

6th United States of America
Junior Mathematical Olympiad

Day II 12:30 PM – 5 PM EDT

April 29, 2015

Note: For any geometry problem, the first page of the solution must be a large, in-scale, clearly labeled diagram made with drawing instruments (ruler, compass, protractor, graph paper). Failure to meet this requirement will result in a 1-point automatic deduction.

JMO 4. Find all functions $f: \mathbb{Q} \rightarrow \mathbb{Q}$ such that

$$f(x) + f(t) = f(y) + f(z)$$

for all rational numbers $x < y < z < t$ that form an arithmetic progression. (\mathbb{Q} is the set of all rational numbers.)

JMO 5. Let $ABCD$ be a cyclic quadrilateral. Prove that there exists a point X on segment \overline{BD} such that $\angle BAC = \angle XAD$ and $\angle BCA = \angle XCD$ if and only if there exists a point Y on segment \overline{AC} such that $\angle CBD = \angle YBA$ and $\angle CDB = \angle YDA$.

JMO 6. Steve is piling $m \geq 1$ indistinguishable stones on the squares of an $n \times n$ grid. Each square can have an arbitrarily high pile of stones. After he is finished piling his stones in some manner, he can then perform *stone moves*, defined as follows. Consider any four grid squares, which are corners of a rectangle, i.e. in positions (i, k) , (i, l) , (j, k) , (j, l) for some $1 \leq i, j, k, l \leq n$, such that $i < j$ and $k < l$. A stone move consists of either removing one stone from each of (i, k) and (j, l) and moving them to (i, l) and (j, k) respectively, or removing one stone from each of (i, l) and (j, k) and moving them to (i, k) and (j, l) respectively.

Two ways of piling the stones are equivalent if they can be obtained from one another by a sequence of stone moves.

How many different non-equivalent ways can Steve pile the stones on the grid?