and the transformed function; f(x) + k, k f(x), f(kx), or f(x + k) • Experiment with cases and illustrate an explanation of the effects on a graph, using technology • Graph a given function by replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative)	4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Building Functions	**Build new functions from existing functions. Use transformations of functions to find models as students consider increasingly more complex situations.** *Extend F.BF.4a , to simple rational, simple radical, and simple exponential functions; connect F.BF.4a to F.LE.4.*	**F.BF.4– Find the inverse functions.** a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example: $f(x) = 2x^3$ or fx = (x+1)/(x-1) for $x \neq 1$. • Define inverse function • Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse	4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure.

Algebra II			
Unit 3: Modeling with Functions			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Linear, Quadratic, and Exponential Models	**Construct and compare linear, quadratic, and exponential models and solve problems.** *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy= log x + log y.*	**F.LE.4 – For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, b, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.** • Recognize the laws and properties of logarithms, including change of base • Recognize and describe the key features of logarithmic functions • Recognize and know the definition of logarithm base b • Evaluate a logarithm using technology • For exponential models, express as a logarithm, the solution to $ab^{ct} = d$, where a, b, and d are numbers and the base b is 2, 10, or e	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra II			
Unit 4: Inferences and Conclusions from Data			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on a single count or measurement variable.** *While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.*	**S.ID.4 – Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables, and American Indian data sources to estimate areas under the normal curve.** • Describe the characteristics of a normal distribution • Recognize that there are data sets for which such a procedure is not appropriate • Use the mean and standard deviation of a data set to fit it to a normal distribution • Use a normal distribution to estimate population percentages • Use a calculator, spreadsheet, and table to estimate areas under the normal curve	5. Use appropriate tools strategically.
Making Inferences and Justifying Conclusions	**Understand and evaluate random processes underlying statistical experiments.**	**S.IC.1 – Understand statistics as a process for making inferences about population parameters based on a random sample form that population.** • Explain that statistics is a process for making inferences about population parameters, or characteristics • Explain that statistical inferences about population characteristics are based on random samples from that population	2. Reason abstractly and quantitatively.
Making Inferences and Justifying Conclusions	**Understand and evaluate random processes underlying statistical experiments.** *For S.IC.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.*	**S.IC.2 – Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?** • Use various, specified data-generating processes/models • Recognize data that various models produce • Identify data or discrepancies that provide the basis for rejecting a statistical model • Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	3. Construct viable arguments and critique the reasoning of others.

Algebra II			
Unit 4: Inferences and Conclusions from Data			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Making Inferences and Justifying Conclusions	**Make inferences and evaluate random processes underlying statistical experiments.** *In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result of random selection in sampling or random assignment in an experiment.*	**S.IC.3 – Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.** • Recognize the purpose of surveys, experiments, and observational studies in making statistical inferences and justifying conclusions and explain how randomization relates to each of these methods of data collection • Recognize the differences among surveys, experiments, and observational studies in making statistical inferences and justifying conclusions and explain how randomization relates to each of the methods of data collection	3. Construct viable arguments and critique the reasoning of others.
Making Inferences and Justifying Conclusions	**Make inferences and evaluate random processes underlying statistical experiments.** *For S.IC.4 and 5, focus on the variability of results from experiments-that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.*	**S.IC.4 – Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.** • Define margin of error • Explain the connection of margin of error to variation within a data set or population • Interpret the data generated by a simulation model for random sampling in terms of the context the simulation models • Develop a margin of error, assuming certain population parameters/characteristics, through the use of simulation models for random sampling • Use a simulation model to generate data for random sampling, assuming certain population parameters/characteristics • Use data from a sample survey to estimate a population mean or proportion	4. Model with mathematics. 7. Look for and make use of structure.
Making Inferences and Justifying Conclusions	**Make inferences and evaluate random processes underlying statistical experiments.** *For S.IC.4 and 5, focus on the variability of results from experiments-that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.*	**S.IC.5 – Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between two parameters are significant.** • Using an established level of significance, determine if the difference between two parameters is significant • Choose appropriate methods to simulate a randomized experiment • Establish a reasonable level of significance • Use data from a randomized experiment to compare two treatments	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra II			
Unit 4: Inferences and Conclusions from Data			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Making Inferences and Justifying Conclusions	**Make inferences and evaluate random processes underlying statistical experiments.**	**S.IC.6 – Evaluate reports bases on data.** • Define the characteristics of experimental design (control randomization, and replication) • Evaluate experimental study design, how data was gathered, and what analysis (numerical or graphical) was used • Draw conclusions based on graphical and numerical summaries • Support with graphical and numerical summaries how appropriate the report of data was	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 6. Attend to precision. 8. Look for an express regularity in repeated reasoning.
Using Probability to Make Decisions	**Use probability to evaluate outcomes of decisions.** *Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.*	**S.MD.6 – (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).** • Compute Theoretical and Experimental Probabilities • Recall previous understandings of probability • Use probabilities to make fair decisions • Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 8. Look for and express regularity in repeated reasoning.
Using Probability to Make Decisions	**Use probability to evaluate outcomes of decisions.** *Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.*	**S.MD.7 – (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).** • Recall previous understandings of probability • Analyze decisions and strategies using probability concepts • Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision.

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because $3/4 + (-3/4) = (-3/4) + 3/4 = 0$.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[4]To be more precise, this defines the *arithmetic mean.* one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a*/*b* or – *a*/*b* for some fraction *a*/*b*. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. **Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

LEARNING PROGRESSIONS BY DOMAIN

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