AP CALCULUS
PREREQUISTE REVIEW PACKET

For students in entering ***AP CALCULUS AB***

Name: ____

Summer Calculus Packet

This summer work is intended to be completed towards the end of the summer. Don't do it now and forget about it! There are examples before each section – read them if you are in doubt as how to do the work. These questions are intended as review (with one or two exceptions) and are due on **loose leaf paper (NO fuzzy edges)** on the **first day of school Wednesday August 22nd regardless of whether you have calculus that day!!** Turn it in to your calculus teacher. Most answers will be posted on the AB Calculus Haiku page the afternoon of August 22. If you have any questions or concerns about how to do these (or even, how to get started or ***anything***), please email me after August 1 at dlissner@orangeusd.org. It is expected that, through any resources available, that you will have **completed AND UNDERSTOOD** these problems **BEFORE** the start of school. AP Calculus Boot Camp will be August 7 – 9 and there may be one Saturday review day to make sure everyone starts at the same place. We will embark on **new** material starting on the first full week of school. Whether you are coming from Pre-Calculus or Honors Pre-Calculus, you can succeed!! Please, please ask all questions so that this material forms a common playing field for everyone. You may help each other with ideas, but NOT the details. Make sure your answers truly make sense and you are not just plugging and chugging since this will certainly come back to haunt you. **ASK ALL YOUR QUESTIONS BEFORE THE START OF SCHOOL. SHOW YOUR WORK IN AN ORGANIZED FASHION WITH PROBLEMS IN NUMERICAL ORDER.**

PLEASE USE LOOSE LEAF PAPER.

Be sure to complete the next to the last page of this assignment.

Have a great summer!

1. This packet is to be handed in the first day of school to your calculus teacher, even if you don't have calculus that day!!
2. **All work** must be shown in the packet OR on separate paper attached to the packet.
3. Completion of this packet is worth one-half of a major test grade and will be counted as your first grade.
4. There will be test on this review material some time the first full week of class.

The AP Calculus Exam
How, not only to Survive, but to Prevail…

The AP Calculus exam is the culmination of all of the years you've spent in high school studying mathematics. It's all led up to this. The calculus you study in the last year completes the prior years of preparation……. Keep these things in mind as you go through the year.

Everything in calculus, and mathematics in general, is best understood verbally, numerically, analytically (that is, through the use of equations and symbols) and graphically. Look at everything from these perspectives. Look at the relationships among them — how the same idea shows up in words, in equations, in numbers and in graphs.

For example: numerically a linear function is one which when written as a table of values, regular changes in the x-values produce regular changes in the y-values. Graphically a linear function has a graph that is a straight line. Analytically it is one whose equation can be written as $y = mx + b$. And the three ways are interrelated: The ratio of the changes in the table is the number m in the equation; the graph can be drawn using the number m by going up and over from one point to the next. The idea of the slope as "rise over run" expresses this verbally. Everything in mathematics and in the calculus works that way.

Learn the concepts — the exam emphasizes concepts.

Learn the procedures and formulae — even though the concepts are more important than the computations you still have to do the computations. Like it or not, learn to do the algebra, the arithmetic and the graphs.

Learn to be methodical — work neatly and carefully all year.

Think about what you are doing. Watch yourself work. It is natural to concentrate on the material you know and can do, but you need to concentrate on the things you do not (yet) know how to do. You can learn much from your mistakes. Look at a wrong answer as a green light to go in that direction until you've reached the right answer.

Excerpt from Lin McMullin's *Teaching AP Calculus*

Complex Fractions (IMPORTANT PAGE)

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}}=\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}}\bullet\frac{x(x-4)}{x(x-4)}=\frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)}=\frac{-2x+8+3x^2}{5x^2-20x-x}=\frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}=\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}\bullet\frac{x+1}{x+1}=\frac{-7x-7-6}{5}=\frac{-7x-13}{5}$$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $\left(f \circ g\right)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

6. $f(2) =$ ____________

7. $g(-3) =$ ____________

8. $f(t+1) =$ ____________

9. $f\left[g(-2)\right] =$ ____________

10. $g\left[f(m+2)\right] =$ ____________

11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ____________

13. $f\left(\dfrac{2\pi}{3}\right) =$ ____________

Let $f(x) = x^2,\ g(x) = 2x + 5,$ *and* $h(x) = x^2 - 1$**. Find each.**

14. $h\left[f(-2)\right] =$ _______

15. $f\left[g(x-1)\right] =$ _______

16. $g\left[h(x^3)\right] =$ _______

Find $\dfrac{f(x+h)-f(x)}{h}$ for the given function f.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$\underline{x-\text{int. } (Let\ y=0)}$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1\ or\ x=3$

$x-\text{intercepts } (-1,0)\ and\ (3,0)$

$\underline{y-\text{int. } (Let\ x=0)}$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept } (0,-3)$

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\qquad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\qquad$ $16 = y^2$

$y = 0$ $\qquad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

25. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
$16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		 +3

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x - 1 \ge 0$

28. $-4 \le 2x - 3 < 4$

29. $\dfrac{x}{2} - \dfrac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x) = x^2 - 5$

31. $f(x) = -\sqrt{x+3}$

32. $f(x) = 3\sin x$

33. $f(x) = \dfrac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite $f\left(x\right)$ as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$\left(x\right)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x))=g(f(x))=x$

Example:

If: $f(x)=\dfrac{x-9}{4}$ *and* $g(x)=4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x))=4\left(\frac{x-9}{4}\right)+9$$
$$=x-9+9$$
$$=x$$

$$g(f(x))=\frac{\left(4x+9\right)-9}{4}$$
$$=\frac{4x+9-9}{4}$$
$$=\frac{4x}{4}$$
$$=x$$

$f(g(x))=g(f(x))=x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

36. $f(x)=\dfrac{x^3}{2}$ $\quad g(x)=\sqrt[3]{2x}$

37. $f(x)=9-x^2,\ x\geq 0$ $\quad g(x)=\sqrt{9-x}$

<u>Equation of a line</u>

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: $x = c$ (slope is undefined)

Horizontal line: $y = c$ (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the *y*- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an *x*-intercept (2, 0) and a *y*-intercept (0, 3).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

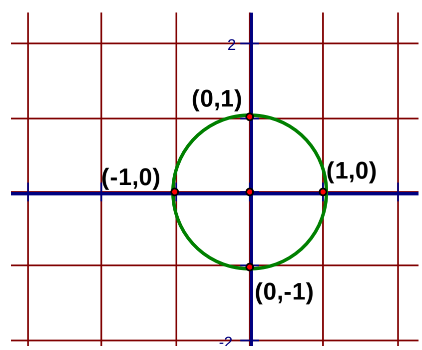
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

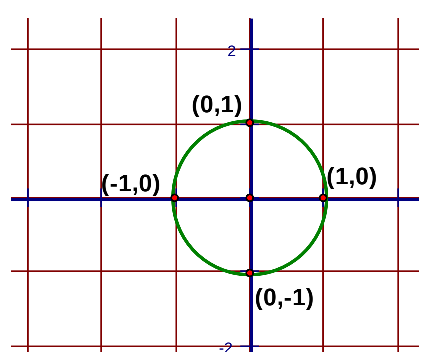
Example: $\sin 90^\circ = 1$ $\qquad$ $\cos\frac{\pi}{2} = 0$



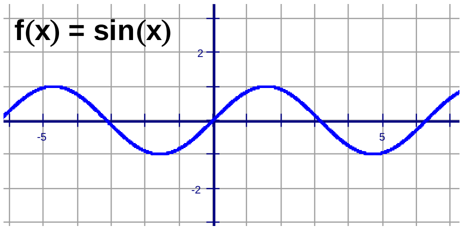
50. $a.)\ \sin 180^\circ$ $\qquad$ $b.)\ \cos 270^\circ$

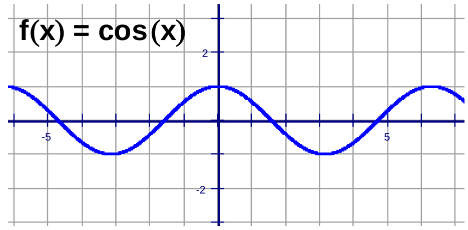
$c.)\ \sin(-90^\circ)$ $\qquad$ $d.)\ \sin\pi$

$e.)\ \cos 360^\circ$ $\qquad$ $f.)\ \cos(-\pi)$



Graphing Trig Functions





y = sin x and y = cos *x* have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51. $f(x) = 5\sin x$

52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

55. $\sin x = -\dfrac{1}{2}$

56. $2\cos x = \sqrt{3}$

57. $\cos 2x = \dfrac{1}{\sqrt{2}}$

58. $\sin^2 x = \dfrac{1}{2}$

59. $\sin 2x = -\dfrac{\sqrt{3}}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

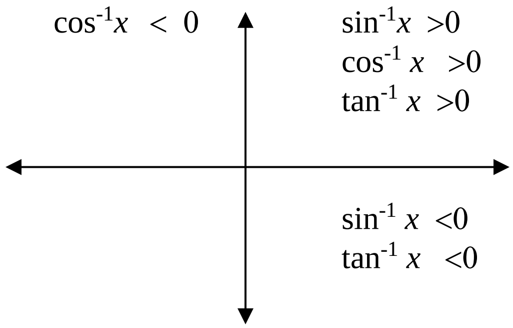
62. $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

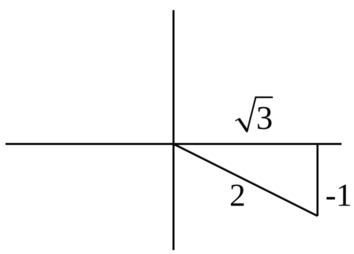
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "*y*" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval

from $\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: $y = -\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

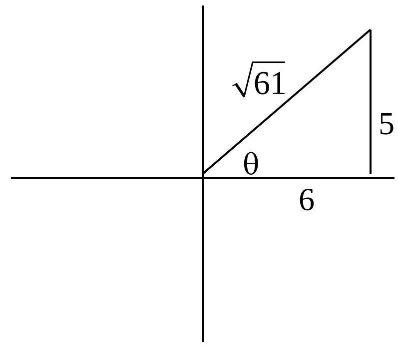
63. $y = \arcsin\frac{-\sqrt{3}}{2}$

64. $y = \arccos(-1)$

65. $y = \arctan(-1)$

Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$



Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

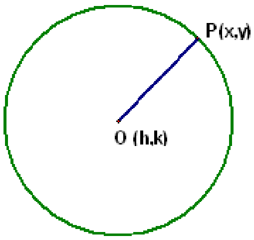
66. $\tan\left(\arccos\frac{2}{3}\right)$

67. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

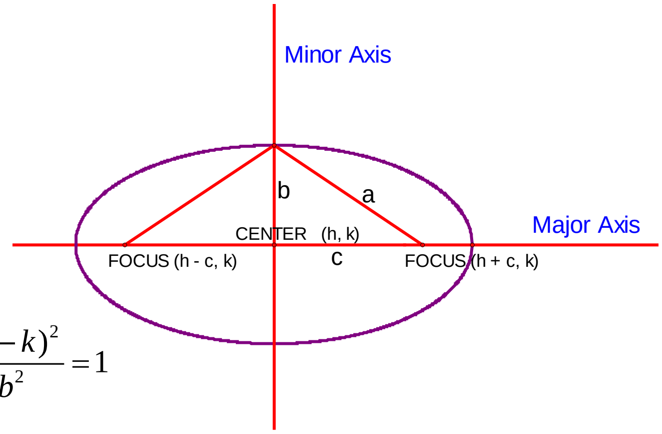
68. $\sin\left(\arctan\frac{12}{5}\right)$

69. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$



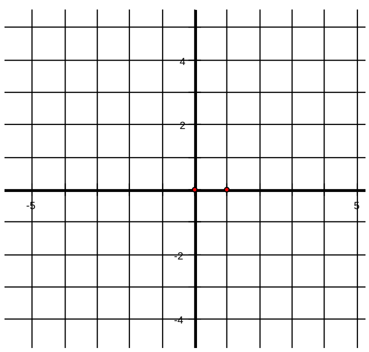
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

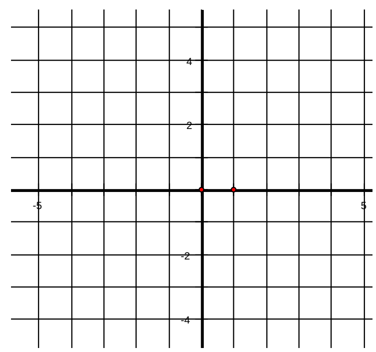
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the *y*-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

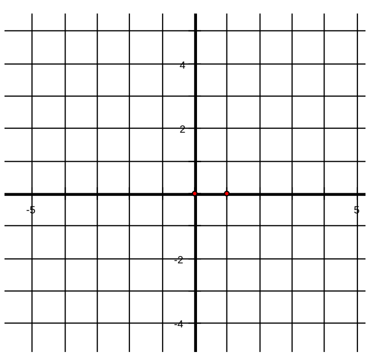
70. $x^2 + y^2 = 16$



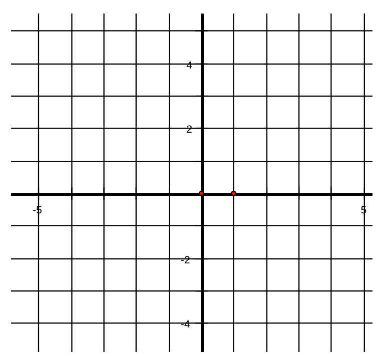
71. $x^2 + y^2 = 5$



72. $\frac{x^2}{1} + \frac{y^2}{9} = 1$



73. $\frac{x^2}{16} + \frac{y^2}{4} = 1$



Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

74. $f(x)=\dfrac{1}{x^2}$

75. $f(x)=\dfrac{x^2}{x^2-4}$

76. $f(x)=\dfrac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

77. $f\left(x\right)=\dfrac{x^2-2x+1}{x^3+x-7}$

78. $f(x)=\dfrac{5x^3-2x^2+8}{4x-3x^3+5}$

79. $f(x)=\dfrac{4x^5}{x^2-7}$

Limits

Finding limits numerically.

Complete the table and use the result to estimate the limit.

80. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

81. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

82. $\lim_{x \to 0} \cos x$

83. $\lim_{x \to 5} \frac{2}{x-5}$

84. $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

85. $\lim_{x \to 2} (4x^2 + 3)$

86. $\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$

87. $\lim_{x \to 0} \sqrt{x^2 + 4}$

88. $\lim_{x \to \pi} \cos x$

89. $\lim_{x \to 1} \left(\frac{x^2 - 1}{x - 1} \right)$ HINT: Factor and simplify.

90. $\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$

Summary. <u>Be sure to include this page in your summer work set.</u>

Roughly how much time did you need to spend on this review in order to complete it?____________

Did you find you remembered most of what you needed? Comment please:

Which topics were the most difficult for you? Comment please:

State the Section Number and Question Number of the problems on this review that you would **MOST** like to see discussed in class:

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p\log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h \to \infty} \dfrac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By + C = 0$