1^{st} United States of America Junior Mathematical Olympiad 2010

Day II 12:30 PM – 5 PM EDT

April 28, 2010

4. A triangle is called a *parabolic triangle* if its vertices lie on a parabola $y = x^2$. Prove that for every nonnegative integer n, there is an odd number m and a parabolic triangle with vertices at three distinct points with integer coordinates with area $(2^n m)^2$.

5. Two permutations $a_1, a_2, \ldots, a_{2010}$ and $b_1, b_2, \ldots, b_{2010}$ of the numbers $1, 2, \ldots, 2010$ are said to *intersect* if $a_k = b_k$ for some value of k in the range $1 \le k \le 2010$. Show that there exist 1006 permutations of the numbers $1, 2, \ldots, 2010$ such that any other such permutation is guaranteed to intersect at least one of these 1006 permutations.

6. Let ABC be a triangle with $\angle A = 90°$. Points D and E lie on sides AC and AB, respectively, such that $\angle ABD = \angle DBC$ and $\angle ACE = \angle ECB$. Segments BD and CE meet at I. Determine whether or not it is possible for segments AB, AC, BI, ID, CI, IE to all have integer lengths.