Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Seventh Grade

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

a. Understand that mathematics is relevant when studied in a cultural context.
b. Explain the meaning of a problem and restate it in their words.
c. Analyze given information to develop possible strategies for solving the problem.
d. Identify and execute appropriate strategies to solve the problem.
e. Evaluate progress toward the solution and make revisions if necessary.
f. Check their answers using a different method, and continually ask "Does this make sense?"

2. Reason abstractly and quantitatively.

Mathematically proficient students:

a. Make sense of quantities and their relationships in problem situations.
b. Use varied representations and approaches when solving problems.
c. Know and flexibly use different properties of operations and objects.
d. Change perspectives, generate alternatives and consider different options.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

a. Understand and use prior learning in constructing arguments.
b. Habitually ask "why" and seek an answer to that question.
c. Question and problem-pose.
d. Develop questioning strategies to generate information.
e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.
f. Justify their conclusions, communicate them to others, and respond to the arguments of others.

g. **Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.**

4. Model with mathematics.

Mathematically proficient students:

a. **Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within cultural context, including those of Montana American Indians.**
b. **Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.**
c. **Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.**
d. **Analyze mathematical relationships to draw conclusions.**

5. Use appropriate tools strategically.

Mathematically proficient students:

a. **Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)**
b. **Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.**

6. Attend to precision.

Mathematically proficient students:

a. **Communicate their understanding of mathematics to others.**
b. **Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.**
c. **Specify units of measure and use label parts of graphs and charts**
d. **Strive for accuracy.**

7. Look for and make use of structure.

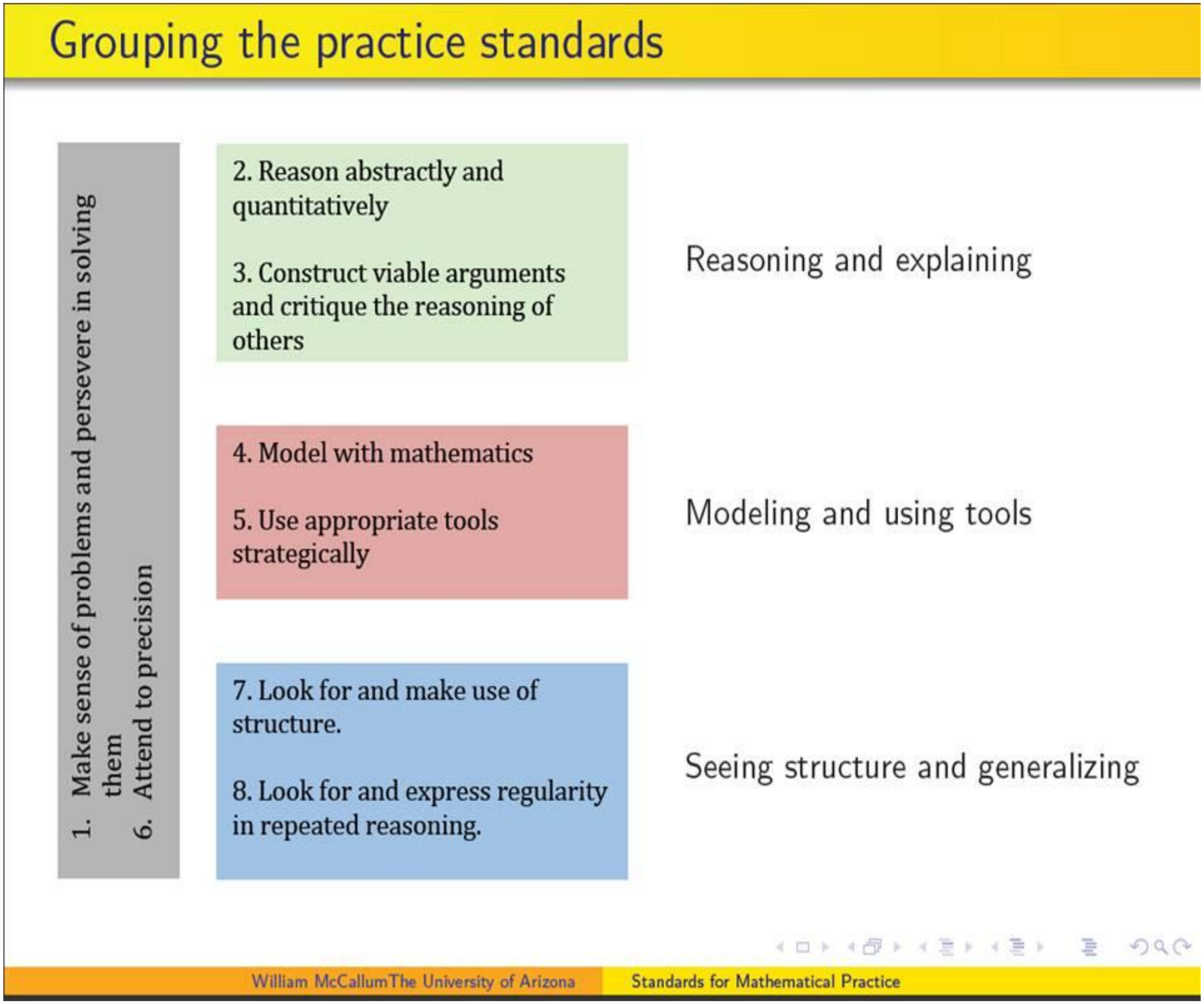
Mathematically proficient students:

a. **Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.**
b. **Apply and discuss properties.**

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

a. **Look for mathematically sound shortcuts.**
b. **Use repeated applications to generalize properties.**


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallumThe University of Arizona
Standards for Mathematical Practice

Standards for Mathematical Practice: Grade 7 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
7.MP.1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”.
7.MP.2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
7.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?”. They explain their thinking to others and respond to others’ thinking.
7.MP.4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
7.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
7.MP.6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
7.MP.8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7 Overview

Ratios and Proportional Relationships
- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry
- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability
- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Ratios of Proportional Relationships — 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rates, ratios, proportional relationships, proportions, constant of proportionality, complex fractions**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

Standards/Learning Objectives	***Explanations and Examples***
7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*	Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. The comparison can be with like or different units. Fractions may be proper or improper. Example 1: If ½ gallon of paint covers 1/6 of a wall, then how much paint is needed for the entire wall? *Solution:* ½ gal / /1/6 wall. 3 gallons per 1 wall

7.RP.2. Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.

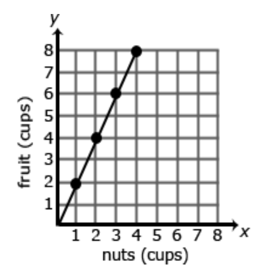
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.

Examples:

- A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

Serving Size	**1**	**2**	**3**	**4**
Cups of Nuts (x)	1	2	3	4
Cups of Fruit (y)	2	4	6	8



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1).

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Ratios of Proportional Relationships **7.RP**

Analyze proportional relationships and use them to solve real-world and mathematical problems.

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Standards/Learning Objectives

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
  - Know that a proportion is a statement of equality between two ratios
  - Define a constant of proportionality as a unit rate
  - Recognize what *(0, 0)* represents on the graph of a proportional relationship
  - Recognize what *(1, r)* on a graph represents, where r is

Explanations and Examples

The constant of proportionality is shown in the first column of the table and by the slope of the line on the graph.

- The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

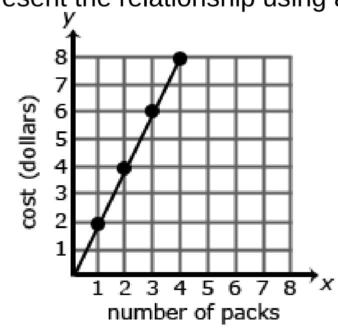

Table:

Number of Packs of Gum (g)	Cost in Dollars (d)
0	0
1	2
2	4
3	6
4	8

Equation: $2g = d$, where d is the cost in dollars and g is the packs of gum

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost ($g \times 2 = d$).

Ratios of Proportional Relationships Analyze proportional relationships and use them to solve real-world and mathematical problems.	**7.RP**

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Standards/Learning Objectives	***Explanations and Examples***
the unit rate ▪ Analyze two ratios to determine if they are proportional to one another with a variety of strategies (e.g., using tables, graphs, pictures, etc.) ▪ Analyze tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships to identify the constant of proportionality ▪ Represent proportional relationships by writing equations ▪ Explain what the points on a graph of a proportional relationship mean in terms of a specific situation	
7.RP.3. Use proportional relationships to solve multistep ratio and percent problems within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land). *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value. Examples: • Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015? A student might say: “The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40.”

Ratios of Proportional Relationships Analyze proportional relationships and use them to solve real-world and mathematical problems.	**7.RP**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **unit rates, ratios, proportional relationships, proportions, constant of proportionality, complex fractions**
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Standards/Learning Objectives	***Explanations and Examples***
▪ Represent the distance between two rational numbers on a number line as the absolute value of their difference and apply this principle in real-world contexts ▪ Apply the principle of subtracting rational numbers in real-world contexts ▪ Apply properties of operations as strategies to add and subtract rational numbers ▪ Demonstrate and explain how adding two numbers, $p + q$, if q is positive, the sum of p and q will be $\|q\|$ spaces to the right of p on the number line ▪ Demonstrate and explain how adding two numbers, $p + q$, if q is negative, the sum of p and q will be $\|q\|$ spaces to the left of p on the number line	See below

$4.17 + 4.17 + (0.24 • 4.17) = 2.24 x 4.17

100%	100%	24%
$4.17	$4.17	?

- A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

37.50 Original Price of Sweater	
33% of 37.50	67% of 37.50 Sale price of sweater

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

- A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

Discount 40% of original price	Sale Price - $12 60% of original price
Original Price (p)	

0.60p = 12

- At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.

Ratios of Proportional Relationships Analyze proportional relationships and use them to solve real-world and mathematical problems.		**7.RP**
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Standards/Learning Objectives		***Explanations and Examples***
		• A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal? After eating at a restaurant, your bill before tax is $52.60 The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill. The amount paid = 0.20 x $52.50 + 0.08 x $52.50 = 0.28 x $52.50

The Number System **7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, integers, additive inverse**

Standards/Learning Objectives

7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

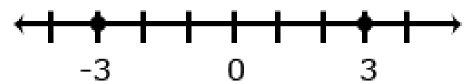
Explanations and Examples

Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.

Examples:

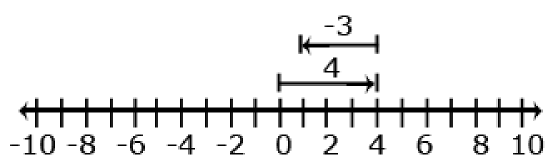
- Use a number line to illustrate:
  - $p - q$
  - $p + (- q)$
  - Is this equation true $p – q = p + (-q)$

- -3 and 3 are shown to be opposites on the number line because they are equal distance from zero and therefore have the same absolute value and the sum of the number and it’s opposite is zero.



- You have $4 and you need to pay a friend $3. What will you have after paying your friend?

$4 + (-3) = 1$ or $(-3) + 4 = 1$

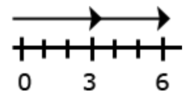
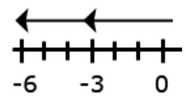
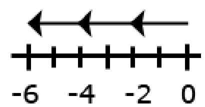
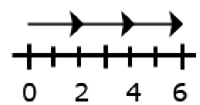


The Number System **7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, integers, additive inverse**

Standards/Learning Objectives		***Explanations and Examples***
7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.		Multiplication and division of integers is an extension of multiplication and division of whole numbers. Examples: • Examine the family of equations. What patterns do you see? Create a model and context for each of the products.

Equation	Number Line Model	Context
2 x 3 = 6		Selling two posters at $3.00 per poster
2 x -3 = -6		Spending 3 dollars each on 2 posters
-2 x 3 = -6		Owing 2 dollars to each of your three friends
-2 x -3 = 6		Forgiving 3 debts of $2.00 each

The Number System	7.NS
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	

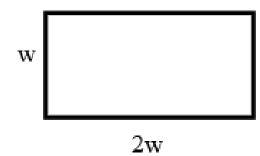
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, integers, additive inverse**

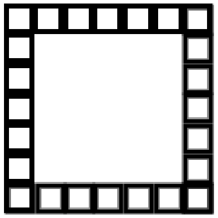
Standards/Learning Objectives		***Explanations and Examples***
7.NS.3. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)		Examples: • Your cell phone bill is automatically deducting \$32 from your bank account every month. How much will the deductions total for the year? -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12 (-32) • It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? $\frac{-100\text{ feet}}{20\text{ seconds}} = \frac{-5\text{ feet}}{1\text{ second}} = -5\text{ ft/sec}$

Expressions and Equations — 7.EE

Use properties of operations to generate equivalent expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coefficients, like terms, distributive property, factor**

Standards/Learning Objectives		***Explanations and Examples***
7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. ▪ Combine like terms with rational coefficients ▪ Factor and expand linear expressions with rational coefficients using the distributive property		Examples: • Write an equivalent expression for $3(x+5)-2$. • Suzanne thinks the two expressions $2(3a-2)+4a$ and $10a-2$ are equivalent? Is she correct? Explain why or why not? • Write equivalent expressions for: $3a+12$. Possible solutions might include factoring as in $3(a+4)$, or other expressions such as $a+2a+7+5$. • A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $w+w+2w+2w$. Write the expression in two other ways. Solution: $6w$ OR $2(w)+2(2w)$.  • An equilateral triangle has a perimeter of $6x+15$. What is the length of each of the sides of the triangle? Solution: $3(2x+5)$, therefore each side is $2x+5$ units long.
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”*		Examples: • Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? Can you write the expression in another way? Students may create several different expressions depending upon how they group the quantities in the problem. One student might say: To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then I would multiply the number of hours Ted worked by 9. I would add these two values with the $27 overtime to find the total wages for the week. The student would write the expression $9J+9T+27$.

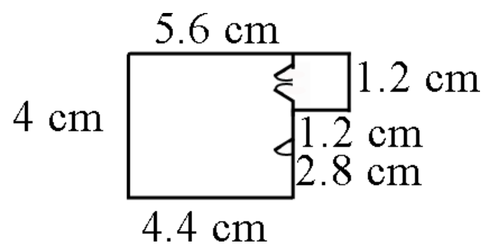
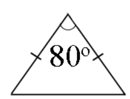
Expressions and Equations Use properties of operations to generate equivalent expressions.		**7.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coefficients, like terms, distributive property, factor**		
Standards/Learning Objectives		***Explanations and Examples***
		Another student might say: To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week. The student would write the expression $9(J+T)+27$ A third student might say: To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, I would multiply the number of hours she worked by 9. To figure out Ted's wages, I would multiply the number of hours he worked by 9 and then add the $27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J)+(9T+27)$ • Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression do you think is most useful? Explain your thinking. 

Expressions and Equations Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	**7.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **numeric expressions, algebraic expressions, maximum, minimum**	

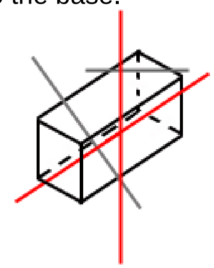
Standards/Learning Objectives		***Explanations and Examples***
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $^{1}/_{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 $^{3}/_{4}$ inches long in the center of a door that is 27 $^{1}/_{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*		Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate). Example: • The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass. [x | x | 11] / [52] $2x + 11 = 52$ $2x = 41$ $x = \$20.5$

Expressions and Equations Solve real-life and mathematical problems using numerical and algebraic expressions and equations.		**7.EE**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **numeric expressions, algebraic expressions, maximum, minimum**		
Standards/Learning Objectives		***Explanations and Examples***
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian Cultural Contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form *px+q=r* and *p*(*x+q*)=*r*, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form *px+q>r* or *px+q* < *r*, where *p*, *q*, and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*		Examples: • Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost? • The sum of three consecutive even numbers is 48. What is the smallest of these numbers? • Solve: $\frac{5}{4}n+5=20$ • Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase. • Steven has $25 dollars. He spent $10.81, including tax, to buy a new DVD. He needs to set aside $10.00 to pay for his lunch next week. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy? Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution. • Solve $\frac{1}{2}x+3>2$ and graph your solution on a number line.

Geometry Draw, construct, and describe geometrical figures and describe the relationships between them.	7.G
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular**	

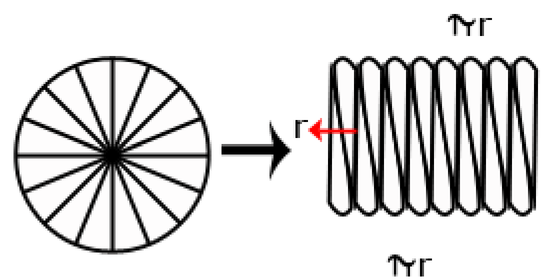
Standards/Learning Objectives	***Explanations and Examples***
7.G.1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. ▪ Identify corresponding sides of scaled geometric figures ▪ Use ratios and proportions to create scale drawing ▪ Compute lengths and areas from scale drawings using strategies such as proportions	Example: • Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size. 
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity. Examples: Is it possible to draw a triangle with a 90˚ angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle? • Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not? • Draw an isosceles triangle with only one 80 degree angle. Is this the only possibility or can you draw another triangle that will also meet these conditions?  • Can you draw a triangle with sides that are 13 cm, 5 cm and 6cm? • Draw a quadrilateral with one set of parallel sides and no right angles.

Geometry Draw, construct, and describe geometrical figures and describe the relationships between them.	**7.G**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular**	

Standards/Learning Objectives	***Explanations and Examples***
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. ▪ Define "slicing" as the cross-section of a 3-D figure	Example: • Using a clay model of a rectangular prism, describe the shapes that are created when planar cuts are made diagonally, perpendicularly, and parallel to the base. 

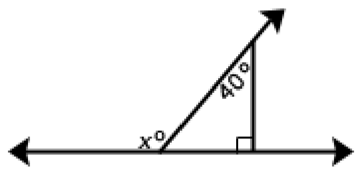
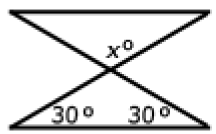
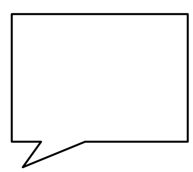
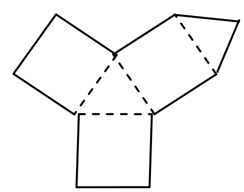
Geometry **7.G**
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inscribed, circumference, radius, diameter, pi,** *Π,* **supplementary, vertical, adjacent, complementary, pyramids, face, base**

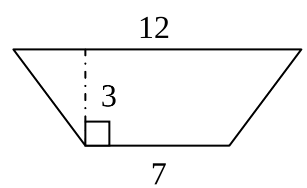
Standards/Learning Objectives	***Explanations and Examples***
7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians; give an informal derivation of the relationship between the circumference and area of a circle. ▪ Know the parts of a circle including radius, diameter, area, circumference, center, and chord ▪ Identify pi (n) ▪ Know the formulas for area and circumference of a circle ▪ Justify that pi (n) can be derived from the circumference and diameter of a circle	Examples: • The seventh grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size? • Students measure the circumference and diameter of several circular objects in the room (clock, trash can, door knob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures. Students write an expression that could be used to find the circumference of a circle with any diameter and check their expression on other circles. • Students will use a circle as a model to make several equal parts as you would in a pie model. The greater number the cuts, the better. The pie pieces are laid out to form a shape similar to a parallelogram. Students will then write an expression for the area of the parallelogram related to the radius (note: the length of the base of the parallelogram is half the circumference, or πr, and the height is r, resulting in an area of πr^2. Extension: If students are given the circumference of a circle, could they write a formula to determine the circle's area or given the area of a circle, could they write the formula for the circumference? 

Geometry	**7.G**
Draw, construct, and describe geometrical figures and describe the relationships between them.	

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular**

Standards/Learning Objectives	***Explanations and Examples***
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. ▪ Identify and recognize types of angles: supplementary, complementary, vertical, adjacent ▪ Determine complements and supplements of a given angle ▪ Determine unknown angle measures by writing and solving algebraic equations based on relationships between angles	Angle relationships that can be explored include but are not limited to: • Same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary. Examples: • Write and solve an equation to find the measure of angle *x.*  • Write and solve an equation to find the measure of angle *x.* 
7.G.6. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students understanding of surface area can be supported by focusing on the sum of the area of the faces. Nets can be used to evaluate surface area calculations. Examples: • Choose one of the figures shown below and write a step by step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?  

Geometry Draw, construct, and describe geometrical figures and describe the relationships between them.	**7.G**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular**	

Standards/Learning Objectives	***Explanations and Examples***
	• A cereal box is a rectangular prism. What is the volume of the cereal box? What is the surface area of the cereal box? (Hint: Create a net of the cereal box and use the net to calculate the surface area.) Make a poster explaining your work to share with the class. • Find the area of a triangle with a base length of three units and a height of four units. • Find the area of the trapezoid shown below using the formulas for rectangles and triangles. 

Statistics and Probability Use random sampling to draw inferences about a population.		**7.SP**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **random sampling, population, representative sample, inferences**		
Standards/Learning Objectives		***Explanations and Examples***
7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. ▪ Know statistics terms such as population, sample, sample size, random sampling, generalizations, valid biased and unbiased ▪ Recognize sampling techniques such as convenience, random, systematic, and voluntary		Example: • The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of sampling used in each survey option. Which survey option should the student council use and why? 1. Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey. 2. Survey the first 20 students that enter the lunch room.

Geometry Draw, construct, and describe geometrical figures and describe the relationships between them.	**7.G**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular**

Standards/Learning Objectives	***Explanations and Examples***
7.SP.2. Use data, including Montana American Indian demographic data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* ▪ Define random sample ▪ Identify an appropriate sample size ▪ Analyze and interpret data from a random sample to draw inferences about a population with an unknown characteristic of interest	Example: • Below is the data collected from two random samples of 100 students regarding student's school lunch preference. Make at least two inferences based on the results.

Lunch Preferences

student sample	hamburgers	tacos	pizza	total
#1	12	14	74	100
#2	12	11	77	100

Statistics and Probability **7.SP**

Draw informal comparative inferences about two populations.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **variation/variability, distribution, measures of center, measures of variability**

Standards/Learning Objectives

7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

- Identify measures of central tendency (mean, median, and mode) in a data distribution
- Identify measures of variation including upper quartile, lower quartile, upper extreme-maximum, lower extreme-minimum, range, interquartile range, and mean absolute deviation
- Compare two numerical data distributions on a graph by visually comparing data displays, and assessing the degree of visual overlap

Explanations and Examples

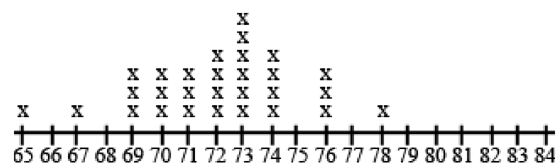
Students can readily find data as described in the example on sports team or college websites. Other sources for data include American Fact Finder (Census Bureau), Fed Stats, Ecology Explorers, USGS, or CIA World Factbook. Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistic functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations.

Example:
Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but doesn't know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists.

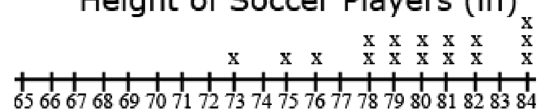
Basketball Team – Height of Players in inches for 2010-2011 Season
75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84

Soccer Team – Height of Players in inches for 2010
73, 73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69

To compare the data sets, Jason creates a two dot plots on the same scale. The shortest player is 65 inches and the tallest players are 84 inches.



Height of Soccer Players (in)



Height of Basketball Players (in)

Statistics and Probability Draw informal comparative inferences about two populations.	**7.SP**

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **variation/variability, distribution, measures of center, measures of variability**

Standards/Learning Objectives	***Explanations and Examples***
	In looking at the distribution of the data, Jason observes that there is some overlap between the two data sets. Some players on both teams have players between 73 and 78 inches tall. Jason decides to use the mean and mean absolute deviation to compare the data sets. Jason sets up a table for each data set to help him with the calculations. The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches. The mean absolute deviation (MAD) is calculated by taking the mean of the absolute deviations for each data point. The difference between each data point and the mean is recorded in the second column of the table. Jason used rounded values (80 inches for the mean height of basketball players and 72 inches for the mean height of soccer players) to find the differences. The absolute deviation, absolute value of the deviation, is recorded in the third column. The absolute deviations are summed and divided by the number of data points in the set. The mean absolute deviation is 2.53 inches for the basketball players and 2.14 for the soccer players. These values indicate moderate variation in both data sets. There is slightly more variability in the height of the soccer players. The difference between the heights of the teams is approximately 3 times the variability of the data sets (7.68 ÷ 2.53 = 3.04). Continued on next page

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **variation/variability, distribution, measures of center, measures of variability**

Standards/Learning Objectives	***Explanations and Examples***

Soccer Players (n = 29)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
65	-7	7
67	-5	5
69	-3	3
69	-3	3
69	-3	3
70	-2	2
70	-2	2
70	-2	2
71	-1	1
71	-1	1
71	-1	1
72	0	0
72	0	0
72	0	0
72	0	0
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
74	+2	2
74	+2	2
74	+2	2
74	+2	2
76	+4	4
76	+4	4
76	+4	4
78	+6	6
Σ = 2090		Σ = 62

Mean = 2090 ÷ 29 =72 inches
MAD = 62 ÷ 29 = 2.14 inches

Basketball Players (n = 16)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
73	-7	7
75	-5	5
76	-4	4
78	-2	2
78	-2	2
79	-1	1
79	-1	1
80	0	0
80	0	0
81	1	1
81	1	1
82	2	2
82	2	2
84	4	4
84	4	4
84	4	4
Σ = 1276		Σ = 40

Mean = 1276 ÷ 16 =80 inches
MAD = 40 ÷ 16 = 2.53 inches

Statistics and Probability Draw informal comparative inferences about two populations.		**7.SP**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **variation/variability, distribution, measures of center, measures of variability**		
<u>Standards/Learning Objectives</u>		***<u>Explanations and Examples</u>***
7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* ▪ Find measures of central tendency (mean, median, and mode) and measures of variability (range, quartile, etc.) ▪ Analyze and interpret data using measures of central tendency and variability		Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range. Example: • The two data sets below depict random samples of the housing prices sold in the King River and Toby Ranch areas of Arizona. Based on the prices below, which measure of center will provide the most accurate estimation of housing prices in Arizona? Explain your reasoning. ○ King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000} ○ Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000}

Statistics and Probability **7.SP**

Investigate chance processes and develop, use, and evaluate probability models.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **sample spaces**

See list from essential standards work.

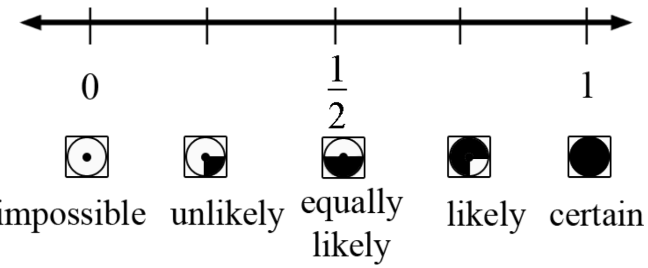
Standards/Learning Objectives

7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Explanations and Examples

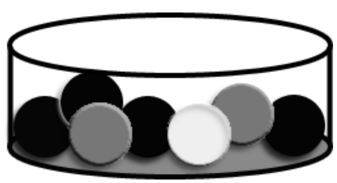
Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line. Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

Marble Mania http://www.sciencenetlinks.com/interactives/marble/marblemania.html
Random Drawing Tool **-** http://illuminations.nctm.org/activitydetail.aspx?id=67

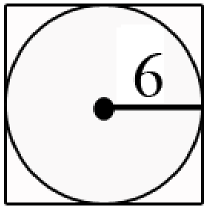


Example:

- The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.



Statistics and Probability Investigate chance processes and develop, use, and evaluate probability models.		7.SP
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **sample spaces** **See list from essential standards work.**		
Standards/Learning Objectives		***Explanations and Examples***
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses.*		Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies. Example: Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities (How many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?). Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, 3 blue marbles.) Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.

Statistics and Probability Investigate chance processes and develop, use, and evaluate probability models.		**7.SP**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **sample spaces** **See list from essential standards work.**		
Standards/Learning Objectives		***Explanations and Examples***
7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*		Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web-based simulations. Students can also develop models for geometric probability (i.e. a target). Example: • If you choose a point in the square, what is the probability that it is not in the circle? 

Statistics and Probability **7.SP**

Investigate chance processes and develop, use, and evaluate probability models.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **sample spaces**

See list from essential standards work.

<u>Standards/Learning Objectives</u>

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

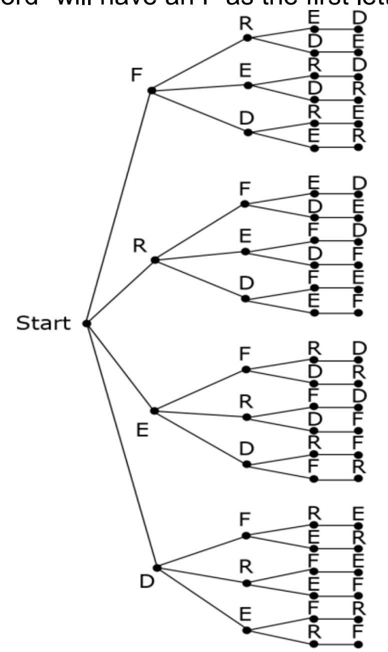
c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

<u>Explanations and Examples</u>

Examples:

- Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles and two purple marbles. Students will draw one marble without replacement and then draw another. What is the sample space for this situation? Explain how you determined the sample space and how you will use it to find the probability of drawing one blue marble followed by another blue marble.

- Show all possible arrangements of the letters in the word FRED using a tree diagram. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your “word” will have an F as the first letter?



GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. 2 numbers whose sum is 0 are additive inverses of one another.Example: 3/4 and – 3/4 are additive inverses of one another because3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

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[3]Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

TABLES

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. **Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here a, b and c stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

<table>
<tr><th colspan="10">Mathematics Learning Progressions by Domain</th></tr>
<tr><td>K</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>HS</td></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">Ratios and Proportional Relationship</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td colspan="2">Functions</td></tr>
<tr><td colspan="10">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="4">Statistics and Probability</td></tr>
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