Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Eighth Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**
- **f. Justify their conclusions, communicate them to others, and respond to the arguments of others.**
- **g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.**

4. Model with mathematics.

Mathematically proficient students:

a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.
b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
d. Analyze mathematical relationships to draw conclusions.

5. Use appropriate tools strategically.

Mathematically proficient students:

a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)
b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.

6. Attend to precision.

Mathematically proficient students:

a. Communicate their understanding of mathematics to others.
b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
c. Specify units of measure and use label parts of graphs and charts
d. Strive for accuracy.

7. Look for and make use of structure.

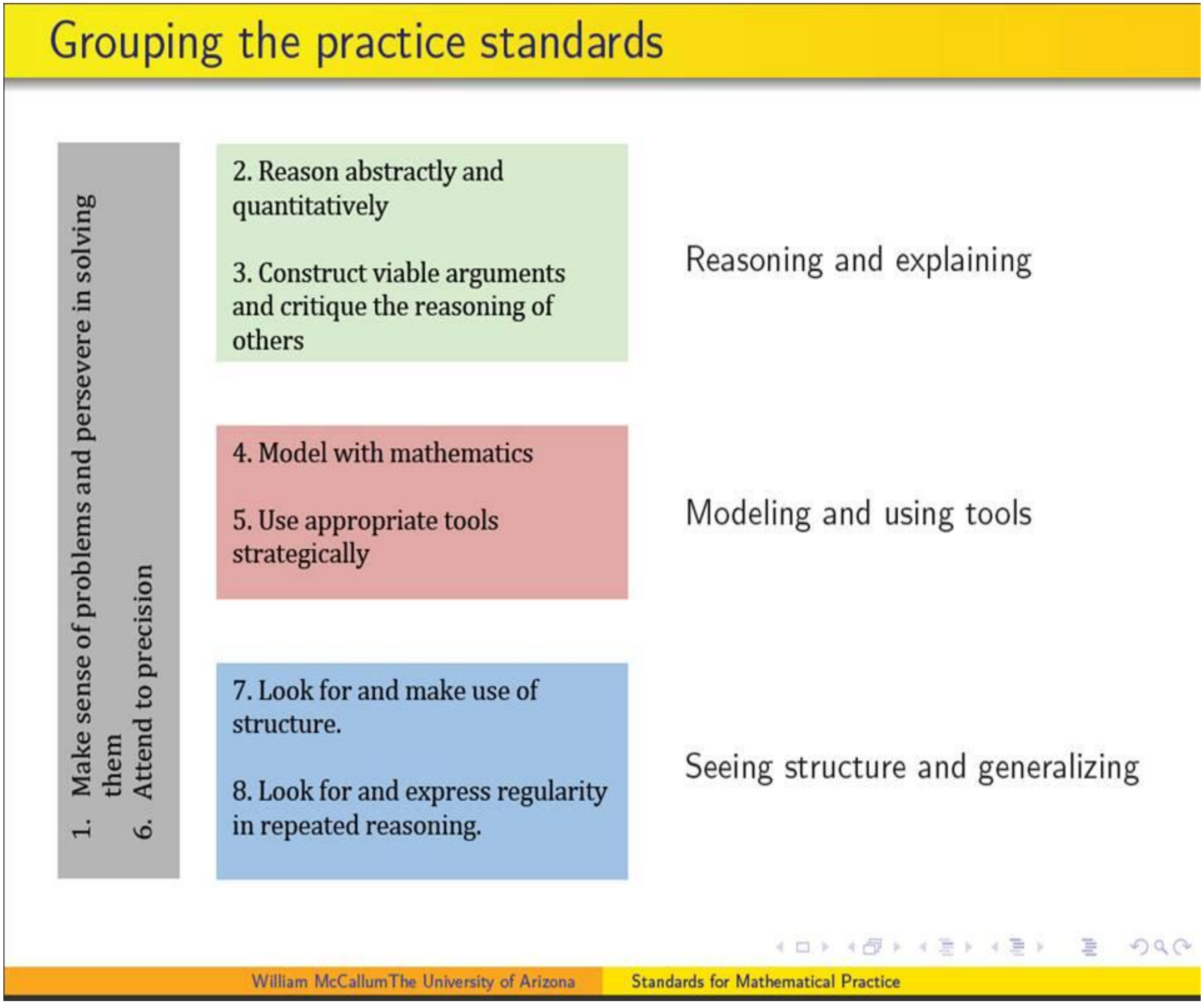
Mathematically proficient students:

a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.
b. Apply and discuss properties.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

a. Look for mathematically sound shortcuts.
b. Use repeated applications to generalize properties.


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallumThe University of Arizona
Standards for Mathematical Practice

Standards for Mathematical Practice: Grade 8 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
8.MP.1. Make sense of problems and persevere in solving them.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
8.MP.2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
8.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
8.MP.4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
8.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
8.MP.6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
8.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8.MP.8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8 Overview

The Number System
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Statistics and Probability
- Investigate patterns of association in bivariate data.

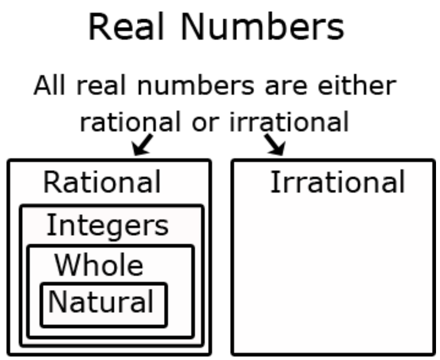
Geometry
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Functions
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

The Number System **8.NS**

Know that there are numbers that are not rational, and approximate them by rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **Real Numbers, Irrational numbers, Rational numbers, Integers, Whole numbers, Natural numbers, radical, radicand, square roots, perfect squares, cube roots, terminating decimals, repeating decimals, truncate**

Standards/Learning Objectives	***Explanations and Examples***
8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. ▪ Define irrational numbers ▪ Show informally that every number has a decimal expansion	Students can use graphic organizers to show the relationship between the subsets of the real number system. 
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	Students can approximate square roots by iterative processes. Examples: • Approximate the value of $\sqrt{5}$ to the nearest hundredth. Solution: Students start with a rough estimate based upon perfect squares. $\sqrt{5}$ falls between 2 and 3 because 5 falls between $2^2 = 4$ and $3^2 = 9$. The value will be closer to 2 than to 3. Students continue the iterative process with the tenths place value. $\sqrt{5}$ falls between 2.2 and 2.3 because 5 falls between $2.2^2 = 4.84$ and $2.3^2 = 5.29$. The value is closer to 2.2. Further iteration shows that the value of $\sqrt{5}$ is between 2.23 and 2.24 since 2.23^2 is 4.9729 and 2.24^2 is 5.0176. Continued on next page

The Number System **8.NS**
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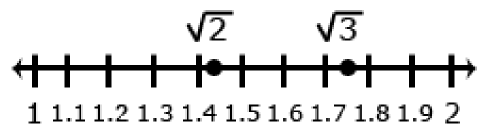
<u>Standards/Learning Objectives</u>		***<u>Explanations and Examples</u>***
		• Compare $\sqrt{2}$ and $\sqrt{3}$ by estimating their values, plotting them on a number line, and making comparative statements.  Solution: Statements for the comparison could include: $\sqrt{2}$ is approximately 0.3 less than $\sqrt{3}$ $\sqrt{2}$ is between the whole numbers 1 and 2 $\sqrt{3}$ is between 1.7 and 1.8

Expressions and Equations **8.EE**
Work with radicals and integer exponents.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **laws of exponents, power, perfect squares, perfect cubes, root, square root, cube root, scientific notation, standard form of a number.** Students should also be able to read and use the symbol: **±**

<u>Standards/Learning Objectives</u>		***<u>Explanations and Examples</u>***
8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.		Examples: • $\frac{4^3}{5^2} = \frac{64}{25}$ • $\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ • $\frac{4^{-3}}{5^2} = 4^{-3} \times \frac{1}{5^2} = \frac{1}{4^3} \times \frac{1}{5^2} = \frac{1}{64} \times \frac{1}{25} = \frac{1}{16,000}$

Expressions and Equations Work with radicals and integer exponents.	**8.EE**

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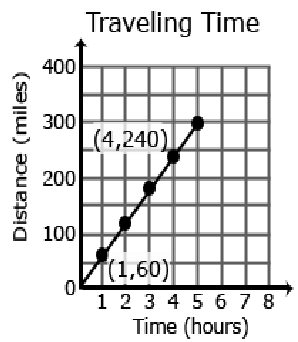
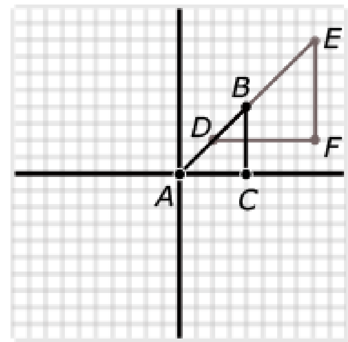
Standards/Learning Objectives	***Explanations and Examples***
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	Examples: • $3^2 = 9$ and $\sqrt{9} = \pm 3$ • $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$ • Solve $x^2 = 9$ Solution: $x^2 = 9$ $\sqrt{x^2} = \pm\sqrt{9}$ $x = \pm 3$ • Solve $x^3 = 8$ Solution: $x^3 = 8$ $\sqrt[3]{x^3} = \sqrt[3]{8}$ $x = 2$

Expressions and Equations Work with radicals and integer exponents.		**8.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **laws of exponents, power, perfect squares, perfect cubes, root, square root, cube root, scientific notation, standard form of a number.** Students should also be able to read and use the symbol: **±**		
Standards/Learning Objectives		***Explanations and Examples***
8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* ▪ Compare quantities to express how much larger one is compared to the other ▪ Use scientific notation to estimate very large and /or very small quantities		
8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.		Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of 2.45E+23 is 2.45×10^{23} and 3.5E-4 is 3.5×10^{-4}. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.

Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations		**8.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rate, proportional relationships, slope, vertical, horizontal, similar triangles, *y*-intercept**		
Standards/Learning Objectives		***Explanations and Examples***
8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*		Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs. Example: • Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation. Scenario 1:  Scenario 2: $y = 50x$ x is time in hours y is distance in miles

Expressions and Equations Understand the connections between proportional relationships, lines, and linear equations		**8.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rate, proportional relationships, slope, vertical, horizontal, similar triangles, *y*-intercept**		
Standards/Learning Objectives		***Explanations and Examples***
8.EE.6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. ▪ Identify characteristics of similar triangles ▪ Analyze patterns for points on a line that passes through the origin ▪ Analyze patterns for points on a line that does not pass through or include the origin ▪ Determine the *y*-intercept) of a line ▪ Find the slope of a line		Example: • Explain why ΔACB is similar to ΔDFE, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation. 

Expressions and Equations Analyze and solve linear equations and pairs of simultaneous linear equations.	**8.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **intersecting, parallel lines, coefficient, distributive property, like terms, substitution, system of linear equations**	

Standards/Learning Objectives	***Explanations and Examples***
8.EE.7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions. When the equation has one solution, the variable has one value that makes the equation true as in $12-4y=16$. The only value for y that makes this equation true is -1. When the equation has infinitely many solutions, the equation is true for all real numbers as in $7x + 14 = 7(x+2)$. As this equation is simplified, the variable terms cancel leaving 14 = 14 or 0 = 0. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution. When an equation has no solutions it is also called an inconsistent equation. This is the case when the two expressions are not equivalent as in $5x - 2 = 5(x+1)$. When simplifying this equation, students will find that the solution appears to be two numbers that are not equal or -2 = 1. In this case, regardless which real number is used for the substitution, the equation is not true and therefore has no solution. Examples: • Solve for x: ○ $-3(x+7)=4$ ○ $3x-8=4x-8$ ○ $3(x+1)-5=3x-2$ • Solve: ○ $7(m-3)=7$ ○ $\frac{1}{4}-\frac{2}{3}y=\frac{3}{4}-\frac{1}{3}y$

Expressions and Equations — 8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **intersecting, parallel lines, coefficient, distributive property, like terms, substitution, system of linear equations**

Standards/Learning Objectives	***Explanations and Examples***
8.EE.8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically. A system of linear equations whose graphs meet at one point (intersecting lines) has only one solution, the ordered pair representing the point of intersection. A system of linear equations whose graphs do not meet (parallel lines) has no solutions and the slopes of these lines are the same. A system of linear equations whose graphs are coincident (the same line) has infinitely many solutions, the set of ordered pairs representing all the points on the line. By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions. Examples: • Find x and y using elimination and then using substitution. $3x + 4y = 7$ $-2x + 8y = 10$ • Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same. Let W = number of weeks Let H = height of the plant after W weeks

Plant A

W	H	
0	4	(0,4)
1	6	(1,6)
2	8	(2,8)
3	10	(3,10)

Plant B

W	H	
0	2	(0,2)
1	6	(1,6)
2	10	(2,10)
3	14	(3,14)

Continued on next page

Expressions and Equations **8.EE**

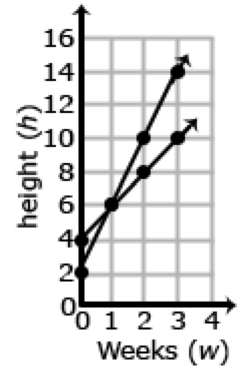
Analyze and solve linear equations and pairs of simultaneous linear equations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **intersecting, parallel lines, coefficient, distributive property, like terms, substitution, system of linear equations**

Standards/Learning Objectives	***Explanations and Examples***

- Given each set of coordinates, graph their corresponding lines.

  Solution:



- Write an equation that represent the growth rate of Plant A and Plant B.

  Solution:

  Plant A $H = 2W + 4$
  Plant B $H = 4W + 2$

- At which week will the plants have the same height?

  Solution:
  The plants have the same height after one week.

Plant A: $H = 2W + 4$	Plant B: $H = 4W + 2$
Plant A: $H = 2(1) + 4$	Plant B: $H = 4(1) + 2$
Plant A: $H = 6$	Plant B: $H = 6$

  After one week, the height of Plant A and Plant B are both 6 inches.

Functions **8.F**

Define, evaluate, and compare functions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **functions, *y*-value, *x*-value, vertical line test, input, output, rate of change, linear function, non-linear function**

Standards/Learning Objectives	***Explanations and Examples***
8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)	For example, the rule that takes x as input and gives x^2+5x+4 as output is a function. Using y to stand for the output we can represent this function with the equation $y = x^2+5x+4$, and the graph of the equation is the graph of the function. Students are not yet expected use function notation such as $f(x) = x^2+5x+4$.

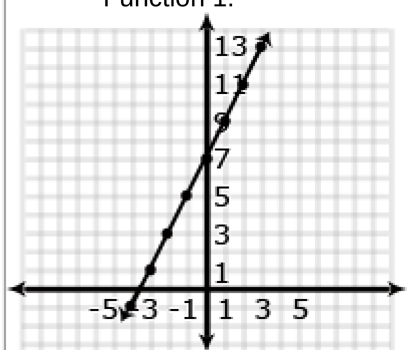
8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

- Identify functions algebraically including slope and *y*-intercept
- Identify functions using graphs, tables and verbal descriptions
- Compare and contrast 2 functions with different representations
- Draw conclusions based on different representations of functions

Examples:

- Compare the two linear functions listed below and determine which equation represents a greater rate of change.

Function 1:



Function 2:

The function whose input x and output y are related by

$y = 3x + 7$

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Functions Define, evaluate, and compare functions.	**8.F**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **functions, *y*-value, *x*-value, vertical line test, input, output, rate of change, linear function, non-linear function**

Standards/Learning Objectives	***Explanations and Examples***
	• Compare the two linear functions listed below and determine which has a negative slope. Function 1: Gift Card Samantha starts with $20 on a gift card for the book store. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, *x*. *x* / *y*: 0 / 20; 1 / 16.50; 2 / 13.00; 3 / 9.50; 4 / 6.00 Function 2: The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*). Solution: Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha's weekly magazine purchase. Function 2 is an example of a function whose graph has positive slope. Students pay a yearly nonrefundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Example 2 could be $c = 5m + 10$.

Function 1 table:

x	*y*
0	20
1	16.50
2	13.00
3	9.50
4	6.00

Functions Define, evaluate, and compare functions.	**8.F**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **functions, *y*-value, *x*-value, vertical line test, input, output, rate of change, linear function, non-linear function**

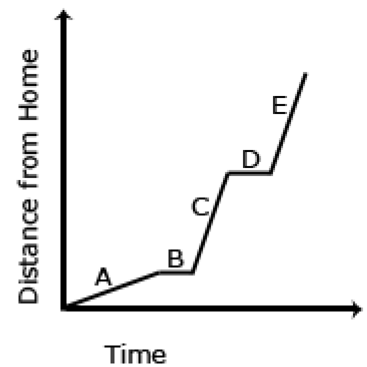
Standards/Learning Objectives	***Explanations and Examples***
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* ▪ Recognize that a linear function is graphed as a straight line ▪ Recognize the equation $y = mx + b$ is the equation of a function whose graph is a straight line where m is the slope and b is the y-intercept ▪ Compare the characteristics of linear and nonlinear functions using various representations ▪ Provide examples of nonlinear functions using multiple representations	Example: • Determine which of the functions listed below are linear and which are not linear and explain your reasoning. ○ $y = -2x^2 + 3$ non linear ○ $y = 2x$ linear ○ $A = \pi r^2$ non linear ○ $y = 0.25 + 0.5(x – 2)$ linear

Functions	**8.F**
Use functions to model relationships between quantities.	

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **linear relationship, rate of change, slope, initial value, y-intercept**

Standards/Learning Objectives	***Explanations and Examples***
8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	Examples: • The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time $25 fee for the car's navigation system (GPS).Write an expression for the cost in dollars, *c,* as a function of the number of days, *d*. Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table. Days (*d*) / Cost (*c*) in dollars: 1 / 70; 2 / 115; 3 / 160; 4 / 205 Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one time fees vs. recurrent fees will help students model contextual situations. • When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t – 100$ shows the relationship between the time of the ascent in seconds (*t*) and the distance from the surface in feet (*d*). ○ Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive? • Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?

Days (*d*)	Cost (*c*) in dollars
1	70
2	115
3	160
4	205

Functions Use functions to model relationships between quantities.		**8.F**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **linear relationship, rate of change, slope, initial value, y-intercept**		
Standards/Learning Objectives		***Explanations and Examples***
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.		Example: • The graph below shows a student's trip to school. This student walks to his friend's house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A-E of the graph relates to the story. 

Geometry	**8.G**
Understand congruence and similarity using physical models, transparencies, or geometry software.	

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **translations, rotations, reflections, line of reflection, center of rotation, clockwise, counterclockwise, parallel lines, betweenness, congruence, reading A' as "A prime", similarity, dilations, pre-image, image, rigid transformations, exterior angles, interior angles, alternate interior angles, angle-angle criterion, deductive reasoning, vertical angles, adjacent, supplementary, complementary, corresponding, scale factor, transversal, parallel**

Standards/Learning Objectives	***Explanations and Examples***
8.G.1. Verify experimentally the properties of rotations, reflections, and translations from a variety of cultural contexts, including those of Montana American Indians: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. ▪ Identify corresponding sides and corresponding angles ▪ Identify center of rotation ▪ Identify direction and degree of rotation ▪ Identify line of reflection ▪ Understand prime notation to describe an image after a translation, reflection, or rotation	Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated. Students are not expected to work formally with properties of dilations until high school.

Geometry **8.G**

Understand congruence and similarity using physical models, transparencies, or geometry software.

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Standards/Learning Objectives

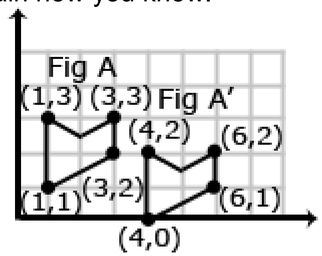
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

- Define congruency
- Identify symbols for congruency
- Describe the sequence of rotations, reflections, translations that exhibits the congruence between 2-D figures using words
- Apply the concept of congruency to write congruent statements
- Reason that a 2-D figure is congruent to another if the second can be obtained by a sequence of rotations, reflections, translations

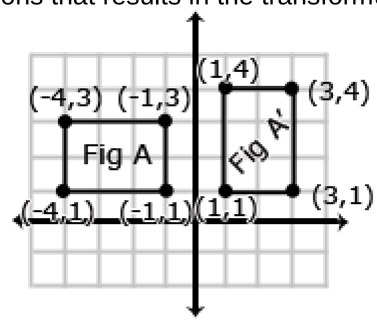
Explanations and Examples

Examples:

- Is Figure A congruent to Figure A'? Explain how you know.



- Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



Standards/Learning Objectives

8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures from a variety of cultural contexts, including those of Montana American Indians,using coordinates.

- Define dilations as a reduction or enlargement of a figure
- Identify scale factor of the dilation

Explanations and Examples

A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is *similar* to its pre-image.

Translation: A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. $\triangle ABC$ has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from x = 1 to x = 8) and 3 units up (from y = 5 to y = 8). Points B + C also move in the same direction (7 units to the right and 3 units up).

Geometry	8.G
Understand congruence and similarity using physical models, transparencies, or geometry software.	

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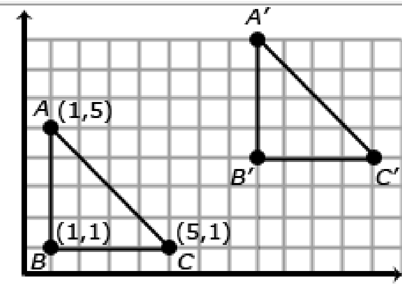
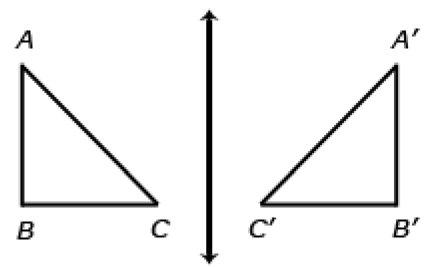
Standards/Learning Objectives	***Explanations and Examples***
	 Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is *congruent* to its pre-image.  $\triangle ABC \cong \triangle A'B'C'$ Continued on next page When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate.

Geometry **8.G**

Understand congruence and similarity using physical models, transparencies, or geometry software.

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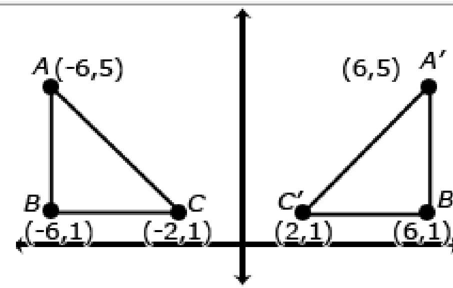
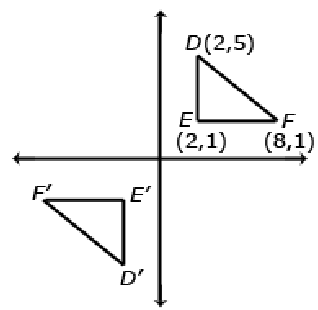
<u>Standards/Learning Objectives</u>	***<u>Explanations and Examples</u>***
	 Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360˚. Rotated figures are congruent to their pre-image figures. Consider when ΔDEF is rotated 180˚ clockwise about the origin. The coordinates of ΔDEF are D(2,5), E(2,1), and F(8,1). When rotated 180˚, $\Delta D'E'F'$ has new coordinates D'(-2,-5), E'(-2,-1) and F'(-8,-1). Each coordinate is the opposite of its pre-image. 

Geometry **8.G**

Understand congruence and similarity using physical models, transparencies, or geometry software.

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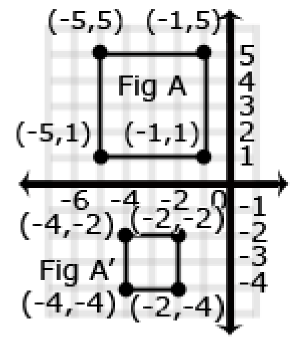
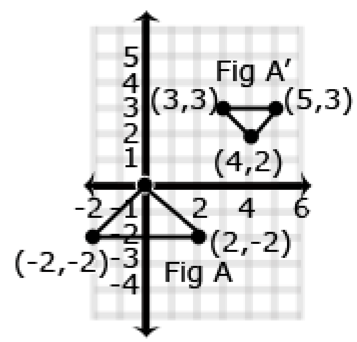
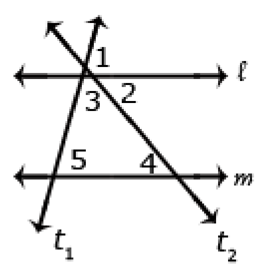
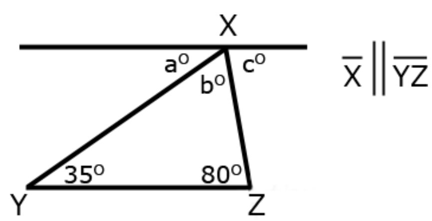
Standards/Learning Objectives	***Explanations and Examples***
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, ▪ Define similar figures as corresponding angles are congruent and corresponding sides are proportional	Examples: • Is Figure A similar to Figure A’? Explain how you know.  • Describe the sequence of transformations that results in the transformation of Figure A to Figure A’. 

Geometry Understand congruence and similarity using physical models, transparencies, or geometry software.	**8.G**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **translations, rotations, reflections, line of reflection, center of rotation, clockwise, counterclockwise, parallel lines, betweenness, congruence, reading A' as "A prime", similarity, dilations, pre-image, image, rigid transformations, exterior angles, interior angles, alternate interior angles, angle-angle criterion, deductive reasoning, vertical angles, adjacent, supplementary, complementary, corresponding, scale factor, transversal, parallel**	
Standards/Learning Objectives	***Explanations and Examples***
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* ▪ Define similar triangles ▪ Define and identify transversals ▪ Justify that the sum of interior angles equals 180 ▪ Justify that the exterior angle of a triangle is equal to the sum of the two remote interior angles	• Examples: Students can informally prove relationships with transversals. Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$ if ℓ and m are parallel lines and t_1 & t_2 are transversals. $\angle 1 + \angle 2 + \angle 3 = 180^\circ$. Angle 1 and Angle 5 are congruent because they are corresponding angles ($\angle 5 \cong \angle 1$). $\angle 1$ can be substituted for $\angle 5$. $\angle 4 \cong \angle 2$: because alternate interior angles are congruent. $\angle 4$ can be substituted for $\angle 2$ Therefore $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$  Continued on next page

Geometry Understand congruence and similarity using physical models, transparencies, or geometry software.	**8.G**

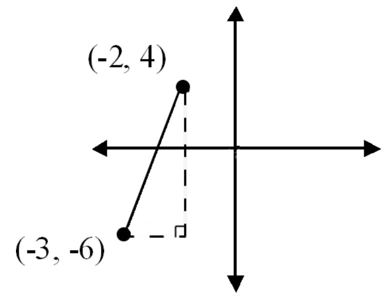
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **translations, rotations, reflections, line of reflection, center of rotation, clockwise, counterclockwise, parallel lines, betweenness, congruence, reading A' as "A prime", similarity, dilations, pre-image, image, rigid transformations, exterior angles, interior angles, alternate interior angles, angle-angle criterion, deductive reasoning, vertical angles, adjacent, supplementary, complementary, corresponding, scale factor, transversal, parallel**

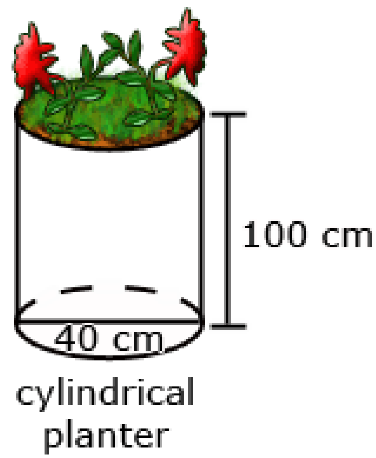
Standards/Learning Objectives	***Explanations and Examples***
	Students can informally conclude that the sum of a triangle is 180º (the angle-sum theorem) by applying their understanding of lines and alternate interior angles. In the figure below, line x is parallel to line *yz*:  Angle *a* is 35º because it alternates with the angle inside the triangle that measures 35º. Angle *c* is 80º because it alternates with the angle inside the triangle that measures 80º. Because lines have a measure of 180º, and angles *a* + *b* + *c* form a straight line, then angle *b* must be 65 º (180 – 35 + 80 = 65). Therefore, the sum of the angles of the triangle are 35º + 65 º + 80 º

Geometry Understand and apply the Pythagorean Theorem.		**8.G**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **right triangle, hypotenuse, legs, Pythagorean Theorem, Pythagorean triple**		
Standards/Learning Objectives		***Explanations and Examples***
8.G.6. Explain a proof of the Pythagorean Theorem and its converse. ▪ Define key vocabulary: *square root, Pythagorean Theorem, right triangle, legs a & b, hypotenuse, sides, right angle, converse, base, height, proof* ▪ Identify the legs and hypotenuse of a right triangle		Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle.
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.		Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets.

Geometry Understand and apply the Pythagorean Theorem.	**8.G**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **right triangle, hypotenuse, legs, Pythagorean Theorem, Pythagorean triple**

Standards/Learning Objectives	***Explanations and Examples***
8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Example: • Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points. 

Geometry Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres		**8.G**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **cones, cylinders, spheres, radius, volume, height, Pi**		
Standards/Learning Objectives		***Explanations and Examples***
8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. ▪ Identify and define vocabulary: *cone, cylinder, sphere, radius, diameter, circumference, area, volume, pi, base, height* ▪ Know formulas for volume of cones, cylinders, and spheres ▪ Compare the volume of cones, cylinders, and spheres ▪ Determine and apply appropriate volume formulas in order to solve mathematical and real-world problems for the given shape	.	Example: • James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume. 

Statistics and Probability **8.SP**

Investigate patterns of association in bivariate data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **bivariate data, scatter plot, linear model, clustering, linear association, non-linear association, outliers, positive association, negative association, categorical data, two-way table, relative frequency**

Standards/Learning Objectives	***Explanations and Examples***
8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Students build on their previous knowledge of scatter plots examine relationships between variables. They analyze scatterplots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. (http://nces.ed.gov/nceskids/createagraph/default.aspx)

Examples:

- Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

- Data for 10 students' Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

Student	1	2	3	4	5	6	7	8	9	10
Math score	64	50	85	34	56	24	72	63	42	93
Dist from school (miles)	0.5	1.8	1	2.3	3.4	0.2	2.5	1.6	0.8	2.5

- Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Number of staff	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

- The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005

Statistics and Probability Investigate patterns of association in bivariate data.	**8.SP**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **bivariate data, scatter plot, linear model, clustering, linear association, non-linear association, outliers, positive association, negative association, categorical data, two-way table, relative frequency**

Standards/Learning Objectives	***Explanations and Examples***
	<table><tr><td>Life Expectancy (in years)</td><td>70.8</td><td>72.6</td><td>73.7</td><td>74.7</td><td>75.4</td><td>75.8</td><td>76.8</td><td>77.4</td></tr></table>
8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Examples: The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon? <table><tr><td>Miles Traveled</td><td>0</td><td>75</td><td>120</td><td>160</td><td>250</td><td>300</td></tr><tr><td>Gallons Used</td><td>0</td><td>2.3</td><td>4.5</td><td>5.7</td><td>9.7</td><td>10.7</td></tr></table>
8.SP.3. Use the equation of a linear	Examples:

Statistics and Probability **8.SP**

Investigate patterns of association in bivariate data.

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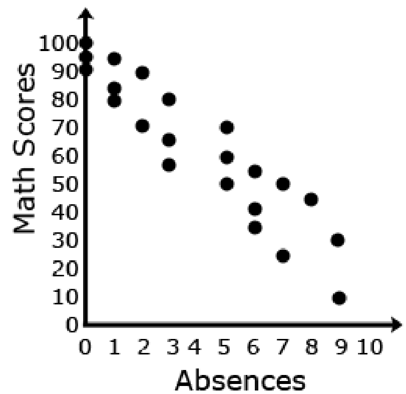
Standards/Learning Objectives

model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*.

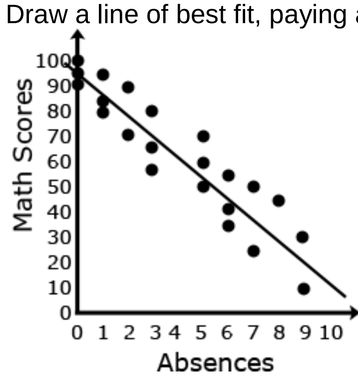
Explanations and Examples

- 1. Given data from students' math scores and absences, make a scatterplot.

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80



2. Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.



Statistics and Probability **8.SP**

Investigate patterns of association in bivariate data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **bivariate data, scatter plot, linear model, clustering, linear association, non-linear association, outliers, positive association, negative association, categorical data, two-way table, relative frequency**

Standards/Learning Objectives	***Explanations and Examples***
	• 3. From the line of best fit, determine an approximate linear equation that models the given data (about y = $-\frac{25}{3}x+95$) • 4. Students should recognize that 95 represents the y intercept and $-\frac{25}{3}$ represents the slope of the line. • 5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.
8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	Example: • The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew? and Do you have assigned chores? Is there evidence that those who have a curfew also tend to have chores? Curfew: Yes / No Chores Yes: 40 / 10 Chores No: 10 / 40 Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.

Chores \ Curfew	Yes	No
Yes	40	10
No	10	40

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another.Example: 3/4 and – 3/4 are additive inverses of one another because3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

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[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a*/*b* or – *a*/*b* for some fraction *a*/*b*. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. **Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a – c = b – c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here a, b and c stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $–a < –b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

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