TONBRIDGE SCHOOL

Scholarship Examination 2016

MATHEMATICS I

Monday 25th April 2016
11.30 a.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.
You may attempt the questions in any order.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. I am thinking of two numbers.
If I double the first and add the second, my answer is 35.
If I double the second and subtract the first, my answer is 15.
By forming and solving two simultaneous equations, find the two numbers.

[8 marks]

2. For a certain journey, I always start at the same time in the morning and arrive later in the day.
If I travel at 10 km per hour, I arrive at 1.00 p.m.
If I travel at 15 km per hour, I arrive at 11.00 a.m.

(a) How far is the journey?

(b) What is my start time?

(c) What should my speed be (with the same start time) if I want to arrive at 12.00 noon?

[8]

3. From the top of a planetary mountain, height H km, the angle A degrees between the horizontal and the horizon is connected to the radius R km of the planet by the formula $R = H\left(\frac{6570}{A^2} - 1\right)$.

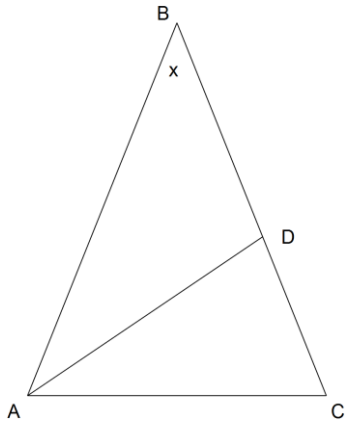
(a) If $H = 9$ and $A = 3.1$, find R.

(b) If $R = 5000$ and $A = 2.3$, find H.

(c) If $R = 8500$ and $H = 7.8$, find A.

[8]

4. In the diagram below, triangle ABC is isosceles with $AB = BC$; triangle ABD is isosceles with $AD = BD$; triangle ADC is isosceles with $AD = AC$.
$x = A\hat{B}C$ in degrees. Use algebra to calculate the size of angle x.



[8]

5. In this question, the angles A, B, C of a triangle are always a whole number of degrees. In the following parts, either give an example to show the statement is true or explain why it is false.

(a) It is possible for A, B, C all to be odd numbers.

(b) It is possible for A, B, C all to be prime numbers.

(c) It is possible for A, B, C all to be square numbers.

[8]

6. (a) One isosceles triangle has sides of length 5 cm, 5 cm, 8 cm ; another has sides of length 5 cm, 5 cm, 6 cm. Show that the two triangles have the same area.

(b) Find another pair of different isosceles triangles with the same area, where the 5 cm in (a) is replaced by a suitable prime number bigger than 5 and the 8 cm and 6 cm are replaced by two appropriate different whole numbers.

[15]

7. A solid cylinder has fixed surface area 500 cm^2. Its height y cm is related to its radius x cm by the formula $y = \frac{250}{\pi x} - x$ where π has its usual meaning.

(a) Find the values of y for $x = 2, 3, 4, 5, 6, 7, 8$.

(b) Choosing sensible scales, use your values in (a) to plot a graph of y against x .

(c) Making your method clear find the value of x when $y = 0$.

(d) Draw the line $y = x$ on the same graph.
What is the radius of the cylinder when the radius is equal to the height?

[15]

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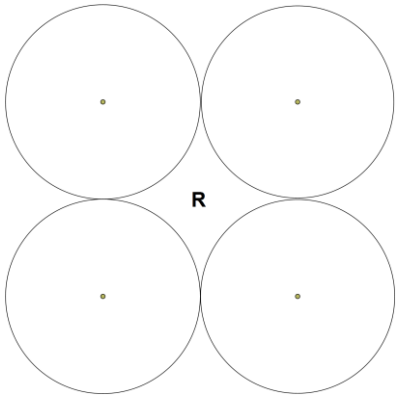
8. Four identical circles of radii 6 cm touch as shown in the figure: their centres form a square.



(a) Find the area of the region between the circles (labelled R above).

(b) Find the radius of the largest circle that will fit in the region labelled R.

[15]

9. Study carefully the pattern of numbers in the table below.
Column B is the answer to the sum shown in Column A. Column C rewrites the number in Column B as a fraction.

	A	**B**	**C**
Row 1	1^2	1	$\frac{1\times 2}{2}$
Row 2	1^2-2^2	-3	$-\frac{2\times 3}{2}$
Row 3	$1^2-2^2+3^2$	6	$\frac{3\times 4}{2}$
Row 4	$1^2-2^2+3^2-4^2$	-10	$-\frac{4\times 5}{2}$
Row 5			
Row 6			
Row *n*			

(a) What are the entries in Columns A, B, C for Rows 5 and 6?

(b) Find a formula in terms of n for the fraction in Column C for Row n.
How can you tell whether a minus sign is required?

(c) What are the entries in Columns B and C: (i) for Row 150, (ii) for Row 251?

(d) Explain whether the following numbers appear in Column B: (i) -1596, (ii) -2145.

(e) How might you write the entry in Column A for Row n?

[15]

END OF PAPER