AP CALCULUS
PREREQUISTE REVIEW PACKET

For students in entering ***AP CALCULUS BC***

Name: ____

Summer Calculus Packet

This summer work is intended to be completed towards the end of the summer. Don't do it now and forget about it! These questions are intended as review (with one or two exceptions) and are due on **loose leaf paper (NO fuzzy edges)** on the **first day of school zero period, Thursday August 24th.** Most answers will be posted on the BC Calculus Haiku page the afternoon of August 24. If you have any questions or concerns about how to do these (or even, how to get started or ***anything***), please email me after August 1 at dlissner@orangeusd.org. It is expected that, through any resources available, that you will have **completed AND UNDERSTOOD** these problems **BEFORE** the start of school. We will have on a couple days of review and at least one Saturday review day to make sure everyone starts at the same place. We will embark on **new** material starting on the first full week of school. Whether you are coming from Calculus AB or Honors Pre-Calculus, you can succeed!! Please, please ask all questions so that this material forms a common playing field for everyone. You may help each other with ideas, but NOT the details. Make sure your answers truly make sense and you are not just plugging and chugging since this will certainly come back to haunt you. **ASK ALL YOUR QUESTIONS BEFORE AUGUST 24. SHOW YOUR WORK IN AN ORGANIZED FASHION WITH PROBLEMS IN NUMERICAL ORDER.**

You may use a calculator on problems marked with an *.

PLEASE USE LOOSE LEAF PAPER.

Be sure to complete the last page of this assignment.

Have a great summer!

1. This packet is to be handed in AS YOU WALK IN THE DOOR the first day of period 0, Thursday Aug. 24th.
2. Completion of this packet is worth one-half of a major test grade and will be counted as your first grade.
3. There will be test on this review material some time the first full week of class.

The AP Calculus Exam
How, not only to Survive, but to Prevail…

The AP Calculus exam is the culmination of all of the years you've spent in high school studying mathematics. It's all led up to this. The calculus you study in the last year completes the prior years of preparation……. Keep these things in mind as you go through the year.

Everything in calculus, and mathematics in general, is best understood verbally, numerically, analytically (that is, through the use of equations and symbols) and graphically. Look at everything from these perspectives. Look at the relationships among them — how the same idea shows up in words, in equations, in numbers and in graphs.

For example: numerically a linear function is one which when written as a table of values, regular changes in the x-values produce regular changes in the y-values. Graphically a linear function has a graph that is a straight line. Analytically it is one whose equation can be written as $y = mx + b$. And the three ways are interrelated: The ratio of the changes in the table is the number m in the equation; the graph can be drawn using the number m by going up and over from one point to the next. The idea of the slope as "rise over run" expresses this verbally. Everything in mathematics and in the calculus works that way.

Learn the concepts — the exam emphasizes concepts.

Learn the procedures and formulae — even though the concepts are more important than the computations you still have to do the computations. Like it or not, learn to do the algebra, the arithmetic and the graphs.

Learn to be methodical — work neatly and carefully all year.

Think about what you are doing. Watch yourself work. It is natural to concentrate on the material you know and can do, but you need to concentrate on the things you do not (yet) know how to do. You can learn much from your mistakes. Look at a wrong answer as a green light to go in that direction until you've reached the right answer.

Excerpt from Lin McMullin's *Teaching AP Calculus*

I. Simplify 1 – 4. Show the work that leads to your answer.

1. $\dfrac{x-4}{x^2-3x-4}$

$\dfrac{x-4}{(x-4)(x+1)}$

$\dfrac{1}{x+1}$

2. $\dfrac{x^3-8}{x-2}$

$\dfrac{(x-2)(x^2+2x+4)}{x-2}$

x^2+2x+4

3. $\dfrac{5-x}{x^2-25}$

$\dfrac{-(x-5)}{(x-5)(x+5)}$

$-\dfrac{1}{x+5}$

4. $\dfrac{x^2-4x-32}{x^2-16}$

$\dfrac{(x-8)(x+4)}{(x-4)(x+4)}$

$\dfrac{x-8}{x-4}$

5. **Expand** $x^{\frac{3}{2}}\left(x+x^{\frac{5}{2}}-x^2\right)$

$x^{\frac{5}{2}}+x^4-x^{\frac{7}{2}}$

6. **Expand and then evaluate the sum** $\displaystyle\sum_{n=0}^{5}\frac{(n-1)^2}{2}$

$\frac{1}{2}+0+\frac{1}{2}+1+\frac{9}{2}+8=\frac{31}{2}$

7. **Factor and answer with only one negative exponent:** $2x^{-5}+4x^{-3}+2x^3$

$2x^{-5}\left(1+2x^2+x^8\right)$

8. **Factor and answer with only one negative exponent:** $2x^{-\frac{3}{4}}+8x^{-\frac{7}{4}}-6x^{\frac{1}{4}}$

$2x^{-\frac{7}{4}}\left(x+4-3x^2\right)=-2x^{-\frac{7}{4}}(3x-4)(x+1)$

II

1. Is $\left\{(x,y):y=\sqrt{4-x^2}\right\}$ a function and if so, what is its domain?

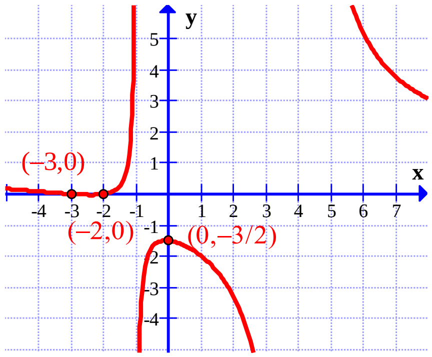
yes, $D:\{x\,|\,x\in\mathbb{R},\,-2\le x\le 2\}$

2. Given $F(x)=\sqrt{x+9}$, find and simplify $\dfrac{F(x+h)-F(x)}{h}$, $h\neq 0$

$$\frac{\sqrt{(x+h)+9}-\sqrt{x+9}}{h}=\frac{x+h+9-(x+9)}{h\left(\sqrt{x+h+9}+\sqrt{x+9}\right)}=\frac{1}{\sqrt{x+h+9}+\sqrt{x+9}}$$

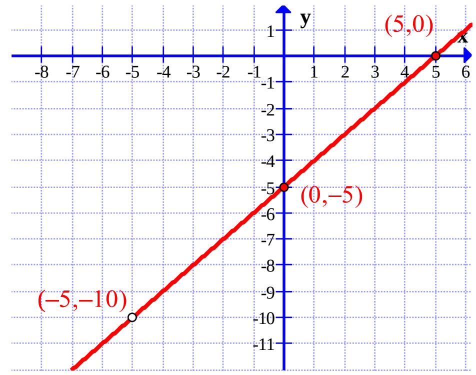
3. **Sketch the graph and determine its DOMAIN:**

a) $\dfrac{(x+3)(x+2)}{(x-4)(x+1)}$



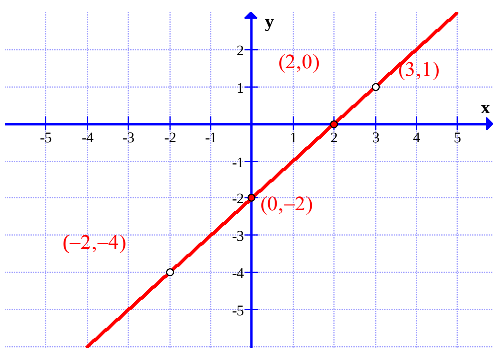
$D: -\infty < x < -1 \cup -1 < x < 4 \cup 4 < x < \infty$

b) $f(x) = \dfrac{x^2 - 25}{x+5}$



$D: \forall \mathbb{R},\ x \neq -5$

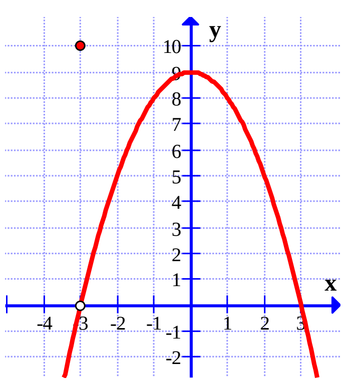
c) $f(x) = \dfrac{(x^2-4)(x-3)}{x^2 - x - 6}$



$D: \forall \mathbb{R},\ x \neq -2,\ x \neq 3$

Include the DOMAIN and RANGE also for the rest:

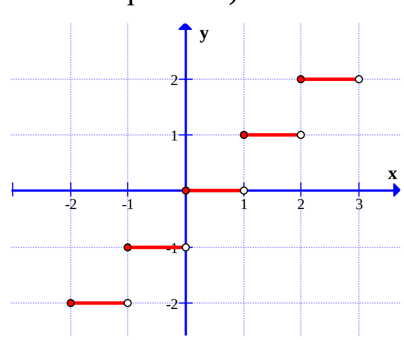
d) $f(x) = \begin{cases} 9 - x^2 & x \neq -3 \\ 10 & x = -3 \end{cases}$



$D: \forall \mathbb{R};$

$R: \forall \mathbb{R}, x \leq 9$ and $x = 10$

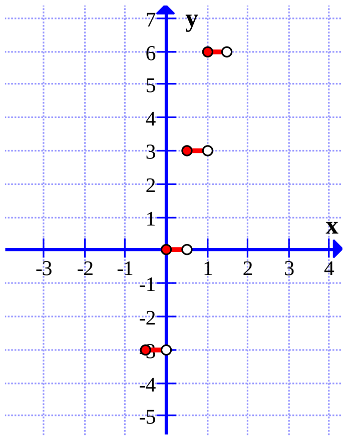
e) $f(x) = \text{int}\, x$ (int x means the greatest integer less than or equal to x)



$D: \forall \mathbb{R}$

R: all integers

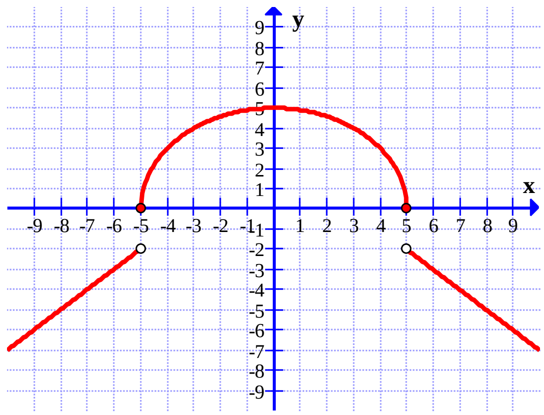
f) $f(x) = 3\operatorname{int}(2x)$



$D: \forall \mathbb{R}$

$R: \text{all } x,\ x = 3k$

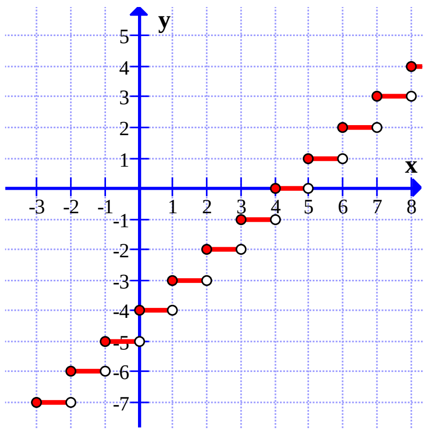
g) $f(x) = \begin{cases} x+3 & x < -5 \\ \sqrt{25-x^2} & -5 \le x \le 5 \\ 3-x & x > 5 \end{cases}$



$D: \forall \mathbb{R}$

$R: -\infty < y < -2 \cup 0 \le y \le 5$

h) $f(x) = \operatorname{int}(x-4)$

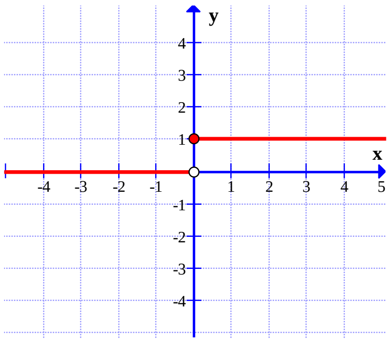
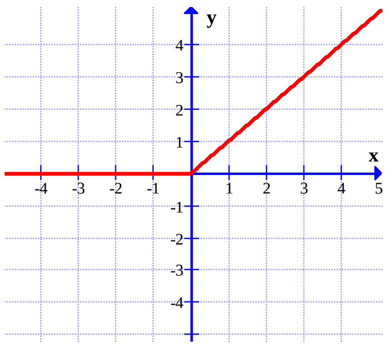
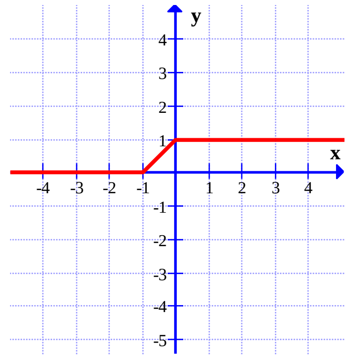


$D: \forall \mathbb{R}$

$R: \text{all integers}$

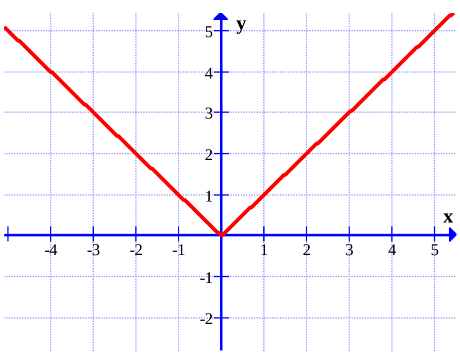
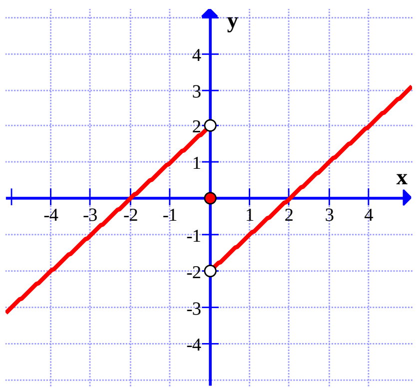
4. The **unit step function** is defined to be $U(x)=\begin{cases}0 & x<0\\ 1 & x\geq 0\end{cases}$. Sketch the graph of

a) $U(x)$ **b)** $xU(x)$ **c)** $(x+1)U(x+1)-xU(x)$

5. The **signum function** is defined to be $\operatorname{sgn}(x)=\begin{cases}-1 & x<0\\ 0 & x=0\\ 1 & x>0\end{cases}$. For each function below, sketch the graph.

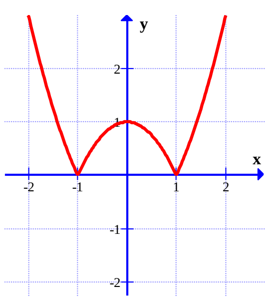
a) $x\operatorname{sgn}(x)$ **b)** $x-2\operatorname{sgn}(x)$

6. Write the following function as a piecewise function (like 3d or 3g) **without** using the absolute value symbol:

$f(x)=|x^2-1|$ Sketch a graph of your function.

$$f(x)=\begin{cases}x^2-1 & x<-1\\ 1-x^2 & -1\leq x\leq 1\\ x^2-1 & x>1\end{cases}$$



III

1. If $f(x)=\frac{x+1}{x-1}$ *and* $g(x)=\frac{1}{x}$, write a **formula** for and give the **domain** of:

a) $f \bullet g$

$$\left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right)=\frac{x+1}{x(x-1)}$$

$D: \forall\mathbb{R},\ x\neq 0,\ x\neq 1$

b) $f \circ g$

$$\frac{\frac{1}{x}+1}{\frac{1}{x}-1}=\frac{1+x}{1-x}$$

$D: \forall\mathbb{R},\ x\neq 1, x\neq 0$

c) $f \circ f$

$$\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}=\frac{x+1+x-1}{x+1-x+1}=\frac{2x}{2}=x$$

$D: \forall\mathbb{R},\ x\neq 1$

2. If $f(x)=\sqrt{x-2}$ *and* $g(x)=x^2-2$, write a **formula** for and give the **domain** of:

a) $f \circ g$

$$\sqrt{(x^2-2)-2}$$

$$\sqrt{x^2-4}$$

$D: x\leq -2 \cup x\geq 2$

b) $g \circ f$

$$\sqrt{x-2}^{\,2}-2$$

$$x-2-2$$

$$x-4$$

$D: x\geq 2$

c) $f \circ f$

$$\sqrt{\sqrt{x-2}-2}$$

$D: x\geq 6$

3. Express $h(x)=\sqrt{x^2-4}$ as the composite of two functions, *f* and *g*, i.e. $(f\circ g)(x)$, in **THREE** different ways.

i) $f(x)=\sqrt{x}$
$g(x)=x^2-4$

ii) $f(x)=\sqrt{x-4}$
$g(x)=x^2$

iii) $f(x)=\sqrt{x^2-4}$
$g(x)=x$

4. Is $f(x)=\sqrt[3]{x}$ even, odd, or neither? Prove analytically (algebraically) using the definition (**NOT** graphically or geometrically). Look up the definition if necessary.

odd:

$$f(-x)=\sqrt[3]{-x}=-\sqrt[3]{x}=-f(x)$$

5. Is $f(x)=\frac{x^2-5}{2x^3+x}$ even, odd, or neither? Prove analytically (**NOT** graphically)

$$f(-x)=\frac{(-x)^2-5}{2(-x)^3+(-x)}=\frac{x^2-5}{-2x^3-x}=-\frac{x^2-5}{2x^3+x}=-f(x)$$ -- therefore, this function is odd

6. Write the function $f(x)=|x-2|-|x+2|$ without using absolute value bars AND state whether $f(x)$ is even, odd, or neither with a justification for your answer.

$$f(x)=\begin{cases}4 & x\le -2\\ -x & -2<x<2\\ -4 & x\ge 2\end{cases}$$

odd because $f(-x)=-f(x)$

7. If f and g are two functions such that when composed in either order, the result is the identity function then f and g are inverses of each other. If $f(x)=x^2,\ x\le 0$ *and* $g(x)=-\sqrt{x}$, show that the functions are inverses analytically via the above definition. Be SURE the details are clear! Look up the definition if necessary.

$f(g(x))=(-\sqrt{x})^2=x>0$

$g(f(x))=-\sqrt{x^2}=-x$

but since the domain of f is $x<0$, this will be a positive number.

8. If $f(x)=x^2$, find TWO functions, g, for which $(f\circ g)(x)=4x^2-12x+9$.

i) $g(x)=2x-3$ ii) $g(x)=3-2x$

9. If $f(x)=\{(3,5),(2,4),(1,7)\}$, $g(x)=\sqrt{x-3}$, $h(x)=\{(3,2),(4,3),(1,6)\}$, $k(x)=x^2+5$, determine each of the following:

a) $(f+h)(1)=$

$(f+h)(1)=7+6=13$

b) $(k-g)(5)=$

$(k-g)(5)=$

$(5^2+5)-\sqrt{5-3}=$

$30-\sqrt{2}$

c) $(f\circ g)(3)=$

$f(g(3))=f(0)$

DNE

d) $(g\circ k)(7)=$

$g(k(7))=\sqrt{k(7)-3}$

$\sqrt{49+5-3}=\sqrt{51}$

e) $f^{-1}(x)=$

$\{(5,3),(4,2),(7,1)\}$

f) $g^{-1}(x)=$

x^2+3

g) $\dfrac{1}{f(x)}=\left\{\left(3,\frac{1}{5}\right),\left(2,\frac{1}{4}\right),\left(1,\frac{1}{7}\right)\right\}$

10. Write the inequality $|A|<B$ without absolute value bars.

$-B<A<B$

11. Write the inequality $|A|>B$ without absolute value bars.

$A<-B$ or $A>B$

12. Solve for *x*: $|2x-3|<5$

$$-5<2x-3<5$$
$$-2<2x<8$$
$$-1<x<4$$

13. Solve for *x*: $|3x-2|>5$

$$3x-2>5 \text{ or } 3x-2<-5$$
$$3x>7 \text{ or } 3x<-3$$
$$x>\frac{7}{3} \text{ or } x<-1$$

IV

1. The period (time for one complete oscillation) of a pendulum is directly proportional to the square root of the length of the pendulum. A pendulum of length 8 ft has a period of 2 sec. Find a mathematical model expressing the period as a function of the length **and** find the number of swings per second made by a pendulum of 2 ft in length.

$$p=k\sqrt{l}$$
$$2=k\sqrt{8}\rightarrow k=\frac{\sqrt{2}}{2}$$
1 complete oscillation per second
$$p=\frac{\sqrt{2}}{2}\sqrt{2}=1$$

2. The surface area of a sphere is given by $A(r)=4\pi r^2$. Suppose a balloon maintains the shape of a sphere as it is being inflated so that the radius is changing at the constant rate of 3 cm per second. If $f(t)$ centimeters is the radius of the balloon after *t* seconds:

   a) Compute $(A\circ f)(t)$ and interpret the result (what does it tell you?)

   b) Find the surface area of the balloon after 4 seconds.

$$(A\circ f)(t)=4\pi(f(t))^2$$
at $t=1$, $(A\circ f)(1)=4\pi(9)=36\pi$

at $t=2$, $(A\circ f)(2)=4\pi(36)=144\pi$

at $t=3$, $(A\circ f)(3)=4\pi(81)=324\pi$

$$(A\circ f)(t)=4\pi(3t)^2=36\pi t^2$$
$$(A\circ f)(4)=36\pi(16)=576\pi$$

On the next two problems, please solve these **without using any calculus** that you may know.

*3. A rectangular field is to be enclosed with 240 m of fence but one side of the rectangle is a river so the fencing only needs to be used on the other three sides. Express the area of the field as a function of the length (the dimension parallel to the river), graph the function and give, to the nearest tenth of a meter, the dimension of the field having the greatest area.

$2x + y = 240 \rightarrow y = 240 - 2x$

$A = (240-2x)(x) = 240x - 2x^2$ 60 m by 120 m

$x = 60$

*4. A manufacturer makes open tin boxes from pieces of tin that are 12 cm by 15 cm by cutting squares out of the corners and bending up the sides. Find the size, to the nearest 0.01 cm, of the cut-out squares in order that the volume of the box is as great as possible.

$V = x(15-2x)(12-2x)$ 2.21 cm by 2.21 cm squares

$x = 2.21$

V

Simplify.

1. $\dfrac{\sqrt{x}}{x}$ 2. $e^{\ln 3}$ 3. $e^{(1+\ln x)}$ 4. $\ln 1$ 5. $\ln e^7$

$x^{-\frac{1}{2}}$ 3 ex 0 7

6. $\log_3\left(\dfrac{1}{3}\right)$ 7. $\log_{1/2} 8$ 8. $\ln\left(\dfrac{1}{2}\right)$ 9. $e^{3\ln x}$

$\left(\dfrac{1}{2}\right)^x = 8$

-1 $2^{-x} = 2^3$ $-\ln 2$ x^3

$x = -3$

10. $\dfrac{4xy^{-2}}{12x^{-1/3}y^{-5}}$ 11. $27^{2/3}$ 12. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$ 13. $\left(4a^{5/3}\right)^{3/2}$

$\dfrac{1}{3}x^{\frac{4}{3}}y^3$ $3^2 = 9$ $20a^{13/6}$ $8a^{5/2}$

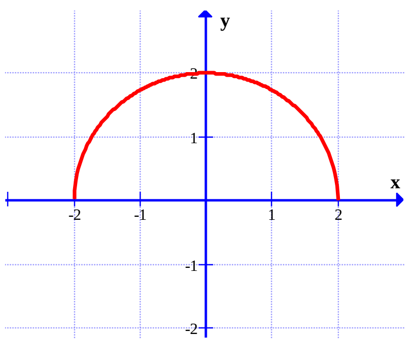
14. $\dfrac{3(n+2)!}{5n!}$ [NOTE: $5n! \neq (5n)!$] $= \dfrac{3(n+2)(n+1)}{5}$

VI

1. Write in slope intercept form the line perpendicular to $2x-3y=7$ and passing through $(5,1)$ by using the **POINT SLOPE** form of a straight line. (Look this up if you need to – it is the most common formula of a line we use.)

$$-3y=-2x+7$$
$$y=\frac{2}{3}x-\frac{7}{3}$$
$$y-1=-\frac{3}{2}(x-5)$$
$$y=-\frac{3}{2}x+\frac{17}{2}$$

2. Find the equation of a straight line (in slope intercept form) that is tangent to the circle of radius 2, centered at the origin at a point that is **in** the center of the second quadrant using the **POINT SLOPE** form of a straight line.



$$y-\sqrt{2}=1\left(x+\sqrt{2}\right)\rightarrow y=x+2\sqrt{2}$$

VII Without a calculator, determine the **exact** value of each expression. (Assume principal inverse values). BE SURE YOU **KNOW** THESE!!!

1. $\sin 0$ — 0
2. $\sin\frac{\pi}{2}$ — 1
3. $\sin\frac{3\pi}{4}$ — $\frac{\sqrt{2}}{2}$
4. $\cos\pi$ — -1
5. $\cos\frac{7\pi}{6}$ — $-\frac{\sqrt{3}}{2}$
6. $\cos\frac{\pi}{3}$ — $\frac{1}{2}$
7. $\tan\frac{7\pi}{4}$ — -1
8. $\tan\frac{\pi}{6}$ — $\frac{\sqrt{3}}{3}$
9. $\sec\frac{2\pi}{3}$ — -2
10. $\cos 0$ — 1
11. $\cos\left(\sin^{-1}\frac{1}{2}\right)$ — $\frac{\sqrt{3}}{2}$
12. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ — $-\frac{\pi}{6}$

VIII Solve for *x*, where *x* is a real number. Show the work that leads to your solution.

1. $x^2+3x-4=14$

$x^2+3x-18=0$

$(x-3)(x+6)=0$

$x=3$ or $x=-6$

2. $\dfrac{x^2-1}{x^3}=0$

$x=1$ or $x=-1$

3. $(x-5)^2=9$

$x-5=\pm 3$

$x-5=3$ or $x-5=-3$

$x=8$ or $x=2$

4. $2x^2+5x=8$

$2x^2+5x-8=0$

$$x=\frac{-5\pm\sqrt{25-4(2)(-8)}}{4}$$

$$x=\frac{-5\pm\sqrt{25+64}}{4}$$

$$x=\frac{-5\pm\sqrt{89}}{4}$$

5. $(x+3)(x-3)>0$

$x>3$ or $x<-3$

6. $x^2-2x-15\le 0$

$(x-5)(x+3)\le 0$

$-3\le x\le 5$

7. $(x+1)^2(x-2)+(x+1)(x-2)^2=0$

$(x+1)(x-2)[x+1+x-2]=0$

$(x+1)(x-2)(2x-1)=0$

$x=-1,\ x=2,\ x=\dfrac{1}{2}$

8. $(x-2)(x+3)^7(x-14)^{18}(x+11)^{29}x^{34}>0$

$-11<-3$ or $2<x<14$ or $x>14$

9. $27^{2x}=9^{x-3}$

$(3^3)^{2x}=(3^2)^{x-3}$

$3^{6x}=3^{2x-6}$

$6x=2x-6$

$4x=-6$

$x=-\dfrac{3}{2}$

10. $\log x+\log(x-3)=1$

$\log[x(x-3)]=1$

$10=x^2-3x$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5$ or ~~$x=-2$~~

11. $e^{3x}=5$

$3x=\ln 5$

$x=\dfrac{1}{3}\ln 5$

12. $\ln y=2x-3$

$\dfrac{3+\ln y}{2}=x$

IX State the following formulae: (from memory if at all possible – only look up what you MUST).
Be sure you know these by the first day of class! They will be assumed.

1. $\sin(A+B)=$ 2. $\cos(A+B)$

$\sin A\cos B+\sin B\cos A$ $\cos A\cos B-\sin A\sin B$

3. $\sin 2A=$ Show the derivation from #1 and #2 – be CLEAR.

$\sin(A+A)=\sin A\cos A+\sin A\cos A=2\sin A\cos A$

4. $\cos 2A=$ Show the derivation from #1 and #2 – be CLEAR. There are three different forms – know or derive them all.

$\cos(A+A)=\cos^2 A-\sin^2 A=1-2\sin^2 A=2\cos^2 A-1$

5. $\sec^2 A=$ Show the derivation from $\sin^2\theta+\cos^2\theta=1$, be CLEAR.

$$\frac{\sin^2 A}{\cos^2 A}+\frac{\cos^2 A}{\cos^2 A}=\frac{1}{\cos^2 A}$$

$$\tan^2 A+1=\sec^2 A$$

6. $\csc^2 A=$ Show the derivation from $\sin^2\theta+\cos^2\theta=1$, be CLEAR.

$$\frac{\sin^2 A}{\sin^2 A}+\frac{\cos^2 A}{\sin^2 A}=\frac{1}{\sin^2 A}$$

$$1+\cot^2 A=\csc^2 A$$

7. In what quadrant is the terminal side of a 100 radian angle that is in standard position?

$$100=2\pi x$$

$$x=\frac{100}{2\pi}=\frac{50}{\pi}\approx 15.915$$

so there are 15.915 rotations.

$\frac{3\pi}{2}<.915\text{ rotation}<2\pi$ **which implies the fourth quadrant**

X Please review these formulae – email me with questions after you have made a good attempt.

1. Write the first five terms of the geometric sequence in which $a=-81$ and $r=\frac{1}{3}$.

$$(-81)\left(\frac{1}{3}\right)^n=-81,\ -27,\ -9,\ -3,\ -1$$

2. Find the common ratio of a geometric series whose third term is -2 and whose sixth term is 54.

$$a_3=a_1r^3=-2\rightarrow a_1=-\frac{2}{r^3}$$

$$a_6=a_1r^6=54\rightarrow\left(-\frac{2}{r^3}\right)r^6=54$$

$$r^3=-27$$

$$r=-3$$

3. Find the sum of the infinite series $60-6+0.6+......$

$$\frac{-6}{60}=-.1 \qquad \frac{.6}{-6}=-.1$$

$$S=\frac{a_1}{(1-r)}=\frac{60}{1+.1}=\frac{600}{11}$$

4. Find the sum of the infinite series $3+\sqrt{3}+1+.......$

$$\frac{\sqrt{3}}{3} \qquad \frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$S=\frac{3}{1-\frac{\sqrt{3}}{3}}=\frac{9}{3-\sqrt{3}}\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)=\frac{9\left(3+\sqrt{3}\right)}{9-3}=\frac{9+3\sqrt{3}}{2}$$

5. Write the rational number $1.234234\overline{234}.....$ as a fraction in lowest terms.

$$1.234234\overline{234}=1+.234+.000234+.000000234.....$$

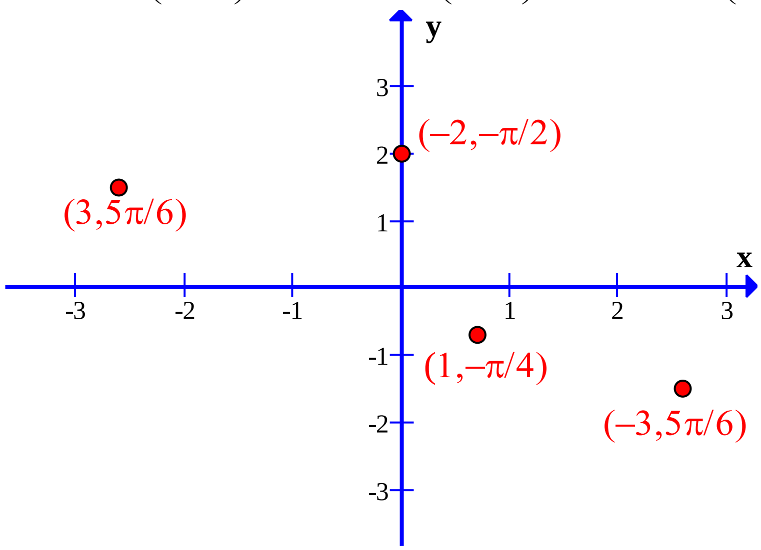
$$a_1=.234 \quad r=10^{-3}$$

$$S+1=\frac{.234}{1-.001}+1=\frac{.234}{.999}+1=\frac{1.233}{.999}=\frac{411}{333}=\frac{137}{111}$$

XI Please review Polar Functions from your Pre-Calculus year – email me with questions after you have made a good attempt.

1. Plot the polar points on a polar coordinate system superimposed on a Cartesian coordinate system:

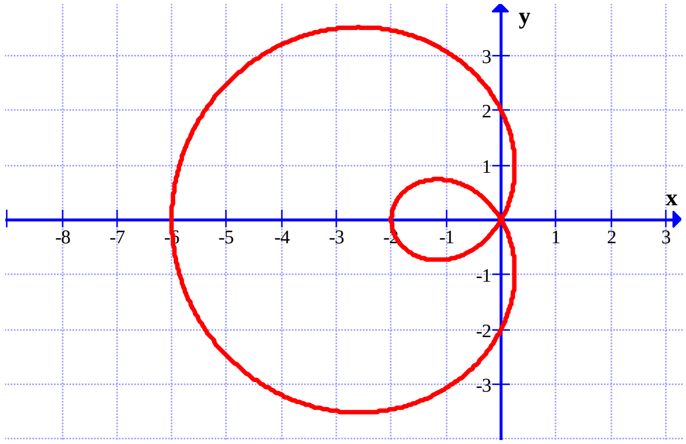
a) $\left(1,-\frac{\pi}{4}\right)$ **b)** $\left(3,\frac{5\pi}{6}\right)$ **c)** $\left(-3,\frac{5\pi}{6}\right)$ **d)** $\left(-2,-\frac{\pi}{2}\right)$



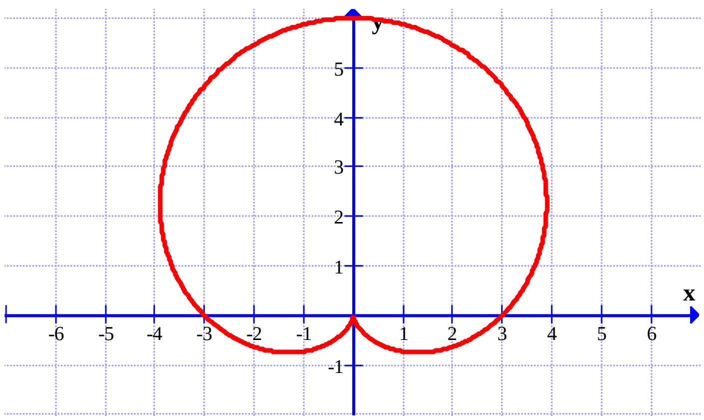
2. Write 5 different (non trivial) polar coordinates for the polar point $\left(-2,-\frac{5\pi}{4}\right)$

$$\left(2,\frac{7\pi}{4}\right),\left(2,-\frac{\pi}{4}\right),\left(-2,\frac{3\pi}{4}\right),\left(2,-\frac{9\pi}{4}\right),\left(-2,\frac{11\pi}{4}\right)$$

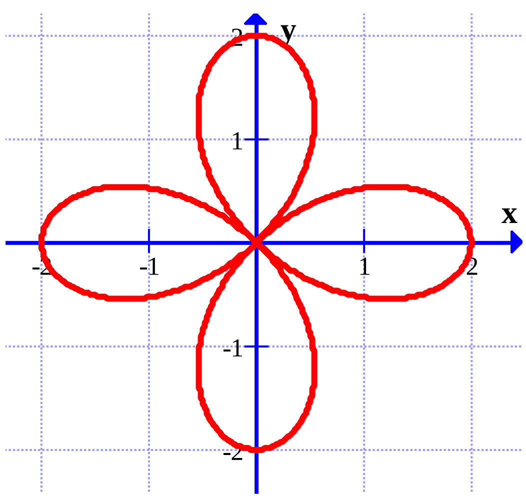
3. Sketch carefully the polar graph of $r = 2 - 4\cos\theta$



4. Sketch carefully the polar graph of $r = 3 + 3\sin\theta$

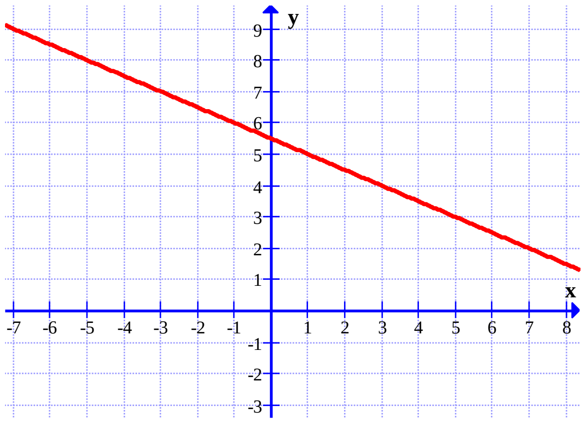


5. Sketch carefully the polar graph of $r = 2\cos 2\theta$

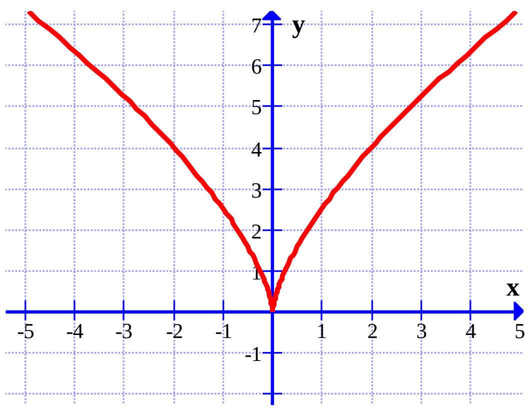


XII Please review Parametric Functions from your Pre-Calculus year – email me with questions after you have made a good attempt.

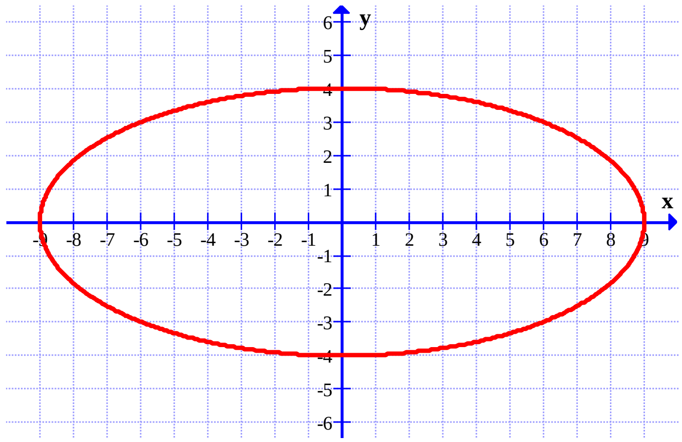
1. Sketch on a Cartesian coordinate system the graph of $x = 3 - 2t,\ y = 4 + t$



2. Sketch on a Cartesian coordinate system the graph of $x = 2t^3,\ y = 4t^2$

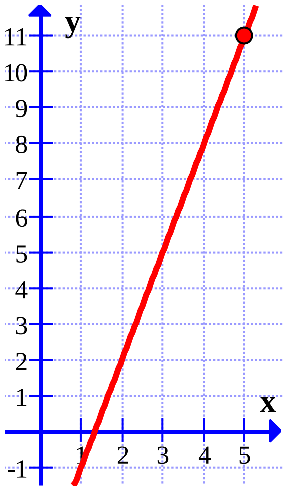


3. Sketch on a Cartesian coordinate system the graph of $x = 9\cos t,\ y = 4\sin t \quad t \in [0, 2\pi]$



XIII Be as analytic as possible but use any **non-calculus method** at your disposal as long as it is clear how you are proceeding. Explain yourself carefully and completely. ANSWER TO FOUR DECIMAL PLACES. **DO NOT USE TRIAL AND ERROR**.

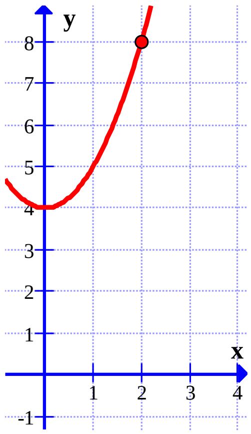
*1. Consider the function $f(x) = 3x - 4$. It is clear that $f(5) = 11$. What is the greatest distance that *x* can be from 5 in order that $f(x)$ is no farther than 0.06 away from 11?



$$11 - .06 \le 3x - 4 \le 11.06$$
$$10.94 \le 3x - 4 \le 11.06$$
$$14.94 \le 3x \le 15.06$$
$$4.98 \le x \le 5.02$$

greatest distance is 0.02

*2. Consider the function $f(x) = x^2 + 4$. What is the greatest distance between *x* and 2 in order that $f(x)$ is no farther than 0.01 from 8 as one considers values of *x* moving away from $x = 2$?



$$7.99 \le x^2 + 4 \le 8.01$$
$$3.99 \le x^2 \le 4.01$$
$$1.9974498 \le x \le 2.002498439$$

greatest distance is 0.002498439

XV

1. Draw a representation of a unit circle in the center of a full sheet of GRAPH paper (download graph paper if you need to) using a compass or something round. Draw a coordinate system with the origin at the center using a straight edge. Draw, with a protractor, all angles that are multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ radians going once around the circle in standard position. For each angle θ, state the first two positive and the first two negative values of θ having that terminal side along with the $\sin\theta$, $\cos\theta$, $\tan\theta$, and $\sec\theta$. EXAMPLE: for the first positive angle you would write $\theta = \frac{\pi}{6}, \frac{13\pi}{6}, -\frac{11\pi}{6}, -\frac{23\pi}{6}$, $\sin\theta = \frac{1}{2}, \cos\theta = ...$ be done carefully and ***from memory***, do NOT look these up! These need to be part of your general knowledge for this year! Look to see how well your values agree with the coordinates on the graph paper.

XVI

From MEMORY, without a calculator, state the exact values of the following:

1. **a)** $\sin\frac{17\pi}{3} = -\frac{\sqrt{3}}{2}$ **b)** $\tan\frac{43\pi}{6} = \frac{\sqrt{3}}{3}$ **c)** $\cos\frac{97\pi}{4} = \frac{\sqrt{2}}{2}$ **d)** $\sin\left(-\frac{55\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

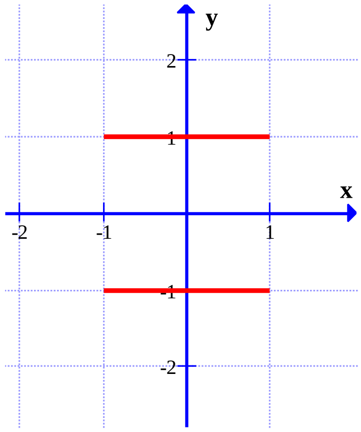
e) $\cos\left(-\frac{71\pi}{6}\right) = \frac{\sqrt{3}}{2}$ **f)** $\cot\left(-\frac{213\pi}{4}\right) = -1$ **g)** $\sec\left(-\frac{137\pi}{3}\right) = 2$ **h)** $\csc\left(\frac{1000\pi}{6}\right) = \frac{2}{\sqrt{3}}$

i) $\tan^{-1}\left(\cot\left(-\frac{13\pi}{4}\right)\right) = -\frac{\pi}{4}$ **j)** $\sin\left(\cot^{-1}\left(-\sqrt{3}\right)\right) = -\frac{1}{2}$ **k)** $\csc\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = $ **2**

l) $\sin^{-1}\left(\frac{\sqrt[3]{65}}{4}\right) =$ **DNE (input value is greater that 1)**

XVII

Suppose that the trigonometric functions were defined in exactly the way you have learned (sine of an angle is the *y* coordinate, cosine of an angle is the *x* coordinate……) based on the unit circle **BUT** instead were based on a **UNIT SQUARE** instead of a unit circle. A unit square in ONE unit on each side, centered on the origin. With this ONE modification, what would the values of these trig functions now be?



1. $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ 2. $\tan\frac{\pi}{4} = 1$ 3. $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

4. $\csc\left(-\frac{19\pi}{4}\right) = -\sqrt{2}$ 5. $\sin 30° = \frac{1}{2}$

6. $\cos(120°) = -\frac{\sqrt{3}}{3}$ 7. $\tan(-210°) = -\frac{\sqrt{3}}{3}$

8. $\csc(1260°) =$ und

9. $\cos(43.72198°) = \frac{1}{1.383696081} = .7227020541$

10. $\sin(-948.6671°) = \sin(-228.6671) = \frac{1}{1.331761805} = .7508850279$

11. $\sin^2 315° + \cos^2 315° = 1$

XVIII Summary. <u>Be sure to include this page in your summer work set.</u>

Roughly how much time did you need to spend on this review in order to complete it?____________

Did you find you remembered most of what you needed? Comment please:

Which topics were the most difficult for you? Comment please:

State the Section Number and Question Number of the problems on this review that you would **MOST** like to see discussed in class: