8^{th} United States of America Junior Mathematical Olympiad
Day 1. 12:30 PM – 5:00 PM EDT
April 19, 2017

Note: For any geometry problem whose statement begins with an asterisk ($*$), the first page of the solution must be a large, in-scale, clearly labeled diagram. Failure to meet this requirement will result in an automatic 1-point deduction.

USAJMO 1. Prove that there are infinitely many distinct pairs (a, b) of relatively prime integers $a > 1$ and $b > 1$ such that $a^b + b^a$ is divisible by $a + b$.

USAJMO 2. Consider the equation

$$(3x^3 + xy^2)(x^2y + 3y^3) = (x - y)^7.$$

(a) Prove that there are infinitely many pairs (x, y) of positive integers satisfying the equation.

(b) Describe all pairs (x, y) of positive integers satisfying the equation.

USAJMO 3. ($*$) Let ABC be an equilateral triangle and let P be a point on its circumcircle. Let lines PA and BC intersect at D; let lines PB and CA intersect at E; and let lines PC and AB intersect at F. Prove that the area of triangle DEF is twice the area of triangle ABC.

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8^{th} United States of America Junior Mathematical Olympiad
Day 2. 12:30 PM – 5:00 PM EDT
April 20, 2017

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USAJMO 4. Are there any triples (a, b, c) of positive integers such that $(a-2)(b-2)(c-2)+12$ is a prime that properly divides the positive number $a^2+b^2+c^2+abc-2017$?

USAJMO 5. ($*$) Let O and H be the circumcenter and the orthocenter of an acute triangle ABC. Points M and D lie on side BC such that $BM = CM$ and $\angle BAD = \angle CAD$. Ray MO intersects the circumcircle of triangle BHC in point N. Prove that $\angle ADO = \angle HAN$.

USAJMO 6. Let $P_1, \ldots, P_{2n}$ be $2n$ distinct points on the unit circle $x^2+y^2=1$ other than $(1,0)$. Each point is colored either red or blue, with exactly n of them red and n of them blue. Let $R_1, \ldots, R_n$ be any ordering of the red points. Let B_1 be the nearest blue point to R_1 traveling counterclockwise around the circle starting from R_1. Then let B_2 be the nearest of the remaining blue points to R_2 traveling counterclockwise around the circle from R_2, and so on, until we have labeled all of the blue points $B_1, \ldots, B_n$. Show that the number of counterclockwise arcs of the form $R_i \to B_i$ that contain the point $(1,0)$ is independent of the way we chose the ordering $R_1, \ldots, R_n$ of the red points.

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