Strategies your Child might Use for...
ADDITION

Count all

Counting Objects - Addition

Look at the number sentence. Make a group of the first number and a group of the second number.
Put the two groups together. Count all. How many altogether?

4 + 2 = 6

Counting Objects – Subtraction

Look at the number sentence. Make a group of the first number. Take away the second number from this group.
How many are left?

6 – 4 = 2

Counting using fingers - Addition

Look at the number sentence. Hold up the first number on your fingers then add the second number on your fingers. Count all. How many altogether?

3 + 2 = 5

Counting using fingers – Subtraction

Look at the number sentence. Hold up the first number on your fingers then put down the second number of fingers. How many left?

5 - 2 = 3

Number Bonds

A number bond is a mental picture of the relationship between a number and the parts that combine to make it.
The concept of number bonds is very basic, an important foundation for understanding how numbers work. Something 'whole' is made up of parts. If you know the parts, you can put them together (add) to find the whole. If you know the whole and one of the parts, you take away the part you know (subtract) to find the other part.
Sometimes known as number 'friends' for Key Stage 1 children, where the children are encouraged to see the relationship between the numbers, who all hold hands within the number sentence (calculation) because of their 'friendship'. Number bonds are initially taught with plenty of practical opportunities for children to explore splitting up and recombining.

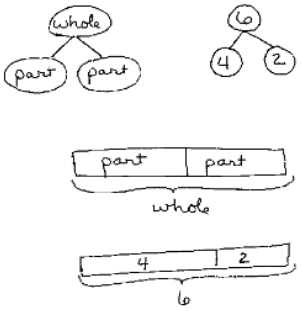
Number bonds are often started to 5, i.e.
0+5=5
1+4=5
2+3=5
3+2=5
4+1=5
etc

Then, children are extended in the same way to other numbers under 10, then, number bonds to 10 i.e.

0+10=10
1+9=10
2+8=10
3+7=10
4+6=10
Etc

2 and 8 are 'friends' with 10, and so on.

When learning their bonds to each number under 10, children are encouraged to use their fingers, counting how many fingers are up and how many are down to explore the relationship.



3+7=10

Some ways of recording bonds

When bonds to 10 are solid, children will then start to explore the 'inverse' relationships between the numbers, consolidating understanding that the biggest number must come first when taking away i.e.

10-7=3
10-3=7
7+3=10

This helps children to see that addition can be done in any order, however, the largest number must come at the end as a result of adding numbers together.

These are basic conventions of adding and taking away that the children will need to use throughout their calculating experiences.

Number bonds then go further, to 20, for example, where children may use what they know about bonds to 10 to help them, i.e.
8+**2**=10 so 8+**12**=20

Again, children can use what they know to derive inverse calculations.

After this, children begin to explore number bonds to other multiples of ten, always starting with the basic knowledge of bonds to 10 (this time, units numbers can often aid calculating) i.e.

7<u>3</u>+<u>7</u>=80
2<u>5</u>+2<u>5</u>=50
6<u>4</u>+3<u>6</u>=100

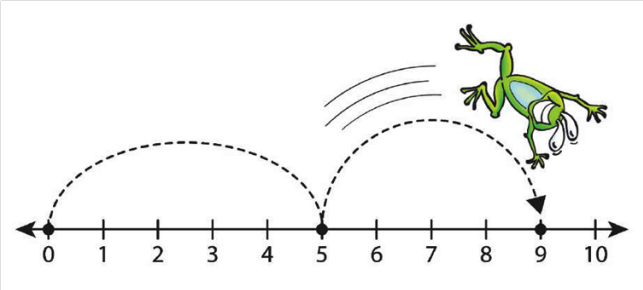
Learning number bonds allows children to calculate at speed, developing mental dexterity and agility, as well as confidence when working in this capacity.

Possible resources used at school include (for bonds to 10 and 20):

- Bead strings,
- Coat hangers with pegs,
- Number line,
- Dice,
- Digit cards,
- Anything where children can physically separate and recombine objects in different ways.

Number Lines for Addition

Number lines are an effective tool to aid simple addition. Number lines usually run from 0 to 20 or 0 to 50, although older children may also use number lines with negative numbers. Many different designs are available to appeal to an individual's preferences.
Children will often use the term 'jumps', 'jumping forward' or 'counting on' or even 'bunny hops'; and will be encouraged to put their finger or an object on the start number and move forward the number of spaces for the number they are adding.



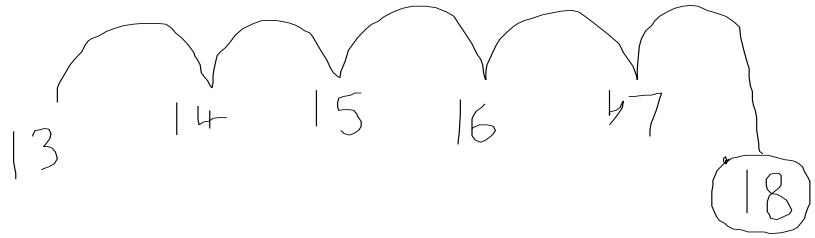
Number lines can also be created in an outside environment (using chalk) where children will physically jump forwards to represent an addition calculation. Physically moving really helps children to understand that addition is moving forward.
A common misconception in early development is 'jumping on the spot' whereby children do not move forward for their initial move.
Empty number lines (when the child fills in the numbers themselves) are a good starting point in assessing children's abilities to order small numbers and to check if numbers are being written back to front or transposed (31 instead of 13).

Next Steps

Blank number lines for addition
As children become more aware of the number system they are encouraged to create their

own number line for an addition calculation. Rather than starting at zero, they will put the first number at the start of their number line and jump forward the number to add before going back to 'fill in the gaps' to find the answer.

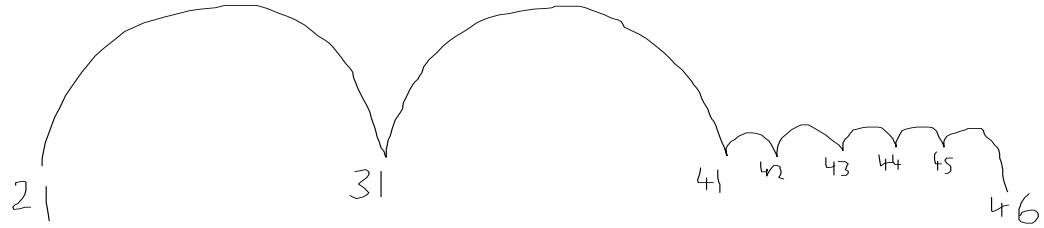
13 + 5



Next Steps

Blank number lines using Tens and Ones (Units)
When children are more familiar with place value they will begin to recognise that jumping forward ten at a time is much more efficient than making ten individual jumps. As they become more confident they will be encouraged to create their own number line to add and will put their starting number first and make a 'big jump' to represent adding a ten and 'little jumps' to show adding the ones. This method requires children to have a sound knowledge of '10 more' and '1 more' than any given number.

21+ 25



A common misconception here is when children draw the number line correctly and fill in the tens numbers, but continue counting in tens rather than reverting to ones.

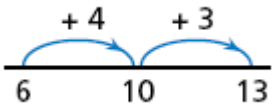
Bridging Multiples of Ten

An important aspect of having an appreciation of number is to know when a number is close to 10 or a multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 92 is 2 away from 90.

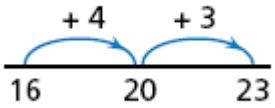
When adding or subtracting mentally, it is often useful to make use of the fact that one of the numbers is close to 10 or a multiple of 10 by splitting up another number to provide the difference. The use of an empty number line where the multiples of 10 are seen as

'landmarks' is helpful and enables children to have an image of jumping forwards or backwards to these 'landmarks'.

For example, 6 + 7 = 6 + 4 + 3



In the case of subtraction, bridging through the next 10 or multiple of 10 is a very useful method (often termed 'shopkeeper's subtraction'; it is the method used almost universally with money). So the change from £1 for a purchase of 37p is carried out thus: '37 and 3 is 40 and 10 is 50 and 50 is £1'. The use of actual coins, or the image of coins, helps to keep track of the subtraction. The empty number line can provide an image for this method when the subtraction does not involve money. The calculation 23 – 16 can be built up as an addition:

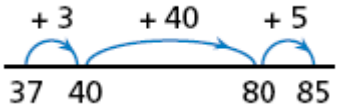


'16 and 4 is 20 and 3 is 23, so add 4 + 3 for the answer.'

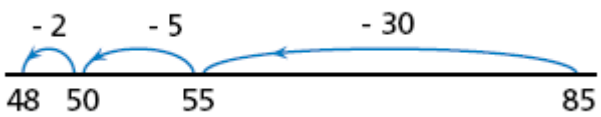
A similar method can be applied to the addition and subtraction of decimals, but here, instead of building up to a multiple of 10, numbers are built up to a whole number or to a tenth.

So, 2.8 + 1.6 can be turned into 2.8 + 0.2 + 1.4 = 3 + 1.4
Children will also be taught to use empty number lines for addition and subtraction, using multiples of 10 as interim numbers:

Example: 85 – 37.
(i) by adding on from 37 to 85 (the shopkeepers' method):



37 and 3 makes 40 and 40 make 80 and 5 makes 85. So add 3 + 40 + 5 to get the answer'
(ii) by counting backwards:



Adding by Partitioning

Partitioning simply means 'splitting' a number into its tens and units (or hundreds, tens and units) components. Understanding what a number is *really* worth or the pieces it is *really* made up of is fundamental in children's ability to understand calculations and to appreciate the method.

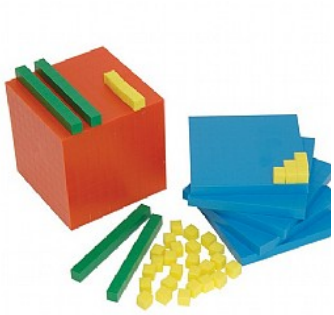
First Steps

Children have a two digit number and identify the tens number and the units number. They will understand that in 13 the first number is not 1, but 1 ten. Children will then progress into identifying the true value of a hundreds number and may label the number with HTU.

Next Steps

Children will be able to write a 3 digit number in words. 145 is 1 hundred, 4 tens and 5 units or as a number sentence.

145 = 100 + 40 + 5



Children may also represent this using HTU apparatus or 'Dienes' (named after the man who invented the apparatus). This apparatus provides an image of "how many" and provides a sense of large numbers by showing them comprised of 1s, 10s, and 100s. It is important for children to have a sense of the size of a number and to be able to have an image of large numbers which is more than just a large collection of 1s. Diene's apparatus provides such an image. Inviting children to count certain quantities using this material forces the children to become more aware of the need to count in 100s, 10s and 1s. Visual tools are always very helpful in understanding a concept. Children are also encouraged to draw a visual picture of what a number 'looks' like in its partitioned form. Children sometimes refer to this as 'chips and peas' as the ten rods look like chips and the units look like peas!

Next Steps

After being able to partition a number into tens and units, children progress to adding by partitioning whereby they combine all the tens in a calculation, then put the units together. So 34 add 22 become 30 add 20 plus 4 add 2. This is often introduced using visual prompts such as Diene's apparatus where children physically move all the ten rods together then group all the units together to then 'see' the total number.

Next Steps

When children are more familiar with place value they will be able to apply this knowledge to add on a number line, recognising that jumping forward ten at a time is much more efficient than making ten individual jumps. As they become more confident, they will be encouraged to create their own number line to add and will put their starting number first and make a 'big jump' to represent adding a ten and 'little jumps' to show adding the units.

Understanding

In early mathematics we always encourage children to write a calculation going across the page. We do not encourage column addition for younger children. Although column addition is a quick and simple way of generating an answer, it does not support a real understanding of the value of numbers. The biggest problem for many children is understanding, and being very secure in this understanding, place value. So they need to totally understand that 21 isn't "a two and a one" - it's 20 and 1 more. If they don't grasp this then there's so much maths they won't understand down the line. We very much work to the ethos that the process is far more important than the getting the correct answer. Encouraging understanding right from the start builds the solid foundations that will help these learners in the future, particularly when they come to perform more complicated multiplication calculations in Key Stage 2 and beyond.

Column Methods of Addition

A column method shows, for example, the addition of the 'tens' to the 'tens' and the 'units' to the 'units' separately. To find the partial sums, either the tens or the ones can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the 'units' digits first always.
The addition of the 'tens' in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.

The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Write the numbers in columns.

Adding the 'tens' first:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$$

Adding the 'ones' first:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

Discuss how adding the 'ones' first gives the same answer as adding the 'tens' first.
The column method is further refined later on in KS2 so that recording is reduced. Any digits that are 'carried over' are recorded below the line, using the language 'carry ten' or 'carry one hundred', not 'carry one'. Such problems are differentiated to include the addition of three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \hline 11 \end{array} \qquad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \hline 11 \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \hline 11 \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.