AREA OF KNOWLEDGE: MATHEMATICS

Introduction

- Mathematics: the rational mind is at work.
- When most abstracted from the world, mathematics stands apart from other areas of knowledge, concerned only with its own internal workings.
- In its practical form it is an integral part of many other fields—engineering, medicine, marketing, architecture.
- And everyday life—budgets to recipes

Definition of Mathematics

- What does calling mathematics a "language" mean?
- Does mathematics function in the same way as our daily written and spoken language?

Definition

- Do mathematical symbols have meaning, in the same sense as words have meaning?

Definition

- Why is it that some claim that mathematics is no more than a “logical game”, such as chess, for example, devoid of particular meaning?
- If this were the case, how do we account for the fact that it seems to apply so well to the world around us?

Definition

- What could Carl Sandburg have meant by the following?
  - "Arithmetic is where the answer is right and everything is nice and you can look out of the window and see the blue sky—or the answer is wrong and you have to start all over and try again and see how it comes out this time."

Math and Language

- Is mathematics "the language of the universe"?
- Math is the study of patterns and relationships between numbers and shapes.
- It is symbolic and abstract.
- It is said that mathematics takes us into our minds and back out to the world? How is that possible?

Origins of Mathematics

- Question: Is math invented or discovered?
- What does math enable us to do as it relates to the world?

Mathematics and the World

- We can use mathematics successfully to model real=world process. Is this because we create mathematics to mirror the world or because the world is intrinsically mathematical?
- Some major advances in physics, for example, discoveries of elementary particles, have come about through arguments involving the beauty, elegance or symmetry of the underlying mathematics. What does this tell us about the relationship between the natural sciences, mathematics and the natural world?

Math and the World

- What is Einstein mean by asking: "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?"
- Is mathematics better defined by its method or by its subject matter?
- Mathematicians marvel at some of the deep connections between disparate parts of their subject. Is this evidence for a simple underlying mathematical reality?

Math and the World

- What are the differences between the **formal** school of thought which regards mathematics as similar to an activity governed by rules, limited only by the rules of logic and the creativity of mathematician, and the **realist** school of thought which regards mathematics as referring to the way the world actually works?

Math and Reality

- What is the foundation on which mathematical knowledge rests?
- Is it discovered or invented?
- What is meant by this distinction?
- Can it be applied usefully in other areas?

Math and Reality

- What is the origin of the axioms of mathematics?
- Are axioms necessarily self-evident to all people?
- How is an axiomatic system of knowledge different from, or similar to, other systems of knowledge?
- Do different geometries (Euclidean and non-Euclidean) refer to or describe different worlds?

Pure and Applied Mathematics

- What is the difference between pure and applied mathematics?
- How would you define the word “pure” as it is used in this context?
- How would you describe the purposes of pure mathematics?
- How would you define the purpose of applied mathematics?

Pure and Applied Mathematics

- It is said that the lines between pure and applied mathematics are fuzzy. Why? How is that the case?
- How are the lines between applied math and the fields it is used in also fuzzy?
- Why is math said to be an abstract science?

Mathematics and Knowledge Claims

- What do mathematicians mean by mathematical proof, and how does it differ from good reasons in other areas of knowledge?
- Is a mathematical statement true only if it has been proved?
- Is the meaning of a mathematical statement dependent on its proof?
- Are there such things as true but unprovable statements in mathematics?

Mathematics and Knowledge Claims

- It has been argued that we come to know the number 3 through examples such as three oranges or three cups. Does this support the independent existence of the number 3 and, by extension, numbers in general? If so, what of numbers such as 0 and -1, (the square root of -1) and a trillion? If not, in what sense do numbers exist?
- In the light of the previous question, why might it be said that mathematics makes true claims about non-existent objects?

Knowledge Claims

- Mathematics has been described as a form of knowledge which requires internal validity or coherence.
- Does this make I self-correcting? What would this mean?
- In what sense might chaos (non-linear dynamical systems) theory suggest a limit to the applicability of mathematics to the real world?

Knowledge Claims

- How is mathematical proof or demonstration different from, or similar to, justifications accepted in other Areas of Knowledge?

Knowledge Claims

- Is mathematical knowledge certain knowledge? Can we claim that 1+1 = 2 is true in mathematics? Does 1 + 1 = 2 hold true in the natural world?

Knowledge Claims

- Does truth exist in mathematical knowledge?
- Could one argue that mathematical truth corresponds to phenomena that we perceive in nature or that it coheres, that is, logically connects, to a designed structure of definitions and axioms?

Knowledge Claims

- Fermat's Last Theorem remained unproved for 358 years, until 1995. Is mathematical knowledge progressive?
- Has mathematical knowledge always grown?
- In this respect how does mathematics compare with other Areas of Knowledge (for example, history, the natural sciences, ethics and the arts)?
- Could there ever be an "end" to mathematics? In other words, could we reach a point where everything important in mathematical sense is known? If so, what might be the consequences of this?

Knowledge Claims

- Has technology, for example, powerful computers and electronic calculators, influenced the knowledge claims made in mathematics?
- Is any technological influence simple a matter of speed and the quantity of data which can be processed?

Mathematics and the Knower

- Can mathematics be characterized as a universal language?
- To what extent is mathematics a product of human social interaction?
- What is the role of the mathematical community in determining the validity of a mathematical proof?
- Why is it that mathematics is considered to be of different value in different cultures?

Mathematics and the Knower

- How would you account for the following features that seem to belong particularly to mathematics: somepeople learn it very easily and outperform their peers by years; some people find it almost impossible to learn, however hard they try; most outstanding mathematicians supposedly achieve their best work before they reach the age of 30?

Mathematics and the Knower

- What counts as understanding in mathematics? Is it sufficient to get the right answer to a mathematical problem to say that one understands the relevant mathematics?
- Are there aspects of mathematics that one can choose whether or not to believe?
- How do we choose the axioms underlying mathematics? Is this an act of faith?

Mathematics and the Knower

- Do the terms “beauty” or “elegance” have a role in mathematical thought?
- Is there a correlation between mathematical ability and intelligence?
- Is there a clear-cut distinction between being good or bad at mathematics?
- How have technological innovation, such as developments in computing, affected the nature and practice of mathematics?

Knowledge Claims

- What impact have major mathematical discoveries and inventions had on conceptions of the world?

Mathematics and Values

- Why do many mathematicians consider their work to be an art form?
- Does mathematics exhibit an aesthetic quality?

Values

- What could be meant by GH Hardy's claim that: "The mathematician's patterns, like the painter's or poet's, must be beautiful; the ideas, like the colors or the words, must fit together in a harmonious way. Beauty is the first test. There is no permanent place in the world for ugly mathematics"?

Values

- What relationships, if any, exist between mathematics and various types of art (for example, music, painting, and dance)?
- How can concepts such as proportion, pattern, iteration, rhythm, harmony and coherence apply both in the arts and in mathematics?

Values

- Is the formation of mathematical knowledge independent of culturalo influence?
- Is it independent of the influence of politics, religion or gender?

Values

- What is meant by S Ramanujan's comment that "Every time you write your student number you are writing Arabic"?
- If mathematics did not exist, what difference would it make?

Math Tasks

- Escher Puzzle Math and art
- Math and music
- Math and poetry
- What is it that mathematics cannot do as a "language"?

Assignment

- Read and Journal
  - It Ain't What You Prove, It's the Way that You Prove It by Chris Binge
  - The Number System is Like Human Life. . . By Peter Hoeg