

4th United States of America Junior Mathematical Olympiad

Day II 12:30 PM – 5 PM EDT

May 1, 2013

Note: For any geometry problem, the first page of the solution must be a large, in-scale, clearly labeled diagram made with drawing instruments (ruler, compass, protractor, graph paper). Failure to meet any of these requirements will result in a 1-point automatic deduction.

JMO 4. Let $f(n)$ be the number of ways to write n as a sum of powers of 2, where we keep track of the order of the summation. For example, $f(4) = 6$ because 4 can be written as 4, $2 + 2$, $2 + 1 + 1$, $1 + 2 + 1$, $1 + 1 + 2$, and $1 + 1 + 1 + 1$. Find the smallest n greater than 2013 for which $f(n)$ is odd.

JMO 5. Quadrilateral $XABY$ is inscribed in the semicircle ω with diameter XY . Segments AY and BX meet at P . Point Z is the foot of the perpendicular from P to line XY . Point C lies on ω such that line XC is perpendicular to line AZ . Let Q be the intersection of segments AY and XC . Prove that

$$\frac{BY}{XP} + \frac{CY}{XQ} = \frac{AY}{AX}.$$

JMO 6. Find all real numbers $x, y, z \geq 1$ satisfying

$$\min(\sqrt{x + xyz}, \sqrt{y + xyz}, \sqrt{z + xyz}) = \sqrt{x - 1} + \sqrt{y - 1} + \sqrt{z - 1}.$$