4^{th} United States of America Junior Mathematical Olympiad

Day I 12:30 PM – 5 PM EDT

April 30, 2013

Note: For any geometry problem, the first page of the solution must be a large, in-scale, clearly labeled diagram made with drawing instruments (ruler, compass, protractor, graph paper). Failure to meet any of these requirements will result in a 1-point automatic deduction.

JMO 1. Are there integers a and b such that a^5b+3 and ab^5+3 are both perfect cubes of integers?

JMO 2. Each cell of an $m \times n$ board is filled with some nonnegative integer. Two numbers in the filling are said to be *adjacent* if their cells share a common side. (Note that two numbers in cells that share only a corner are not adjacent.) The filling is called a *garden* if it satisfies the following two conditions:

(i) The difference between any two adjacent numbers is either 0 or 1.

(ii) If a number is less than or equal to all of its adjacent numbers, then it is equal to 0.

Determine the number of distinct gardens in terms of m and n.

JMO 3. In triangle ABC, points P, Q, R lie on sides BC, CA, AB, respectively. Let $\omega_A, \omega_B, \omega_C$ denote the circumcircles of triangles AQR, BRP, CPQ, respectively. Given the fact that segment AP intersects $\omega_A, \omega_B, \omega_C$ again at X, Y, Z respectively, prove that $YX/XZ = BP/PC$.