TO: Next Year's Calculus Students
FROM: Carlos Roman, Calculus Teacher

Attached is a summer homework packet, which will be due the first day of AP Calculus BC class in August. The material in the packet should be material you learned in Algebra II and Precalculus.

<u>You will turn in the packet the first day of Calculus class, and it will count as a daily grade</u>. <u>During the first week of school, you will take a test on the material in the packet</u>.

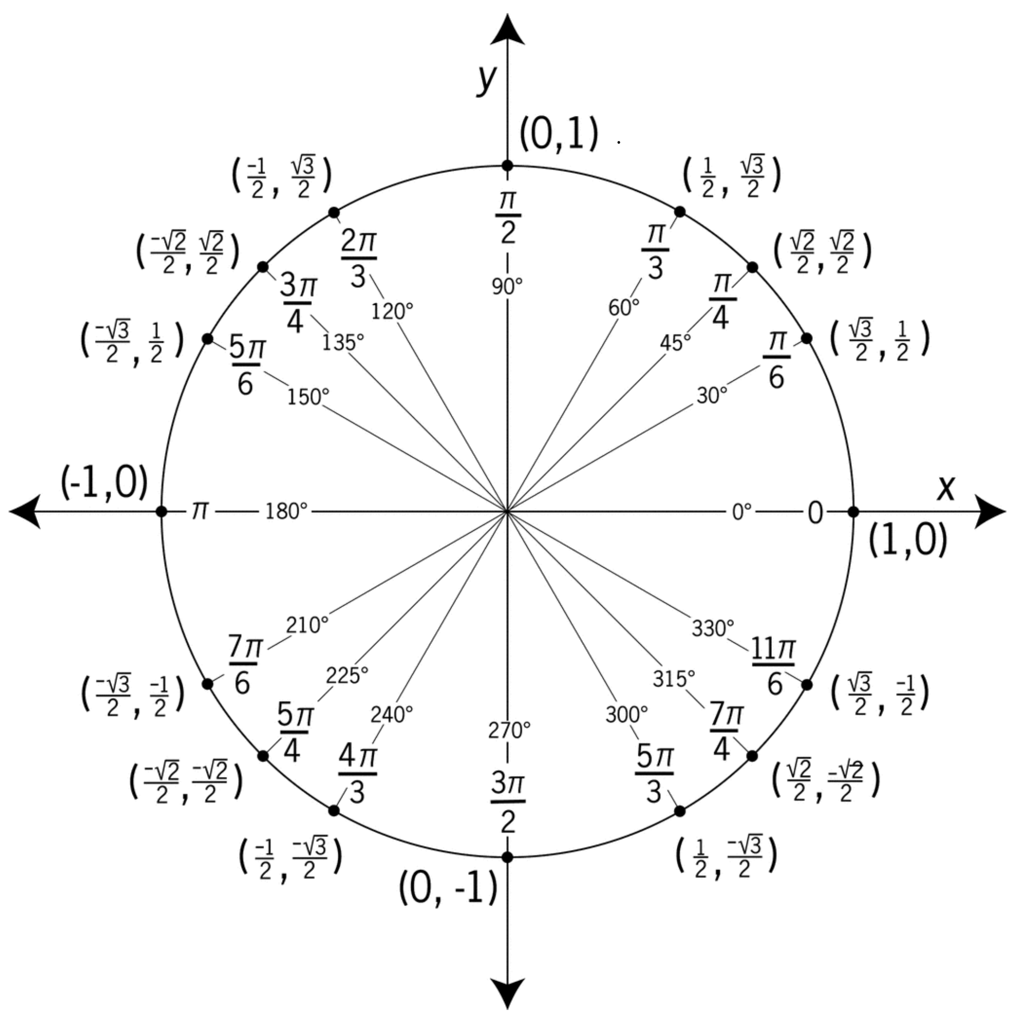
My recommendation is that you look over the problems in the packet when you receive it but that you wait until the week before school starts to work the problems so that you will remember the material very well when school starts.

Remember that we will be using a graphing calculator in Calculus. A TI is preferable.
NOTE: <u>You will be expected to have it the first week of school.</u>

I am looking forward to seeing you in AP Calculus BC in August.

Trig Identities You Should Know:

$\sin^2\theta+\cos^2\theta=1$	$\sin(2\theta)=2\sin\theta\cos\theta$
$1+\tan^2\theta=\sec^2\theta$	$\cos(2\theta)=\cos^2\theta-\sin^2\theta$
$1+\cot^2\theta=\csc^2\theta$	$\cos(2\theta)=1-2\sin^2\theta$
	$\cos(2\theta)=2\cos^2\theta-1$



AP CALCULUS BC
SUMMER HOMEWORK

<u>This homework packet is due the first day of school.</u> <u>It will be turned in the FIRST DAY of AP Calculus BC class and will count as a daily grade.</u> <u>You will take a test on the material in the packet during the third week of school.</u>

Work these problems on <u>notebook paper</u>. <u>All work must be shown</u>. In many sections I have included the site on the Khan Academy web page where you can review the topic
Use your graphing calculator only on problems <u>44 - 55</u>.

Find the x- and y-intercepts and the domain and range, and sketch the graph. <u>No calculator</u>.

1. $y=\sqrt{x-1}$
2. $y=\sqrt{9-x^2}$
3. $y=\dfrac{|x|}{x}$
4. $y=\sin x,\ -2\pi\le x\le 2\pi$
5. $y=\cos x,\ -2\pi\le x\le 2\pi$
6. $y=\tan x,\ -2\pi\le x\le 2\pi$
7. $y=\cot x,\ -2\pi\le x\le 2\pi$
8. $y=\sec x,\ -2\pi\le x\le 2\pi$
9. $y=\csc x,\ -2\pi\le x\le 2\pi$
10. $y=e^x$
11. $y=\ln x$
12. $y=\begin{cases}-1, & \text{if } x\le -1\\ 3x+2, & \text{if } |x|<1\\ 7-2x, & \text{if } x\ge 1\end{cases}$
13. $y=\begin{cases}x^2+1, & \text{if } x>0\\ -2x+2, & \text{if } x\le 0\end{cases}$

Find the asymptotes (horizontal, vertical, and slant), symmetry (x-axis symmetry, y-axis symmetry, or origin symmetry), and intercepts, and sketch the graph. https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions#graphs-of-rational-functions

<u>No calculator</u>.

14. $y=\dfrac{1}{x-1}$
15. $y=\dfrac{1}{(x+2)^2}$
16. $y=\dfrac{2(x^2-9)}{x^2-4}$
17. $y=\dfrac{x^2-2x+4}{x-1}$

Use a number line graph to solve. <u>No calculator</u>.

18. $x^2-x-12>0$
19. $(x-2)^2(x+1)^3(x-5)\le 0$
20. $\dfrac{3x-2}{x+4}\le 0$
21. $\dfrac{(2x+5)(x-1)^2}{(x+2)^3}\ge 0$

Evaluate. <u>No calculator</u>.

22. $\cos\dfrac{5\pi}{6}$
23. $\sin\dfrac{3\pi}{2}$
24. $\tan\dfrac{5\pi}{4}$
25. $\sin\dfrac{7\pi}{4}$
26. $\cos\pi$
27. $\tan\dfrac{2\pi}{3}$
28. $\sec\dfrac{4\pi}{3}$
29. $\csc\dfrac{\pi}{4}$
30. $\cot\dfrac{2\pi}{3}$

Evaluate. <u>No calculator</u>.

31. $\tan\left(\text{Cos}^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right)$
32. $\sec\left(\text{Arc}\sin\left(-\dfrac{\sqrt{2}}{2}\right)\right)$
33. $\cos\left(\text{Sin}^{-1}(2x)\right)$
34. $\sec\left(\text{Arc}\tan(4x)\right)$

Solve. Give exact answers in radians, $0 \le x \le 2\pi$. No calculator.
https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities#advanced-sinusoidal-equations

35. $2\cos^2 x + 3\cos x - 2 = 0$ 36. $2\sin^2 x - \cos x = 1$ 37. $\sin(2x) = \cos x$

38. $2\cos(2x) + 1 = 0$ 39. $2\csc^2 x + 3\csc x - 2 = 0$ 40. $\tan^2 x - \sec x = 1$

41. $2\cos\left(\frac{x}{3}\right) - \sqrt{3} = 0$ 42. $\tan(2x) = -\sqrt{3}$ 43. $2\sin(3x) - \sqrt{3} = 0$

Solve. Show all steps. Use your calculator, and give decimal answers correct to **three** decimal places.
https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions

44. $e^{2x+3} = 37$ 45. $e^{2x} - 5e^{x} + 6 = 0$ 46. $e^{x} - 12e^{-x} - 1 = 0$

47. $\frac{50}{4+e^{2x}} = 11$ 58. $\log_4(x^2 - 3x) = 1$ 49. $\ln(5x-1) = 3$

50. $\log_2(x+3) + \log_2(x-1) = \log_2 12$ 51. $\log_8(x+5) - \log_8(x-2) = 1$

52. $\log_6(\log_4(\log_2 x)) = 0$ 53. $\log_3(\log_2(\log_5 25)) = x$

54. The number of students in a school infected with the flu t days after exposure is modeled by the function $P(t) = \frac{300}{1+e^{4-t}}$.
(a) How many students were infected after three days?
(b) When will 100 students be infected?

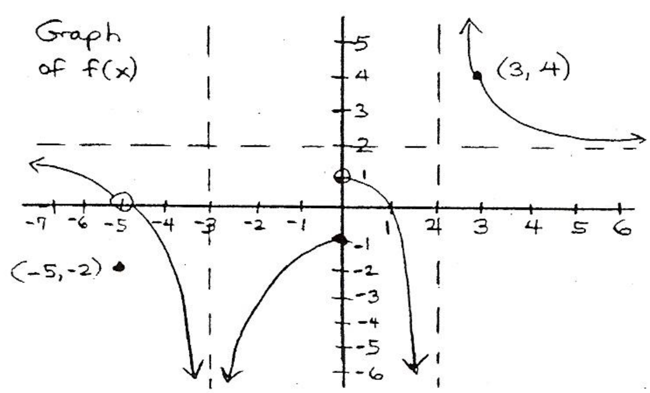
55. Exponential growth is modeled by the function $n = n_0 e^{kt}$. A culture contains 500 bacteria when $t = 0$. After an hour, the number of bacteria is 1200.
(a) How many bacteria are there after four hours?
(b) After how many hours will there be 8000 bacteria?

Use the figure to find the limit. No calculator.
https://www.khanacademy.org/math/differential-calculus/limit-basics-dc

56. $\lim_{x\to 3} f(x)$ 57. $\lim_{x\to \infty} f(x)$

58. $\lim_{x\to 2^+} f(x)$ 59. $\lim_{x\to 0} f(x)$

60. $\lim_{x\to -\infty} f(x)$ 61. $\lim_{x\to -5} f(x)$



Evaluate. Show supporting work for each problem (algebraic steps or sketch). No calculator.

62. $\lim\limits_{x\to -3} \dfrac{x^2+x-6}{x+3}$

63. $\lim\limits_{x\to 0} \dfrac{(x-5)^2-25}{x}$

64. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

65. $\lim\limits_{x\to -6} \dfrac{x+6}{x^2+3x-18}$

66. $\lim\limits_{x\to -2} \dfrac{x^3+8}{x+2}$

67. $\lim\limits_{x\to \infty} \dfrac{3x-5x^2}{4x^2+1}$

68. $\lim\limits_{x\to 3^+} \dfrac{1}{x-3}$

69. $\lim\limits_{x\to 3^-} \dfrac{1}{x-3}$

70. $\lim\limits_{x\to 3} \dfrac{1}{x-3}$

71. $\lim\limits_{x\to 3} \dfrac{1}{(x-3)^2}$

72. $\lim\limits_{x\to 3^+} [\![x-1]\!]$

73. $\lim\limits_{x\to 3^-} [\![x-1]\!]$

74. $f(x)=\begin{cases} 1-x, & x\le 1 \\ x^2, & x>1 \end{cases}$ (a) $\lim\limits_{x\to 1^-} f(x)$ (b) $\lim\limits_{x\to 1^+} f(x)$ (c) $\lim\limits_{x\to 1} f(x)$

75. $f(x)=\begin{cases} \dfrac{x^2-x-6}{x-3} & \text{if } x\ne 3 \\ 4 & \text{if } x=3 \end{cases}$ (a) $\lim\limits_{x\to 3} f(x)$ (b) $f(3)$

Use the **definition of the derivative** to find the derivative. No calculator.
https://www.khanacademy.org/math/differential-calculus/derivative-intro-dc/derivative-as-a-limit-dc/v/calculus-derivatives-1-new-hd-version

Definition of the derivative: $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$. (You must **know** this formula.)

76. $f(x)=x^2-8x$

77. $f(x)=\sqrt{x+9}$

78. $f(x)=\dfrac{3}{x-4}$

79. $f(x)=x^3+2x^2-x+4$

Use the differentiation rules (power rule, product rule, quotient rule) to find the derivative. Do not leave negative exponents or complex fractions in your answers. No calculator.
https://www.khanacademy.org/math/differential-calculus/basic-differentiation-dc

80. $f(x)=3x^4-5x^3+\dfrac{2}{x}+6x^{2/3}-12$

81. $f(x)=\dfrac{2x^2-3x+1}{x}$

82. $f(x)=\sqrt{x}+\sqrt[3]{x}$

83. $f(x)=(6x+5)(x^3-2)$

84. $f(x)=\dfrac{x^3+5x-3}{x^2-1}$

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85. Given the function $f(x)=x^4-3x^2+7$.

(a) Use the differentiation rules to find $f'(x)$.

(b) Write the equation of the tangent line to f at (1, 5). Leave your equation in point-slope form.