ADVANCED MATHEMATICAL ANALYSIS LEVEL 1 - IB MATH ANALYSIS AND APPROACHES HIGHER LEVEL YEAR 1 FRAMEWORK

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INTRODUCTION

Mathematics: analysis and approaches is for students who enjoy developing their mathematics to become fluent in the construction of mathematical arguments and develop strong skills in mathematical thinking. They will also be fascinated by exploring real and abstract applications of these ideas, with and without technology. Students who take Mathematics: analysis and approaches will be those who enjoy the thrill of mathematical problem solving and generalization and for students interested in mathematics, engineering, physical sciences, and some economics.

Students who choose Mathematics: analysis and approaches should be comfortable in the manipulation of algebraic expressions and enjoy the recognition of patterns and understand the mathematical generalization of these patterns. Students who wish to take Mathematics: analysis and approaches will have strong algebraic skills and the ability to understand simple proof. They will be students who enjoy spending time with problems and get pleasure and satisfaction from solving challenging problems.

PRIOR LEARNING TOPICS

It is expected that all students have extensive previous mathematical experiences, but these will vary.
In order to enroll in year one, CAISL's students are expected to have successfully completed Geometry and Algebra II. If a student is coming from another school, we expect equivalent courses.

EXPECTATIONS

INFORMATION TECHNOLOGY EXPECTATIONS

Graphic Display Calculator and Computer

Students are expected to use a graphic display calculator both in class and during assessments. The math department recommends the use of the TI – 84 plus model.

Students are also expected to use Geogebra or Excel to produce graphical representations or table of values. (Geogebra: https://www.geogebra.org/?lang=pt-PT)

PERFORMANCE INDICATORS

MATH PRACTICES

Explanations of Math Practices: By the end of the year students will be expected to problem solve, reason mathematically, and communicate efficiently according to grade level expectations. See link below:
https://www.caislisbon.org/uploaded/Curriculum_links/Math/Math_Practice_Progressions_5-12.pdf

PROBLEM SOLVING

Make sense of problems and persevere in solving them
Look for and make use of structure (Deductive Reasoning)
Look for and express regularity in repeated reasoning (Inductive Reasoning)
Reason abstractly and quantitatively
Construct viable arguments and critique the reasoning of others
Model with mathematics
Use appropriate tools strategically
Attend to precision

MATH CONCEPTS

Regarding the Analysis and Approaches course, students will demonstrate an understanding of and solve a variety of problems with the topics below:

Topic 1 – Number and Algebra

Operations with numbers in scientific notation.

Arithmetic sequences and series.

Use of the formulae for the nth term and the sum of the first n terms of the sequence.

Use of sigma notation for sums of arithmetic sequences.

Applications.

Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.

Geometric sequences and series.

Use of the formulae for the nth term and the sum of the first n terms of the sequence.

Use of sigma notation for the sums of geometric sequences.

Applications.

Financial applications of geometric sequences and series: compound interest;

annual depreciation.

Laws of exponents with integer exponents.

Introduction to logarithms with base 10 and e.

Numerical evaluation of logarithms using technology.

Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof.

The symbols and notation for equality and identity.

Laws of exponents with rational exponents.

Laws of logarithms.

Change of base of a logarithm.

Solving exponential equations, including using logarithms.

Sum of infinite convergent geometric sequences.

The binomial theorem. Expansion of the binomial expansion.

Use of Pascal's triangle and nCr.

Counting principles, including permutations and combinations.
Extension of the binomial theorem to fractional and negative indices.
Partial fractions.
Complex numbers: the number i. Cartesian form z=a+bi; the terms real part, imaginary part, conjugate, modulus and argument.
The complex plane.
Modulus–argument (polar) form.
Euler's form.
Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation.

Complex conjugate roots of quadratic and polynomial equations with real coefficients.
De Moivre’s theorem and its extension to rational exponents.
Powers and roots of complex numbers.
Proof by mathematical induction.
Proof by contradiction.
Use of a counterexample to show that a statement is not always true.
Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.

Topic 2 – Functions

Different forms of the equation of a straight line.

Gradient; intercepts.

Lines with gradients.

Parallel lines.

Perpendicular lines.

Concept of a function, domain, range and graph.

Function notation, for example f(x), v(t), C(n).

The concept of a function as a mathematical model.

Informal concept that an inverse function reverses or undoes the effect of a function.

Inverse function as a reflection in the line y=x, and the notation f−1(x).

The graph of a function; its equation y=f(x).

Creating a sketch from information given or a context, including transferring a graph from screen to paper.

Using technology to graph functions including their sums and differences.

Determine key features of graphs.

Finding the point of intersection of two curves or lines using technology.

Composite functions.

Identity function. Finding the inverse function.

The standard form of a quadratic function; its graph, y -intercept (0,c). Axis of symmetry.

The x- intercept form with x - intercepts (p,0) and (q,0).

The vertex form and vertex (h,k).

Solution of quadratic equations and inequalities.

The quadratic formula.

The discriminant and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.

The reciprocal function: its graph and self-inverse nature.

Rational functions and their graphs.

Equations of vertical and horizontal asymptotes.

Exponential functions and their graphs.

Logarithmic functions and their graphs.

Solving equations, both graphically and analytically.

Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.

Applications of graphing skills and solving equations that relate to real-life situations.

Transformations of graphs.

Translations.

Reflections (in both axes).

Vertical stretch with scale factor p.

Horizontal stretch with scale factor $\frac{1}{q}$.

Composite transformations.

Polynomial functions, their graphs and equations; zeros, roots and factors.

The factor and remainder theorems.

Sum and product of the roots of polynomial equations.

Rational functions of the form f(x)=ax+bcx^2+dx+e, and f(x)=ax^2+bx+cx+d

Odd and even functions.

Finding the inverse function, including domain restriction.

Self-inverse functions.

Solutions of g(x)≥f(x), both graphically and analytically.

The graphs of the functions, y=|f(x)|

and y=f(|x|), y=1f(x), y=f(ax+b), y=[f(x)]^2.

Solution of modulus equations and inequalities.

Topic 3 – Geometry and Trigonometry

The distance between two points in three-dimensional space, and their midpoint.

Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combir

The size of an angle between two intersecting lines or between a line and a plane.

Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.

The sine rule.

The cosine rule.

Area of a triangle.

Applications of right and non-right angled trigonometry, including Pythagoras's theorem.

Angles of elevation and depression.

Construction of labelled diagrams from written statements.

The circle: radian measure of angles; length of an arc; area of a sector.

Definition of cosθ, sinθ in terms of the unit circle.

Definition of tanθ as sinθcosθ.

Exact values of trigonometric ratios of 0, π/6, π/4, π/3, π/2 and their multiples.

Extension of the sine rule to the ambiguous case.

The Pythagorean identity cos2θ+sin2θ=1.

Double angle identities for sine and cosine.

The relationship between trigonometric ratios.

The circular functions sinx, cosx, and tanx; amplitude, their periodic nature, and their graphs.

Composite functions of the form f(x)=asin(b(x+c))+d.

Transformations.

Real-life contexts.

Solving trigonometric equations in a finite interval, both graphically and analytically.

Equations leading to quadratic equations in sinx, cosx or tanx.

Definition of the reciprocal trigonometric ratios secθ, cosecθ and cotθ.

Pythagorean identities: 1+tan2θ=sec2θ1+cot2θ=cosec2θ

The inverse functions f(x)=arcsinx,f(x)=arccosx,f(x)=arctanx; their domains and ranges; their graphs.

Compound angle identities.

Double angle identity for tan.

Relationships between trigonometric functions and the symmetry properties of their graphs.

Concept of a vector; position vectors; displacement vectors.

Representation of vectors using directed line segments.

Base vectors i, j, k.

Components of a vector.

Algebraic and geometric approaches to the following: the sum and difference of two vectors, the zero vector 0, the vecto

vector.

Proofs of geometrical properties using vectors.

The definition of the scalar product of two vectors.

The angle between two vectors.

Perpendicular vectors; parallel vectors.

Vector equation of a line in two and three dimensions: $r=a+\lambda b$.

The angle between two lines.

Simple applications to kinematics.

Coincident, parallel, intersecting and skew lines, distinguishing between these cases.

Points of intersection.

The definition of the vector product of two vectors.

Properties of the vector product.

Geometric interpretation of $|\mathbf{v}\times\mathbf{w}|$

Vector equations of a plane:

$r=a+\lambda b+\mu c$, where b and c are non-parallel vectors within the plane.

$r\cdot n=a\cdot n$, where n is a normal to the plane and a is the position vector of a point on the plane.

Cartesian equation of a plane $ax+by+cz=d$.

Intersections of: a line with a plane; two planes; three planes.

Angle between: a line and a plane; two planes.

Topic 5 – Calculus

Introduction to the concept of a limit.

Derivative interpreted as gradient function and as rate of change.

Increasing and decreasing functions.

Graphical interpretation of $f'(x)>0$, $f'(x)=0$, $f'(x)<0$.

Derivative of f(x)=ax^n is f'(x)=anx^(n-1), $n\in\mathbb{Z}$

The derivative of functions of the form f(x)=ax^n +bx^(n-1),.... where all exponents are integers.

Tangents and normals at a given point, and their equations.

Introduction to integration as anti-differentiation of functions of the form f(x)=axn+bxn−1+...., where $n\in\mathbb{Z}$, $n\neq-1$.

Anti-differentiation with a boundary condition to determine the constant term.

Definite integrals using technology.

Area of a region enclosed by a curve y=f(x) and the x -axis, where f(x)>0.

Derivative of xn($n\in\mathbb{Q}$), sinx, cosx, ex and lnx.

Differentiation of a sum and a multiple of these functions.

The chain rule for composite functions.

The product and quotient rules.

The second derivative.

Graphical behavior of functions, including the relationship between the graphs of f, f' and f".

Local maximum and minimum points.

Testing for maximum and minimum.

Optimization.

Points of inflexion with zero and non-zero gradients.

Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.

Indefinite integral of $x^n (n \in \mathbb{Q})$, sinx, cosx, $1/(x)$ and e^x.

The composites of any of these with the linear function ax+b.

Integration by inspection (reverse chain rule) or by substitution for expressions of the form:

$\int k g'(x) f(g(x)) dx$.

Definite integrals, including analytical approach.

Areas of a region enclosed by a curve y=f(x) and the x-axis, where f(x) can be positive or negative, without the use of technology.

Areas between curves.

Informal understanding of continuity and differentiability of a function at a point.

Understanding of limits (convergence and divergence).

Definition of derivative from first principles.

Higher derivatives.

The evaluation of limits of the form limx→af(x)g(x) and limx→∞f(x)g(x) using l'Hôpital's rule or the Maclaurin series.

Repeated use of l'Hôpital's rule.

Implicit differentiation.

Related rates of change.

Optimization problems.

Derivatives of tanx, secx, cosecx, cotx, ax, logax, arcsinx, arccosx, arctanx.

Indefinite integrals of the derivatives of any of the above functions.

The composites of any of these with a linear function.

Use of partial fractions to rearrange the integrand.

Integration by substitution.

Integration by parts.

Repeated integration by parts.

Area of the region enclosed by a curve and the y-axis in a given interval.

Volumes of revolution about the x-axis or y-axis.

First order differential equations.

Numerical solution of dydx=f(x,y) using Euler's method.

Variables separable.

Homogeneous differential equation dy dx=f(yx) using the substitution y=vx.

Solution of y'+P(x)y=Q(x), using the integrating factor.

Maclaurin series to obtain expansions for ex, sinx, cosx, ln(1+x), (1+x) p, p∈ℚ.

Use of simple substitution, products, integration and differentiation to obtain other series.

Maclaurin series developed from differential equations.

ASSESSMENT

For students to receive a credit towards their High School Diploma and/or IB Diploma, they must demonstrate proficiency on:

Summative assessments set by the class teacher which may take the form of

- in-class or out-of-class projects
- tests and quizzes which assess both knowledge and skill acquisition
- A final exam at the end of the year which covers material from the syllabus.

FURTHER CURRICULAR EXPECTATIONS

Notebook

- Math notebooks are an independent responsibility of the student.
- Students are expected to keep an organized notebook with notes from class, work done at home.

Scientific Writing

Students are expected to use the equation tool from word office to write all mathematical notation.