45th United States of America Junior Mathematical Olympiad

Day I 12:30PM — 5PM EDT

April 19, 2016

Note: For any geometry problem, the first page of the solution must be a large, in-scale, clearly labeled diagram made with drawing instruments (ruler, compass, protractor, graph paper). Failure to meet this requirement will result in an automatic 1-point deduction.

USAJMO 1. The isosceles triangle $\triangle ABC$, with $AB = AC$, is inscribed in the circle ω. Let P be a variable point on the arc $\overset{\frown}{BC}$ that does not contain A, and let I_B and I_C denote the incenters of triangles $\triangle ABP$ and $\triangle ACP$, respectively.

Prove that as P varies, the circumcircle of triangle $\triangle PI_BI_C$ passes through a fixed point.

USAJMO 2. Prove that there exists a positive integer $n < 10^6$ such that 5^n has six consecutive zeros in its decimal representation.

USAJMO 3. Let $X_1, X_2, \ldots, X_{100}$ be a sequence of mutually distinct non-empty subsets of a set S. Any two sets X_i and X_{i+1} are disjoint and their union is not the whole set S, that is, $X_i \cap X_{i+1} = \emptyset$ and $X_i \cup X_{i+1} \neq S$, for all $i \in \{1, \ldots, 99\}$. Find the smallest possible number of elements in S.