

# Summer Refresher For Incoming PreCalculus Students

## LINEAR EQUATIONS

### Exercise 31. One Variable

Written

**Example 1:** Solve:  $\frac{x+3}{4} - \frac{x+4}{6} = \frac{x}{2} - 2$ .

Multiply both sides by the L.C.D. of 12. Indicate the multiplications, then do it in the next step.

$$3(x+3) - 2(x+4) = 6x - 24$$

$$3x + 9 - 2x - 8 = 6x - 24$$

$$-5x = -25$$

$$x = 5$$

Solve each of the following on the domain of rational numbers.

1.  $\frac{x}{5} + \frac{x}{2} = 7$

2.  $\frac{a}{3} - 2a = -10$

3.  $\frac{m}{6} = \frac{m}{2} - \frac{1}{3}$

4.  $\frac{x}{4} - x = \frac{x}{3} + \frac{1}{2}$

5.  $\frac{x}{7} - \frac{x}{3} = \frac{2}{3}$

6.  $\frac{b}{15} - \frac{2b}{5} + 3 = 0$

7.  $\frac{a}{11} + \frac{3a}{2} = \frac{7}{4}$

8.  $\frac{x+1}{6} - \frac{x}{8} = -\frac{3}{4}$

9.  $\frac{x+2}{12} = \frac{x-1}{9}$

10.  $\frac{2m-1}{3} - \frac{m}{2} = \frac{m-2}{5}$

11.  $2a + 3 - \frac{a-1}{6} - \frac{a}{4} = 0$

12.  $\frac{7x-4}{3} - x + \frac{x}{5} = 4 - \frac{x}{3}$

13.  $\frac{4x}{9} - \frac{10}{3} = \frac{x-7}{6}$

14.  $\frac{2p-7}{13} - \frac{p}{2} - \frac{16}{13} = 0$

15.  $\frac{9a-3}{2} = -\frac{3a-6}{9} - \frac{13}{6}$

**Example 2:** Solve  $\frac{3}{4}(x-1) = \frac{1}{3}(x-2)$ , on the domain

$$\{-2, -\frac{1}{2}, \frac{1}{10}, \frac{5}{6}, 2, 4, 1, 6, 14, 46, 5\}.$$

Remove the parentheses.

$$\frac{3x}{4} - \frac{3}{4} = \frac{x}{3} - \frac{2}{3}$$

Clear of fractions, using the L.C.D. of 12.

$$9x - 9 = 4x - 8$$

$$5x = 1$$

$$x = \frac{1}{5}$$

Since  $\frac{1}{5} \notin D$ , the solution set is empty.

EXERCISES IN ELEMENTARY ALGEBRA

Written

**Example 1:** Solve for x and y:  $\begin{cases} 3x - 2y = 5 \\ 4x + 5y = -1 \end{cases}$

*Addition and Subtraction Method.* Multiply both equations to make the coefficients of x (or y if it is easier) the same, then add or subtract the two equations to eliminate that variable. In this case we multiply the first equation by 5 and the second by 2, then add to eliminate y.

$$\begin{array}{r} 15x - 10y = 25 \\ 8x + 10y = -2 \\ \hline 23x = 23 \\ x = 1 \end{array}$$

Substitute in either equation to find y.

$$\begin{array}{r} 3 \cdot 1 - 2y = 5 \\ -2y = 2 \\ y = -1 \end{array}$$

**Ans.** (1, -1)

**Example 2:** Solve for x and y:  $\begin{cases} 2x - 3y = 0 & (1) \\ x + y = 5 & (2) \end{cases}$

*Substitution Method.* If it easy to do so, solve one equation for x or y, (whichever is easier) and substitute into the other equation.

$$\begin{array}{r} (2) \quad y = 5 - x \\ (1) \quad 2x - 3(5 - x) = 0 \\ \quad 2x - 15 + 3x = 0 \\ \quad 5x = 15 \\ \quad x = 3 \end{array}$$

Substitute in either equation to find y.

$$\begin{array}{r} 2 \cdot 3 - 3y = 0 \\ -3y = -6 \\ y = 2 \end{array}$$

**Ans.** (3, 2)

Solve for x and y by the easiest method.

1.  $\begin{cases} x + 3y = 6 \\ 2x - y = 5 \end{cases}$

2.  $\begin{cases} 5x - 3y = 14 \\ 2x - y = 6 \end{cases}$

3.  $\begin{cases} -x + 4y = 7 \\ 3x - 2y = 9 \end{cases}$

4.  $\begin{cases} 2x - 7y = -11 \\ -5x + 3y = 13 \end{cases}$

5.  $\begin{cases} -6x - 7y = -3 \\ 4x - 3y = 25 \end{cases}$

6.  $\begin{cases} 7x - 3y = 1 \\ x + 4y = 9 \end{cases}$

7.  $\begin{cases} 12x - 6y = -2 \\ -9x - 7y = -10 \end{cases}$

8.  $\begin{cases} -6x + 10y = 3 \\ 18x - 10y = 1 \end{cases}$

9.  $\begin{cases} 8x = 3y \\ 5x - 2y = -1 \end{cases}$

Written

1. a) Plot the points (0, 0), (1, 2), (3, 6), (-2, -4).
- b) Plot the points (1, 2), (2, 0), (3, -2), (4, -4).
- c) Plot the points (-6, -5), (-3, -2), (0, 1), (3, 4).

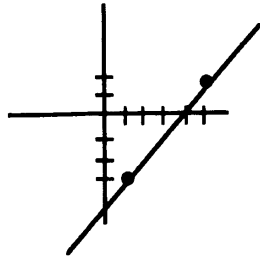
**Example 1:** Consider the points (5,2), and (2,-3). Determine:  
a) the slope of the line through the two points,  
b) an equation of the line and  
c) its graph.

a) Slope:  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-3)}{5 - 2} = \frac{5}{3}$

b) Use the point-slope formula:  $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{5}{3}(x - 5)$$

c)



For each of the pairs of points in problems 2-15:

- a) Determine the slope.
  - b) Determine the equation of the line through the two points.
  - c) Draw the graph.
2. (2,3), (0,0)
  3. (-1,5), (0,0)
  4. (6,2), (5,2)
  5. (6,2), (5,1)
  6. (-1,2), (3,5)
  7. (-2,-3), (-5,-4)
  8. (-2,-5), (1,-7)
  9. (9,-3), (7,-1)
  10. (73,27), (23,-13)
  11. (17,-5), (6,-13)
  12. (5,-2), (5,3)
  13. (-2,3), (4,-3)
  14. (0,5), (3,-7)
  15. (-2,1), (-2,0)

## GRAPHS

For each of the equations in problems 16–25:

- a) determine the slope of the graph.
- b) determine the y-intercept of the graph.
- c) determine the x-intercept of the graph.
- d) draw the graph.

16.  $y = 2x + 3$

17.  $y = 2x + 5$

18.  $y = -3x + 6$

19.  $2x - 3y = 12$

20.  $\frac{x}{3} + \frac{y}{2} = 1$

21.  $x = 2y + 4$

22.  $\frac{2x}{3} + \frac{3y}{4} = 1$

23.  $3x + 2y = 5$

24.  $2x - 3y = 0$

25.  $4x - 6y = 10$

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### EXERCISES IN ELEMENTARY ALGEBRA

*Written*

Factor completely. Some are prime.

1.  $x^2 + 16x + 64$

2.  $x^2 + x - 6$

3.  $12x^2 + 11x + 2$

4.  $36a^2 - 49b^2$

5.  $a^3 - 8$

6.  $a^4 - 81$

7.  $4x^2 + 7x - 36$

8.  $6x^2 - x - 77$

9.  $a^2b^2 + 19a^2b - 42a^2$

10.  $16 + a^3b^3$

11.  $(a^2 + ac) + (3ab + 3bc)$

12.  $(4x^2 - 25y^2) + (2x - 5y)$

13.  $p^4 - 17p^2 + 16$

14.  $(a + b)^2 + 2(a + b) + 1$

15.  $(x + a)(y - b) - (x - a)(y - b)$

16.  $(a - 1)(a^2 - 2) + (a - 1)a$

17.  $4ab - 4ac - 8b^2 + 8bc$

18.  $x^2 - x - 110$

19.  $121a^4 - 44a^2 + 4$

20.  $10a^2 + 28a - 6$

21.  $-1 - a + 6a^2$

22.  $a^4 - (a - 2)^2$

23.  $16x^4 - 8x^2 + 1$

24.  $a^4 - 6a^2 + 9 - 4a^2$

25.  $9p^2 - 13p + 4$

26.  $(a^2 - 9) - (a^2 - 6a + 9)$

Determine an equation for each of the following lines.

34. The line through  $(0, 0)$  with slope  $-1$ .

35. The line through  $(2, 3)$  with slope  $-1$ .

36. The line through  $(-2, 3)$  with slope  $-1$ .

37. The line through  $(2, 3)$  with slope  $3$ .

APC